VOL. 17, NO. 3, FEBRUARY 2022 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

STABILITY ANALYSIS ON THE MODEL OF RICE BLAST DISEASE UNDER CHANGING OF RICE CULTIVAR, VIRULENCE OF FUNGUS AND GROWTH STAGE OF RICE

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ABSTRACT

The global asymptotic stability is considered in the model of rice blast disease under the difference of rice cultivar susceptibility, virulence of fungus and growth stage of rice. A new global as-ymptotic stability criterion of equilibrium point for susceptible-exposed-infections-recovered (SEIR) epidemic model is derived by constructing a suitable function. Then, this research shows the points of free-disease equilibrium and endemic equilibrium to be able to apply for study on the preparation to prevent the rice blast disease or plan rice planting.

Keywords: endemic equilibrium, equilibrium analysis, free-disease equilibrium, model of rice blast disease, stability analysis.

INTRODUCTION

Mathematical modeling is considered an effective tool for describing the dynamical behavior of infections [1-5]. Many researchers have formulated models for realizing and controlling the outbreak of transmissible diseases. They applied mathematical modeling for the study of transmissible infectious diseases. Several researchers have developed dif-ferent mathematical models depend on their study for these epidemic diseases and then studied the stability analysis and optimal control of these epidemic models [6-12]. Then, mathematical modeling is one of the powerful tools for describing the dynamic behavior of many diseases.

This research aims to study the stability analysis, where the stability of the state tra-jectory or equilibrium state is examined. It will formulate the fundamental matrix, the so-lution of systems of differential equations and the computation of the eigenvalues from the model of rice blast disease under the changing of rice cultivar susceptibility, virulence of fungus and growth stage of rice. This research will be considered the disease free-equilibrium, endemic equilibrium and basic reproduction number by analyzing the global stability.

MODEL FORMULATION

Kirtphaiboon *et al.* [13] developed the model of rice blast disease on the mathematic model by considering the change of susceptible (S), exposed (E), infectious (I), and removal (R) site as known as SEIR. The model is considered the pathogen life cycle together with the disease development that affects the weather parameters change. According to three optimal factors for occurring the rice blast disease, consists of host, pathogen and environment. This research analyzed the stability of the extended model of rice blast disease by considering the difference of rice cultivar susceptibility, virulence of fungus and rice growth stage in term of infection efficiency (I_E). The canopy growth is considered from $f(t) = R_{RG}A\left(1-\frac{A}{M}\right)$, where A is the total sites (A = H+L+I+R), R_{RG} is the maximum growth rate and M is the maximum canopy size. The senescence is calculated from $\sigma(t) = \sigma_0 e^{r_s(t-t_{\text{sen}})}$ where r_s is the senescence increase rate and t_{sen} is the date of senescence. Then, the governing equations of the extended model of rice blast disease are:

$$\frac{dH}{dt} = \sigma - \sigma H - \alpha I_E SIH \tag{1}$$

$$\frac{dL}{dt} = \alpha I_E SIH - \frac{L}{p} - \sigma L \tag{2}$$

$$\frac{dI}{dt} = \frac{L}{p} - \frac{I}{i} - \sigma I \tag{3}$$

$$\frac{dR}{dt} = \frac{I}{i} - \sigma R \tag{4}$$

To construct the new system, is dimensionless by letting, $\overline{H} = \frac{H}{N}$, $\overline{L} = \frac{L}{N}$, $\overline{I} = \frac{I}{N}$, $\overline{R} = \frac{R}{N}$.

The simplified model becomes:

$$\frac{d\overline{H}}{dt} = \sigma - \left(\sigma - \alpha I_E S\overline{I}\,\overline{H}\right)N \tag{5}$$

$$\frac{d\overline{L}}{dt} = \alpha I_E S\overline{I}\,\overline{H}N - \left(\frac{1}{p} - \sigma\right)\overline{L} \tag{6}$$

$$\frac{d\overline{I}}{dt} = \frac{\overline{L}}{p} - \left(\frac{1}{i} - \sigma\right)\overline{I} \tag{7}$$

$$\frac{d\bar{R}}{dt} = \frac{\bar{I}}{i} - \sigma\bar{R} \tag{8}$$

STABILITY ANALYSIS

Equilibrium Analysis

Based on Equation (5)-(8), stability analysis is carried out to determine the disease-free equilibrium point and endemic equilibrium point.

To determine the two equilibrium points, each equation in Equation (5)-(8), must be equal to zero, or $\frac{d\bar{H}}{dt} = 0$, $\frac{d\bar{L}}{dt} = 0$, $\frac{d\bar{I}}{dt} = 0$ and $\frac{d\bar{R}}{dt} = 0$.

Then obtained:

$$\sigma - \left(\sigma - \alpha I_E S\bar{I}\bar{H}\right)N = 0 \tag{9}$$

$$\alpha I_E S\bar{I}\bar{H}N - \left(\frac{1}{p} - \sigma\right)\bar{L} = 0 \tag{10}$$

$$\frac{\bar{L}}{p} - \left(\frac{1}{i} - \sigma\right)\bar{I} = 0 \tag{11}$$

$$\frac{\bar{I}}{i} - \sigma\bar{R} = 0 \tag{12}$$

Then, we found the equilibrium point of $\bar{H}, \bar{L}, \bar{I}$ and $\bar{R}$.

Free-Disease Equilibrium

Equilibrium points are conditions where there is no spread of disease. Then, $L = I = 0$.

From Equation (9); $\bar{H} = \frac{\sigma}{\left(\sigma - \alpha I_E S\bar{I}N\right)}$

Then obtained, $\bar{H} = 1$

From Equation (10); $\bar{L} = \frac{\alpha I_E S\bar{I}\bar{H}N}{\left(\frac{1}{p} - \sigma\right)}$

Then obtained, $\bar{L} = 0$

From Equation (11); $\bar{I} = \frac{\bar{L}}{p\left(\frac{1}{i} - \sigma\right)}$

Then obtained, $\bar{I} = 0$

From Equation (12); $\bar{R} = \frac{\bar{I}}{i\sigma}$

Then obtained, $\bar{R} = 0$

Thus, the equilibrium points of disease-free are

$$E_f = (H, L, I, R) = (1, 0, 0, 0) \tag{13}$$

Endemic Equilibrium

Endemic equilibrium points are used to indicate the possibility of disease spread. Because in the endemic conditions and disease spread, the populations $S \neq 0, L \neq 0, I \neq 0$ and $R \neq 0$.

Equation (5)-(8) obtained the endemic equilibrium points are:

$$\bar{H} = \frac{(1-\sigma p)\left(\frac{1}{i} - \sigma\right)}{\alpha I_E SN} \tag{14}$$

$$\bar{L} = \left(\frac{\sigma p\left(\frac{1}{i} - \sigma\right)}{\alpha I_E SN}\right) - \left(\frac{\sigma p}{(1-\sigma p)}\right) \tag{15}$$

$$\bar{I} = \left(\frac{\sigma}{\alpha I_E SN}\right) - \left[\frac{\sigma}{(1-\sigma p)\left(\frac{1}{i} - \sigma\right)}\right] \tag{16}$$

$$\bar{R} = \left[\left(\frac{1}{\alpha I_E SN}\right) - \left(\frac{1}{(1-\sigma p)\left(\frac{1}{i} - \sigma\right)}\right)\right]\left[\frac{1}{i}\right] \tag{17}$$

Thus, the equilibrium points of endemic are

$$E_e = (H, L, I, R)$$

$$E_e = \begin{pmatrix} \frac{(1-\sigma p)\left(\frac{1}{i} - \sigma\right)}{\alpha I_E SN}, \\ \left(\frac{\sigma p\left(\frac{1}{i} - \sigma\right)}{\alpha I_E SN}\right) - \left(\frac{\sigma p}{(1-\sigma p)}\right), \\ \left(\frac{\sigma}{\alpha I_E SN}\right) - \left[\frac{\sigma}{(1-\sigma p)\left(\frac{1}{i} - \sigma\right)}\right], \\ \left[\left(\frac{1}{\alpha I_E SN}\right) - \left(\frac{1}{(1-\sigma p)\left(\frac{1}{i} - \sigma\right)}\right)\right]\left[\frac{1}{i}\right] \end{pmatrix} \tag{18}$$

Basic Reproduction Number

The basic reproduction number (R_0) is determined using the matrices generation method, Based on Equation (5) to (8), to determine (R_0):

Let $F = \begin{bmatrix} 0 & \alpha I_E SN\bar{H} \\ \frac{1}{p} & 0 \end{bmatrix}$ and $V = \begin{bmatrix} -\left(\frac{1}{p}-\sigma\right) & 0 \\ 0 & -\left(\frac{1}{i}-\sigma\right) \end{bmatrix}$

Then we found,

$$V^{-1} = \begin{bmatrix} -\frac{1}{\left(\frac{1}{p}-\sigma\right)} & 0 \\ 0 & -\frac{1}{\left(\frac{1}{i}-\sigma\right)} \end{bmatrix} \quad (19)$$

The generation matrix is :

$$\rho = \begin{bmatrix} 0 & -\frac{\alpha I_E SN\bar{H}}{\left(\frac{1}{i}-\sigma\right)} \\ -\frac{1}{p\left(\frac{1}{p}-\sigma\right)} & 0 \end{bmatrix} \quad (20)$$

To consider the characteristic equation, $\det\left(\lambda I - FV^{-1}\right) = 0$

Thus, the basic reproduction number is given the spectral radius of matrix FV^{-1} is

$$R_0 = \rho \sqrt{\frac{\alpha I_E SN\bar{H}}{p\left(\frac{1}{p}-\sigma\right)\left(\frac{1}{i}-\sigma\right)}} \quad (21)$$

Stability Analysis

Theorem 1.

1. If $R_0 \le 1$, then Equation (5)-(8) of the model is global asymptotic stable.
2. If $R_0 > 1$, then Equation (5)-(8) is unstable.

Proof of Theorem 1.

Base on the Equation (5)-(8), found the Jacobian matrices

$$J = \begin{bmatrix} \alpha I_E S\bar{I}N & 0 & \alpha I_E S\bar{H}N & 0 \\ \alpha I_E S\bar{I}N & -\left(\frac{1}{p}-\sigma\right) & \alpha I_E S\bar{H}N & 0 \\ 0 & \frac{1}{p} & -\left(\frac{1}{i}-\sigma\right) & 0 \\ 0 & 0 & \frac{1}{i} & -\sigma \end{bmatrix} \quad (22)$$

Then, to find the eigenvalue of the Jacobian matrix in Equation (22):

$$\lambda I - J = \begin{bmatrix} \lambda - \alpha I_E S\bar{I}N & 0 & -\alpha I_E S\bar{H}N & 0 \\ -\alpha I_E S\bar{I}N & \lambda + \left(\frac{1}{p}-\sigma\right) & -\alpha I_E S\bar{H}N & 0 \\ 0 & -\frac{1}{p} & \lambda + \left(\frac{1}{i}-\sigma\right) & 0 \\ 0 & 0 & -\frac{1}{i} & \lambda+\sigma \end{bmatrix}$$

To consider the characteristic equation: $\det(\lambda I - J) = 0$, then

$$(\lambda+\sigma)\begin{bmatrix} \lambda^2 \\ +\left\{\begin{matrix} \left(\frac{1}{p}-\sigma\right)-\left(\alpha I_E SN\bar{I}\right) \\ -\frac{1}{p}\left(\alpha I_E SN\bar{I}\right) \\ +\frac{1}{p}\left(\alpha I_E SN\bar{H}\right) \end{matrix}\right\}\lambda \\ +\sigma\left(\alpha I_E SN\bar{I}\right)-\frac{1}{p}\left(\alpha I_E SN\bar{I}\right) \\ -\frac{1}{p}\left(\alpha I_E SN\bar{H}\right)\left(\alpha I_E SN\bar{I}\right) \end{bmatrix} = 0$$

$$(\lambda+\sigma)\begin{bmatrix} \lambda^2 + \left(C_1 - C_3 - C_2C_3 + C_2C_4\right)\lambda \\ +\sigma C_3 - C_2C_3 - C_2C_3C_4 \end{bmatrix} = 0 \quad (23)$$

where $C_1 = \left(\frac{1}{p}-\sigma\right)$, $C_2 = \frac{1}{p}$, $C_3 = \alpha I_E SN\bar{I}$ and $C_4 = \alpha I_E SN\bar{H}$.

So, $\lambda = -\sigma$ and $\lambda^2 + D_1\lambda + D_2 = 0$.

Due to the characteristic values of equation system are negative, the equilibrium point is stable global asymptotic.

To consider $\lambda^2 + \left(C_1 - C_3 - C_2C_3 + C_2C_4\right)\lambda + \sigma C_3 - C_2C_3 - C_2C_3C_4 = 0$,

From the Routh-Hurwitz criterion, the equilibrium point is stable when it satisfied the following conditions:

1. $\left(C_1 + C_2C_4\right) > (C_3 + C_2C_3)$
2. $\sigma C_3 > (C_2C_3 + C_2C_3C_4)$.

This concludes that if $\left(C_1 + C_2C_4\right) > (C_3 + C_2C_3)$ and $\sigma C_3 > (C_2C_3 + C_2C_3C_4)$. Then $\lambda < 0$.

Due to the characteristic values of the equation system in the model are negative, Then, the equilibrium point is stable global asymptotic.

Thus, the disease equilibrium point is global asymptotically stable when $R_0 < 1$.

CONCLUSION AND DISCUSSIONS

This research presented the stability analysis of the model of rice blast disease that is considered under the changing of rice cultivar susceptibility, virulence of fungus and growth stage of rice. It is found that the disease free-equilibrium are $E_f = (H, L, I, R) = (1, 0, 0, 0)$ and found the equilibrium points of endemic. The analysis of the model uses the generation matrix method to obtain the basic reproduction number and the global stability.

Kermack and Mckendrick [14] initiated a fundamental mathematical model by proposing the SIR model as a tool for epidemic analysis and control of communicable diseases. The basic formulation are

$$\frac{dS}{dt} = -kSI$$
$$\frac{dI}{dt} = kSI - \gamma I$$
$$\frac{dR}{dt} = \gamma I$$

where S is the susceptible population, I is the infected population and R is the removed population from disease or die, k is rate of infection and γ is the rate of recovery of the infected population.

Capasso and Serio [15] developed the general formulation of the SIR model as follows:

$$\frac{dS}{dt} = -g(I)S$$
$$\frac{dI}{dt} = g(I)S - \gamma I$$
$$\frac{dR}{dt} = \gamma I$$

They considered that $g : R^+ \to R^+$ is the continuous and bounded function corresponding with the following conditions:

i) $\forall x \in R^+;\ g(x) \geq 0$

ii) $g(0) = 0$

iii) $\exists c \in R^+ - \{0\};\ \forall x \in R^+; g(x) \leq c$

iv) $g' : R^+ \to R^+$ is exist and has close bounded on R^+ where $g'(0) > 0$

v) $\forall x \in R^+;\ g(x) \leq g'(0)x$ where $R^+ = [0, +\infty)$

Wanblao, *et al*. [16] studied the asymptomatic linear stability with the damper of the SIR model as follows:

$$\frac{dS}{dt} = -\beta S(t)I(t-h) - \mu_1 S(t) + b$$
$$\frac{dI}{dt} = \beta S(t)I(t-h) - \mu_2 I(t) - \lambda I(t)$$
$$\frac{dR}{dt} = \lambda I(t) - \mu_3 R(t)$$

They classified the newly born people as those at risk of infection where $\mu_1 > 0$, $\mu_2 > 0$ and $\mu_3 > 0$ is senescence rate of susceptible, infectious and removed population, respectively. In biology, they assumed $\mu_1 \leq \min\{\mu_2, \mu_3\}$ and $h \geq 0$ is the damper. The SIR model corresponded to the initial conditions as $S(\theta) = \varphi_1(\theta),\ I(\theta) = \varphi_2(\theta),\ R(\theta) = \varphi_1(\theta),\ (-h \leq \theta \leq 0)$ where $\varphi = (\varphi_1, \varphi_2, \varphi_3)^T \in C$ is the continuous function and $\varphi_i(\theta) \geq 0$ where $-h \geq \theta \leq 0,\ i = 1, 2, 3$, C is Banach spaces $C([-h, 0], R^3)$ by assuming $\varphi_i(0) > 0$ for $i = 1, 2, 3$.

They found that the asymptomatic stability system showed that the endemic equilibrium point was stable when $S_0 > S^*$ using the Lipunov function as a stability analysis tool.

ACKNOWLEDGEMENTS

This work was supported by the Post-Doctoral Fellowship of King Mongkut's University of Technology Thonburi and National Higher Education Science Research and Innovation Policy Council [number B16F630087].

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