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ROBUSTNESS OF THE AIRCRAFT LANDING CONTROL YSTEM BUILT IN THE CLASS OF SINGLE PARAMETER STRUCTURALLY STABLE MAPS

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ABSTRACT

The article describes a method for investigating the robustness of an aircraft landing control system built in the form of a MIMO system.As a research apparatus, the gradient-velocity method of the Lyapunov function is used. The study assumes that wind gusts are equal to zero, the regulator is selected in the form of single-parameter structurally stable maps ("fold" catastrophe).The constructed function is investigated for stability in three stationary states. The results of numerical experiments confirm the existence of asymptotic stability of the system.

Keywords: control systems, robust stability, one-parameter structural lystablemapping, lyapunov function, stationary states, the aircraft landing process, fold catastrophe.

INTRODUCTION

It is known that the key criterion for the appearance of chaos can be the stability of formations arising in the system with respect to small disturbances. If there is no such stability, the deterministic description loses its meaning, and statistical methods must be used.

As numerous studies have shown, statistical laws, as well as statistical description, apply not only to significantly complex systems with a large number of degrees of freedom [1]. The problem lies not in the complexity of the system under study and not in external noise, but in the appearance of exponential instability of motion at certain values of the parameters.

Nowadays, methods for classifying various types of chaos have been developed, patterns of its development have been found, techniques have been created to distinguish chaos from white noise, etc. [1, 2-7]. Moreover, it was discovered and rigorously substantiated that the complex space-time behavior of distributed media with a huge number of degrees of freedom can be adequately described by nonlinear systems of small dimension [8, 9-11].

As is known, the mathematical image of steady periodic oscillations is the limit cycle. [12-13, 14-15]. Stable cycles are examples of attractors, since they "attract" all close trajectories. Physically, this means that when deviating from such fluctuations, the system returns to them after a while. If the system exhibits chaotic properties, this corresponds to the presence in its phase space of a more complex formation than the cycle: a strange (chaotic) attractor. In addition, the movement in the parameter space along the corresponding direction makes it possible to determine the sequence of bifurcations, as a result of which a chaotic attractor is formed [2-7].

It should be noted that the theory of catastrophes defines the area of existence of various structures, the boundaries of their stability. To study the dynamics of systems, it is necessary to know exactly how new solutions of equations "branch off" from the known solution. The answer to such questions is given by the theory of bifurcations, that is, the emergence of a new solution at a critical value of the parameter. The moment of transition (catastrophic jump) depends on the properties of the system and the level of fluctuations [12-13].

Studies of nonlinear dynamic processes in mathematics and physics have shown that chaotic behavior in systems with a small number of degrees of freedom is quite typical [14, 15]. Thus, the problem of predictability has become common to many areas of modern science. In this regard, recently it has become urgent to study the problems of predictability of the behavior of chaotic systems, control of their dynamics and the possibility of suppressing chaos.

Chaotic dynamical systems are absolutely malleable and extremely sensitive to external influences [1]. Moreover, the dynamics of chaotic systems can be controlled, that is, by means of weak influences, such systems can be transferred from the mode of chaotic oscillations to the required dynamic mode (thereby, stabilizing their behavior). Consequently, such systems can be classified as self-organizing. One of the approaches is based on the fact that chaotic attractors contain, as a rule, an infinite number of unstable cycles. For a number of systems, methods have been developed that make it possible to either stabilize these cycles or create new ones. This is the key to solving the problem based on the use of systems with suppressed chaos [2-7, 8-13].

The phenomenon of self-organization is always associated with the loss of stability of a less organized or disordered state of motion and is always nonlinear in nature [12, 13]. To ensure the safe functioning of real technical systems, it is necessary that the mathematical models describing the dynamics of these complex systems possess stability. Frequently, it is impracticable to achieve the desired result by classical methods, since they are based on the assumption that the mathematical model of an object absolutely accurately describes its behavior and the model is known in advance. In addition, circumstances such as the presence of parametric uncertainty and external uncontrolled disturbances are frequently ignored [1, 14, 15]. The problem of finding an appropriate algorithm which would solve the problem posed to the specialist appears. An adequate solution is the construction of a robust controller capable of ensuring the stability and acceptable quality of the control system with small deviations of the plant parameters and external disturbances from the nominal models.

The research is devoted to the construction of a control system with an increased potential for robust stability by a dynamic object with uncertain parameters, with an approach to the construction of control systems in the class of fold catastrophe [12-13, 16-19].

To study the robustness of automatic control systems, the methods of Lyapunov functions [20-21, 22-25] are used, which is based on the geometric interpretation of the theorem on asymptotic stability in the state space. The Lyapunov function is synthesized in the form of a vector function, the antigradient, which is given by the components of the velocity vector of the system in the form of a tensor.

RESEARCH METHOD

Consider the problem of investigating stability of control systems with increased potential of robust stability constructed in the class of single-parameter structurally stable maps for objects with m inputs and n outputs.
The control system defined by the equation [16-19].

$$\dot{x} = Ax + Bu \\ y = Cx \tag{1}$$

here $x(t) \in R^n$ - state vector of control object, $A \in R^{n \times n}$, $C \in R^{l \times n}$ and $B \in R^{n \times n}$ - matrix of the control object. Suppose that all components of the state vector are measurable, and for simplicity, the matrix of the system can be represented in the following form

$$A = \begin{Vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{Vmatrix}, B = \begin{Vmatrix} b_{11} & b_{12} & \dots & b_{1m} \\ b_{21} & b_{22} & \dots & b_{2m} \\ \dots & \dots & \dots & \dots \\ b_{n1} & b_{n2} & \dots & b_{nm} \end{Vmatrix}$$

An Algorithm for Studying the Robust Stability of a MIMO System Using the Lyapunov Function

Choosing Control Law

The control law is described by a vector-function in the form of single-parameter structurally stable maps [16-19]

$$u_i = -x_i^3 + k_i x_i, i = 1, \dots, n \tag{2}$$

Stationary States of the System

The system (1) can be written in expanded form

$$\begin{cases} \dot{x}_1 = -b_{11}x_1^3 + (a_{11} + b_{11}k_1)x_1 + a_{12}x_2 + \dots + a_{1n}x_n \\ \dot{x}_2 = -a_{21}x_1 - b_{22}x_2^3 + (a_{22} + b_{22}k_2)x_2 + \dots + a_{2n}x_n \\ \dots \\ \dot{x}_n = -a_{n1}x_1 + a_{n2}x_2 + \dots - b_{nn}x_n^3 + (a_{nn} + b_{nn}k_n)x_n \end{cases} \tag{3}$$

The stationary state $x_{is}, i = 1, \dots, n$ of the system (3) is determined by solving the equations

$$\begin{cases} -b_{11}x_{1s}^3 + (a_{11} + b_{11}k_1)x_{1s} + a_{12}x_{2s} + \dots + a_{1n}x_{ns} = 0 \\ -a_{21}x_{1s} - b_{22}x_{2s}^3 + (a_{22} + b_{22}k_2)x_{2s} + \dots + a_{2n}x_{ns} = 0 \\ \dots \\ -a_{n1}x_{1s} + a_{n2}x_{2s} + \dots - b_{nn}x_n^3 + (a_{nn} + b_{nn}k_n)x_{ns} = 0 \end{cases} \tag{4}$$

From (4) the system's stationary state can be determined as (5)

$$x_{is}^1 = 0, \ i = 1, \dots, n \tag{5}$$

Other stationary states can be determined as solutions of equations

$$-x_{is}^2 + (k_i - a_{n-i+1}) = 0, \ i = 1, \dots\dots, n \tag{6}$$

For negative $k_i - a_{n-i+1} (k_i - a_{n-i+1} < 0), i = 1,2, \dots, n$, this equation has an false solution, which cannot correspond to any physically possible situation. For $k_i - a_{n-i+1} > 0, i = 1,2, \dots n$, equation (6) admits the following stationary states

$$x_{is}^{2,3} = \pm\sqrt{\frac{a_{ii}}{b_{ii}} + k_i}, \ x_{js} = 0, \\ i \neq j, \ i = 1, \dots, n, \ j = 1, \dots, n \tag{7-8}$$

The states (7-8) merge with (5) at the control parameter $k_i - a_{n-i+1} = 0$ and branch off from it at $k_i - a_{n-i+1} > 0$

INVESTIGATION OF THE CONTROL SYSTEM FOR STABILITY IN ZERO STATIONARY STATES

Finding the Components of the Gradient Vector

It is known, that on the basis of Lyapunov's theorem [6; 14, 16-19; 23; 26]: if Lyapunov function V (x) is given in the form of a vector-function V (x), then according to geometric interpretation of the second Lyapunov method the components of the velocity vector are equal to [20-22, 25]

$$-\frac{dx_i}{dt} = \frac{\partial V_i(x)}{\partial x_1} + \frac{\partial V_i(x)}{\partial x_2} + \dots + \frac{\partial V_i(x)}{\partial x_n}, \ i = 1, \dots, n \tag{9}$$

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From (8a) the components of the gradient vector of the Lyapunov vector function can be found in the form:

$$\begin{cases} \frac{\partial V_1(x)}{\partial x_1} = b_{11}x_1^3 - (a_{11}+b_{11}k_1)x_1, \frac{\partial V_1(x)}{\partial x_2} = -a_{12}x_2, ..., \frac{\partial V_1(x)}{\partial x_n} = -a_{1n}x_n \\ \frac{\partial V_2(x)}{\partial x_1} = -a_{21}x_1, \frac{\partial V_2(x)}{\partial x_2} = b_{22}x_2^3 - (a_{22}+b_{22}k_2)x_2, ..., \frac{\partial V_2(x)}{\partial x_n} = -a_{2n}x_n \\ \cdots\cdots\cdots \\ \frac{\partial V_n(x)}{\partial x_1} = -a_{n1}x_1, \frac{\partial V_n(x)}{\partial x_2} = -a_{n2}x_2, ..., \frac{\partial V_n(x)}{\partial x_n} = b_{nn}x_n^3 - (a_{nn}+b_{nn}k_n)x_n \end{cases}$$

Finding the Components of the Velocity Vector

From (9) the expansion of velocity vector components in the coordinates can be represented as follows:

$$\begin{cases} \left(\frac{dx_1}{dt}\right)_{x_1} = -b_{11}x_1^3 + (a_{11}+b_{11}k_1)x_1, \left(\frac{dx_1}{dt}\right)_{x_2} = a_{12}x_2, ..., \left(\frac{dx_1}{dt}\right)_{x_n} = a_{1n}x_n \\ \left(\frac{dx_2}{dt}\right)_{x_1} = a_{21}x_1, \left(\frac{dx_2}{dt}\right)_{x_2} = -b_{22}x_2^3 + (a_{22}+b_{22}k_2)x_2, ..., \left(\frac{dx_2}{dt}\right)_{x_n} = a_{2n}x_n \\ \cdots\cdots\cdots \\ \left(\frac{dx_n}{dt}\right)_{x_1} = a_{n1}x_1, \left(\frac{dx_n}{dt}\right)_{x_2} = a_{n2}x_2, ..., \left(\frac{dx_n}{dt}\right)_{x_n} = -b_{nn}x_n^3 + (a_{nn}+b_{nn}k_n)x_n \end{cases}$$

Finding the Common Time Derivative of the Lyapunov Function

Common time derivative of the Lyapunov vector-function V (x), with considering the state equation (3) can be defined as the scalar product of the gradient vector from the Lyapunov vector-function and the velocity vector, i.e.,

$$\begin{aligned} \frac{dV(x)}{dt} &= \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial V_i(x)}{\partial x_j}\left(\frac{dx_i}{dt}\right)_{x_j} = \\ &= -\left(b_{11}x_1^3 - (a_{11}+b_{11}k_1)x_1\right)^2 - \\ &- a_{12}^2x_2^2 - ... - a_{1n}^2x_n^2 - \\ &- a_{21}^2x_1^2 - \left(b_{22}x_2^3 - (a_{22}+b_{22}k_2)x_2\right)^2 - ... - a_{2n}^2x_n^2 - ... \\ &... - a_{n1}^2x_1^2 - a_{n2}^2x_2^2 ... - \left(b_{nn}x_n^3 - (a_{nn}+b_{nn}k_n)x_n\right)^2 \end{aligned} \tag{10}$$

Construction the Lyapunov Vector-Function

It is obvious from (10) that the total time derivatives of the vector-function V(x) are negative. Components of Lyapunov vector-function can be found from gradient-vector. Lyapunov function in the scalar form can be written as [6; 8; 14, 16-19; 23; 26]:

$$\begin{aligned} V(x) &= \sum_{i=1}^{n} V_i(x) = \\ &= \frac{1}{4}x_1^4 - \frac{1}{2}(a_{11}+b_{11}k_1+a_{21}+a_{31}+...+a_{n1})x_1^2 + \\ &+ \frac{1}{4}x_2^4 - \frac{1}{2}(a_{12}+a_{22}+b_{22}k_2+a_{32}+...+a_{n2})x_2^2 + \\ &+ \frac{1}{4}x_3^4 - \frac{1}{2}(a_{13}+a_{23}+a_{33}+b_{33}k_3+...+a_{n3})x_3^2 + ... \\ &... + \frac{1}{4}x_n^4 - \frac{1}{2}(a_{1n}+a_{2n}+a_{3n}+...+a_{nn}+b_{nn}k_n)x_n^2 \end{aligned} \tag{11}$$

Finding Conditions for the Positive Definiteness of the Lyapunov Function, if the Gradient Vector is Negative

The conditions of positive definiteness of the function (11) are determined by the inequalities:

$$\begin{cases} a_{11}+b_{11}k_1+a_{21}+a_{31}+...+a_{n1} < 0 \\ a_{12}+a_{22}+b_{22}k_2+a_{32}+...+a_{n2} < 0 \\ a_{13}+a_{23}+a_{33}+b_{33}k_3+...+a_{n3} < 0 \\ ... \\ a_{1n}+a_{2n}+a_{3n}+...+a_{nn}+b_{nn}k_n < 0 \end{cases} \tag{12}$$

Thus, the area of system stability (3) for the steady state (5) is determined by the system of inequalities (12).

INVESTIGATION THE CONTROL SYSTEM FOR STABILITY IN STATIONARY STATES 2 AND 3

Develop a similar algorithm for investigating the stability of the stationary states (7-8) based on the method of Lyapunov functions.

The formally described expansion [15] can be represented in the form

$$\begin{aligned} F(X_s+x) &= F(X_s) + \left(\frac{\partial F}{\partial x}\right)_{X_s} x + \frac{1}{2}\left(\frac{\partial^2 F}{\partial x \partial x}\right)_{X_s} xx + \\ &+ \frac{1}{6}\left(\frac{\partial^3 F}{\partial x \partial x \partial x}\right)_{X_s} xxx + \frac{1}{24}\left(\frac{\partial^4 F}{\partial x \partial x \partial x \partial x}\right)_{X_s} xxxx + ... \end{aligned} \tag{13}$$

The values of the derivatives in the stationary states X_s are determined as

$$\left(\frac{\partial F_i}{\partial x_1}\right)_{X_s} = \pm a_{i1}, \left(\frac{\partial F_i}{\partial x_2}\right)_{X_s} = \pm a_{i2},$$

$$\left(\frac{\partial F_i}{\partial x_3}\right)_{X_s} = a_{i3}, ..., \left(\frac{\partial F_i}{\partial x_i}\right)_{X_s} = -(a_{ii}+b_{ii}k_i), ..., \left(\frac{\partial F_i}{\partial x_n}\right)_{X_s} = a_{in}$$

$$\left(\frac{\partial^2 F_i}{\partial x_i \partial x_j}\right)_{X_s} = 0, \text{ when } k \neq j, j = 1,...,n; k = 1,...,n$$

$$\left(\frac{\partial^2 F_i}{\partial x_i^2}\right)_{X_s} = \mp 6 b_{ii}\sqrt{\frac{a_{ii}}{b_{ii}} + k}, i = 1,...,n$$

$$\left(\frac{\partial^3 F_i}{\partial x_j \partial x_k \partial x_i}\right)_{X_s} = 0, i \neq j \neq k, i = 1,...,n, j = 1,...,n; k = 1,...,n$$

$$\left(\frac{\partial^3 F_i}{\partial x_i^3}\right)_{X_s} = \mp 6b_{ii}, i = 1,...,n$$

Describing the Control System for Relatively Stationary States

Equations of state (3) in the deviations with respect to the stationary states x_S^2 and x_S^3 (7-8) can be determined as

$$\begin{cases} \dot{x}_1 = -b_{11}x_1^3 \mp 3b_{11}\sqrt{\frac{a_{11}}{b_{11}} + k_1 x_1^2} - (a_{11} + b_{11}k_1)x_1 + a_{12}x_2 + ... + a_{1n}x_n \\ \dot{x}_2 = a_{21}x_1 - b_{22}x_2^3 \mp 3b_{22}\sqrt{\frac{a_{22}}{b_{22}} + k_2 x_2^2} - (a_{22} + b_{22}k_2)x_2 + ... + a_{2n}x_n \\ ... \\ \dot{x}_n = a_{n1}x_1 + a_{n2}x_2 + ... - b_{nn}x_n^3 \mp 3b_{nn}\sqrt{\frac{a_{nn}}{b_{nn}} + k_n x_n^2} - (a_{nn} + b_{nn}k_n)x_n \end{cases} \quad (14)$$

The Components of the Gradient Vector

To study the stability of the system in stationary states (7-8), based on the geometric interpretation, from equation (14) the components of the Lyapunov vector-function V(x)=(V_1(x),…,V_n(x)) should be determined:

$$\begin{cases} \frac{\partial V_1(x)}{\partial x_1} = b_{11}x_1^3 \pm 3b_{11}\sqrt{\frac{a_{11}}{b_{11}} + k_1 x_1^2} + (a_{11} + b_{11}k_1)x_1, \\ \frac{\partial V_1(x)}{\partial x_2} = -a_{12}x_2, \frac{\partial V_1(x)}{\partial x_3} = -a_{13}x_3, ..., \frac{\partial V_1(x)}{\partial x_n} = -a_{1n}x_n, \\ \frac{\partial V_2(x)}{\partial x_1} = -a_{21}x_1, \frac{\partial V_2(x)}{\partial x_2} = b_{22}x_2^3 \pm 3b_{22}\sqrt{\frac{a_{22}}{b_{22}} + k_2 x_2^2} + (a_{22} + b_{22}k_2)x_2, \\ \frac{\partial V_2(x)}{\partial x_3} = -a_{23}x_3, ..., \frac{\partial V_2(x)}{\partial x_n} = -a_{2n}x_n, \\ ... \\ \frac{\partial V_n(x)}{\partial x_1} = -a_{n1}x_1, \frac{\partial V_n(x)}{\partial x_2} = -a_{n2}x_2 ... \\ ... \frac{\partial V_n(x)}{\partial x_n} = b_{nn}x_n^3 \pm 3b_{nn}\sqrt{\frac{a_{nn}}{b_{nn}} + k_n x_n^2} + (a_{nn} + b_{nn}k_n)x_n \end{cases}$$

The Components of the Velocity Vector

From the state equation (14), the expansion of the components of the velocity vector in coordinates is obtained in the form

$$\begin{cases} \left(\frac{dx_1}{dt}\right)_{x_1} = -b_{11}x_1^3 \mp 3b_{11}\sqrt{\frac{a_{11}}{b_{11}} + k_1 x_1^2} - (a_{11} + b_{11}k_1)x_1, \\ \left(\frac{dx_1}{dt}\right)_{x_2} = a_{12}x_2, \left(\frac{dx_1}{dt}\right)_{x_3} = a_{13}x_3, ..., \left(\frac{dx_1}{dt}\right)_{x_n} = a_{1n}x_n \\ \left(\frac{dx_2}{dt}\right)_{x_1} = a_{21}x_1, \left(\frac{dx_2}{dt}\right)_{x_2} = -b_{22}x_2^3 \mp 3b_{22}\sqrt{\frac{a_{22}}{b_{22}} + k_2 x_2^2} - (a_{22} + b_{22}k_2)x_2, \\ \left(\frac{dx_2}{dt}\right)_{x_3} = a_{23}x_3, ..., \left(\frac{dx_2}{dt}\right)_{x_n} = a_{2n}x_n, \\ .... \\ \left(\frac{dx_n}{dt}\right)_{x_1} = a_{n1}x_1, \left(\frac{dx_n}{dt}\right)_{x_2} = a_{n2}x_2, \left(\frac{dx_n}{dt}\right)_{x_3} = a_{n3}x_3, ... \\ ..., \left(\frac{dx_n}{dt}\right)_{x_n} = -b_{nn}x_n^3 \mp 3b_{nn}\sqrt{\frac{a_{nn}}{b_{nn}} + k_n x_n^2} - (a_{nn} + b_{nn}k_n)x_n \end{cases}$$

The Total Time Derivative of the Lyapunov Function

Full time derivative of the Lyapunov function V (x), given the state equations in deviations (14) relative to the stationary states (7-8) is defined as the scalar product

$$\frac{dV(x)}{dt} = \sum_{i=1}^{n}\left(\sum_{j=1}^{n}\frac{\partial V_i(x)}{\partial x_j}\left(\frac{dx_i}{dt}\right)_{X_j}\right) = \quad (15)$$

$$= -(b_{11}x_1^3 + 3b_{11}\sqrt{\frac{a_{11}}{b_{11}} + k_1 x_1^2} + (a_{11} + b_{11}k_1)x_1)^2 - a_{12}^2x_2^2 - a_{13}^2x_3^2 - ... - a_{1n}^2x_n^2 -$$

$$- a_{21}^2x_1^2 - (b_{22}x_2^3 + 3b_{22}\sqrt{\frac{a_{22}}{b_{22}} + k_2 x_2^2} + (a_{22} + b_{22}k_2)x_2)^2 - a_{23}^2x_3^2 - ... - a_{2n}^2x_n^2 - ...$$

$$... - a_{n1}^2x_1^2 - a_{n2}^2x_2^2 - a_{n3}^2x_3^2 - ... - (b_{nn}x_n^3 + 3b_{nn}\sqrt{\frac{a_{nn}}{b_{nn}} + k_n x_n^2} + (a_{nn} + b_{nn}k_n)x_n)^2$$

The function (15) is negative, i.e. a sufficient stability condition will permanently be satisfied.

Lyapunov Vector-Function

$$V(x) = V_1(x) + V_2(x) + ... + V_n(x) =$$

$$= \frac{1}{4}b_{11}x_1^4 \pm b_{11}\sqrt{\frac{a_{11}}{b_{11}} + k_1 x_1^3} + \frac{1}{2}(a_{11} + b_{11}k_1 - a_{21} - ... - a_{n1})x_1^2 +$$

$$+ \frac{1}{4}b_{22}x_2^4 \pm b_{22}\sqrt{\frac{a_{22}}{b_{22}} + k_2 x_2^3} + \frac{1}{2}(a_{22} + b_{22}k_2 - a_{12} - ... - a_{n2})x_2^2 + ...$$

$$... + \frac{1}{4}b_{nn}x_n^4 \pm b_{nn}\sqrt{\frac{a_{nn}}{b_{nn}} + k_n x_n^3} + \frac{1}{2}(a_{nn} + b_{nn}k_n - a_{1n} - a_{2n} - ... - a_{n-1,n})x_n^2$$

or

$$V(x) = \sum_{i=1}^{n}(\frac{1}{4}b_{ii}x_i^4 \pm b_{ii}\sqrt{\frac{a_{ii}}{b_{ii}} + k_i x_i^3}) +$$

$$+ \frac{1}{2}\sum_{i=1}^{n}(-a_{1i} - a_{2i} - ... + (a_{ii} + b_{ii}k_i) - ... - a_{ni})x_i^2 \quad (16)$$

The function (16) at the origin is zero, is a continuously differentiable function and has as variables the terms with odd powers. Therefore, on the basis of Morse lemma [8, 11, 15], the function (16) around stationary states can be represented in the quadratic form

$$V(x)=\frac{1}{2}\sum_{i=1}^{n}(-a_{1i}-a_{2i}-a_{3i}-...+(a_{ii}+b_{ii}k_i)-...-a_{ni})x_i^2$$

Finding Conditions for the Positive Definiteness of the Lyapunov Function

Hence, the positive definiteness of the Lyapunov function is determined by the inequality

$$(a_{ii}+b_{ii}k_i)>a_{1i}+a_{2i}+...+a_{ni},\ i=1,...,n; \quad (17)$$

Thus, the control system built in the class of single-parameter structurally stable maps, is stable indefinitely in wide limits changes of uncertain parameters of the control object $a_i (i=1,...,n)$. The stationary state x_S^1 (5) exists and is stable when the indeterminate parameters of the object change in the region (12), and the stationary states x_S^2 and x_S^3 (7-8) appear when the stability of the state x_S^1 (5) is lost and they do not exist simultaneously. The stationary states x_S^2 and x_S^3 are stable when the system of inequalities (17) is satisfied.

CASE STUDY

Construction Aircraft Landing Control System in the Class of "Fold" Catastrophe

According to the developed technique, the stability of the aircraft landing control system constructed in the class of single-parameter structurally stable maps [7-8] by the Lyapunov function method [14, 16-20; 23, 26] is investigated.

On the considering the aircraft landing system, the following variables are represented as state variables: x_1 - the height, x_2 - the rate of change in altitude, x_3 - the pitch angle, and x_4 - the angular velocity of the pitch, these variables can be easily measured with a radio altimeter and hydro sensors. The dynamics of the aircraft is characterized by the equations [26]:

$$\begin{cases}\frac{dx_1}{dt}=x_2\\ \frac{dx_2}{dt}=a_{22}x_2+a_{23}x_3\\ \frac{dx_3}{dt}=x_4\\ \frac{dx_4}{dt}=a_{42}x_2+a_{43}x_3+a_{44}x_4-k_p\delta\end{cases} \quad (18)$$

in equation (1), δ (t) characterizes the deviation of the elevator, and the coefficients of the state variables are [26]:

$$a_{22}=-\frac{1}{T_0};a_{23}=\frac{V}{T_0};a_{42}=\frac{1}{VT_0^2}-\frac{2\xi\omega_0}{VT_0}+\frac{\omega_0^2}{V};$$
$$a_{43}=\frac{1}{T_0^2}+\frac{2\xi\omega_0}{T_0}-\omega_0^2;a_{44}=\frac{1}{T_0}-2\xi\omega_0$$

ξ,K,ω_0,T_0 – coefficient of damping of short-period oscillations, amplification coefficient of short-period oscillations, resonance frequency of oscillations, trajectory time constant, respectively. These parameters depend on the aircraft structure. In the study it's assumed that $\xi=0.5;K=-0.95\sec^{-1};\omega_0=1rad/\sec;T_0=2.5\sec$

To examine the system for stability, the control law is chosen in the form of single-parameter structurally stable maps [14, 16-20; 23, 26]:

$$\begin{aligned}U=k_p\left[\left(-x_1^3+k_1x_1\right)+\left(-x_2^3+k_2x_2\right)\right]+\\ +k_p\left[\left(-x_3^3+k_3x_3\right)+\left(-x_4^3+k_4x_4\right)\right]\end{aligned} \quad (19)$$

In the expanded form, the equation of state (18) is written

$$\begin{cases}\frac{dx_1}{dt}=x_2\\ \frac{dx_2}{dt}=a_{22}x_2+a_{23}x_3\\ \frac{dx_3}{dt}=x_4\\ \frac{dx_4}{dt}=a_{42}x_2+a_{43}x_3+a_{44}x_4-k_p(x_1^3-k_1x_1)-\\ -k_p(x_2^3-k_2x_2)-k_p(x_3^3-k_3x_3)-k_p(x_4^3-k_4x_4)\end{cases} \quad (20)$$

Investigate the stability of the system (20), constructed in the class of single-parameter structurally stable maps, according to the developed method [16-19]. System's steady states are determined as:

$$\begin{cases}x_{2s}=0\\ a_{22}x_{2s}+a_{23}x_{3s}=0\\ x_{4s}=0\\ a_{42}x_{2s}+a_{43}x_{3s}+a_{44}x_{4s}-\\ -k_p(x_{1s}^3-k_{1s}x_{1s})-k_p(x_{2s}^3-k_{2s}x_{2s})-\\ -k_p(x_{3s}^3-k_{3s}x_{3s})-k_p(x_{4s}^3-k_{4s}x_{4s})=0\end{cases} \quad (21)$$

From (21) the stationary states of system (20) can be found as:

$$x_{1s}^1=x_{2s}^1=x_{3s}^1=x_{4s}^1=0 \quad (22)$$

Other stationary states of system (20) will be determined by solving equations $-x_{1s}^2+k_1=0,-x_{2s}^2+k_2=0,-x_{3s}^2+k_3=0,-x_{4s}^2+k_4=0$ For negative $k_1\,(k_1<0)$, $k_2\,(k_2<0)$, $k_3\,(k_3<0)$ и $k_4\,(k_4<0)$ this equation has an false solution, which cannot correspond to any physically possible situation. For $k_1\,(k_1>0)$, k_2

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$(k_2>0)$, $k_3\ (k_3>0)$ и $k_4\ (k_4>0)$ the equation admits the following stationary states

$$x_{1s}^2=\sqrt{k_1},\ x_{2s}^2=\sqrt{k_2},\ x_{3s}^2=\sqrt{k_3},\ x_{4s}^2=\sqrt{k_4} \tag{23}$$

$$x_{1s}^2=-\sqrt{k_1},\ x_{2s}^2=-\sqrt{k_2},\ x_{3s}^2=-\sqrt{k_3},\ x_{4s}^2=-\sqrt{k_4} \tag{24}$$

The states (23) and (24) for system (21) merge with (22) at the value of the parameters $k_1=0, k_2=0, k_3=0, k_4=0$ and branch off from it at $k_1>0, k_2>0, k_3>0, k_4>0$

Investigation of the Stability of the System in the Zero Stationary State

To research the robust stability of the stationary states (22), (23) and (24), the basic propositions of the developed Lyapunov function method are used, here the components of the gradient vector can be given as:

$$\begin{cases}
\dfrac{\partial V_1(x_1,x_2,x_3,x_4)}{\partial x_1}=0, \dfrac{\partial V_1(x_1,x_2,x_3,x_4)}{\partial x_2}=-x_2,\\
\dfrac{\partial V_1(x_1,x_2,x_3,x_4)}{\partial x_3}=0, \dfrac{\partial V_1(x_1,x_2,x_3,x_4)}{\partial x_4}=0\\
\dfrac{\partial V_2(x_1,x_2,x_3,x_4)}{\partial x_1}=0, \dfrac{\partial V_2(x_1,x_2,x_3,x_4)}{\partial x_2}=-a_{22}x_2,\\
\dfrac{\partial V_2(x_1,x_2,x_3,x_4)}{\partial x_3}=-a_{23}x_3, \dfrac{\partial V_2(x_1,x_2,x_3,x_4)}{\partial x_4}=0\\
\dfrac{\partial V_3(x_1,x_2,x_3,x_4)}{\partial x_1}=0, \dfrac{\partial V_3(x_1,x_2,x_3,x_4)}{\partial x_2}=0,\\
\dfrac{\partial V_3(x_1,x_2,x_3,x_4)}{\partial x_3}=0, \dfrac{\partial V_3(x_1,x_2,x_3,x_4)}{\partial x_4}=-x_4\\
\dfrac{\partial V_4(x_1,x_2,x_3,x_4)}{\partial x_1}=k_p x_1^3-k_p k_1 x_1,\\
\dfrac{\partial V_4(x_1,x_2,x_3,x_4)}{\partial x_2}=-a_{42}x_2+k_p x_2^3-k_p k_2 x_2,\\
\dfrac{\partial V_4(x_1,x_2,x_3,x_4)}{\partial x_3}=-a_{43}x_3+k_p x_3^3-k_p k_3 x_3,\\
\dfrac{\partial V_4(x_1,x_2,x_3,x_4)}{\partial x_4}=-a_{44}x_4+k_p x_4^3-k_p k_4 x_4
\end{cases}$$

The total time derivative of the Lyapunov scalar function can be written

$$\begin{aligned}\frac{dV(x)}{dt}&=-x_2^2-a_{22}^2x_2^2-a_{23}^2x_3^2-x_4^2-k_p^2(x_1^3-k_1x_1)^2-\\
&-a_{42}^2x_2^2-k_p^2(x_2^3-k_2x_2)^2-\\
&-a_{43}^2x_3^2-k_p^2(x_3^3-k_3x_3)^2-\\
&-a_{44}^2x_4^2-k_p^2(x_4^3-k_4x_4)^2\end{aligned} \tag{25}$$

The total time derivative of the Lyapunov function is a negative function. The components of the Lyapunov vector-function can be obtained in the form

$$V_1(x)=-\frac{1}{2}x_2^2, V_2(x)=-\frac{1}{2}a_{22}x_2^2-\frac{1}{2}a_{23}x_3^2, V_3(x)=-\frac{1}{2}x_4^2,$$

$$\begin{aligned}V_4(x)&=\frac{1}{4}k_px_1^4-\frac{1}{2}k_pk_1x_1^2-\frac{1}{2}a_{42}x_2^2+\frac{1}{4}k_px_2^4-\frac{1}{2}k_pk_2x_2^2-\\
&-\frac{1}{2}a_{43}x_3^2+\frac{1}{4}k_px_3^4-\frac{1}{2}k_pk_3x_3^2-\frac{1}{2}a_{44}x_4^2+\frac{1}{4}k_px_4^4-\frac{1}{2}k_pk_4x_4^2\end{aligned}$$

The Lyapunov function in the scalar form can be represented in the form

$$\begin{aligned}V(x)&=\frac{1}{4}k_px_1^4-\frac{1}{2}k_pk_1x_1^2+\\
&+\frac{1}{4}k_px_2^4-\frac{1}{2}(1+a_{22}+a_{42}+k_pk_2)x_2^2+\\
&+\frac{1}{4}k_px_3^4-\frac{1}{2}(a_{23}+a_{43}+k_pk_3)x_3^2+\\
&+\frac{1}{4}k_px_4^4-\frac{1}{2}(1+a_{44}+k_pk_4)x_4^2\end{aligned} \tag{26}$$

The stability conditions for the zero stationary state (22) are obtained with allowance for the negative definiteness of the function (25), from the condition of positive definiteness of the function (26), the stability condition can be found as:

$$\begin{cases}k_pk_1<0\\ 1+a_{22}+a_{42}+k_pk_2<0\\ a_{23}+a_{43}+k_pk_3<0\\ 1+a_{44}+k_pk_4<0\end{cases} \Rightarrow \begin{cases}k_p<0, k_1>0\\ k_2<\dfrac{(1+a_{22}+a_{42})}{k_p}\\ k_3<\dfrac{(a_{23}+a_{43})}{k_p}\\ k_4<\dfrac{(1+a_{44})}{k_p}\end{cases} \Rightarrow$$

$$\left|\begin{aligned}&a_{22}=-\frac{1}{T_0}=-\frac{1}{2.5}=-0.4;\\
&a_{23}=\frac{V}{T_0}=\frac{280}{2.5}=112;\\
&a_{42}=\frac{1}{VT_0^2}-\frac{2\xi\omega_0}{VT_0}+\frac{\omega_0^2}{V}\approx 0.003;\\
&a_{43}=\frac{1}{T_0^2}+\frac{2\xi\omega_0}{T_0}-\omega_0^2=-0.44;\\
&a_{44}=\frac{1}{T_0}-2\xi\omega_0=-0.6;\end{aligned}\right| \Rightarrow \begin{cases}k_p=-2.375\\ k_1>0\\ k_2<-0.3\\ k_3<-47\\ k_4<-0.17\end{cases} \tag{27}$$

$$k_p=KV\omega_0^2=-0.95*2.5*1=-2.375$$

Stability of the Control System in a Stationary State (23-24)

Observe the stability of the stationary states (23-24) and for this equation the state of the aircraft landing process (21) is written in the deviations with respect to the stationary state (23-24):

$$\begin{cases} \dfrac{dx_1}{dt} = x_2 \\ \dfrac{dx_2}{dt} = a_{22}x_2 + a_{23}x_3 \\ \dfrac{dx_3}{dt} = x_4 \\ \dfrac{dx_4}{dt} = a_{42}x_2 + a_{43}x_3 + a_{44}x_4 - \\ -(k_p x_1^3 \pm 3k_p\sqrt{k_1}x_1^2 + 2k_1k_px_1) - \\ -(k_p x_2^3 \pm 3k_p\sqrt{k_2}x_2^2 + 2k_2k_px_2) \\ -(k_p x_3^3 \pm 3k_p\sqrt{k_3}x_3^2 + 2k_3k_px_3) - \\ -(k_p x_4^3 \pm 3k_p\sqrt{k_4}x_4^2 + 2k_4k_px_4) \end{cases}$$

The components of the gradient vector from the components of the Lyapunov vector-function are represented as

$$\begin{cases} \dfrac{\partial V_1(x_1,x_2,x_3,x_4)}{\partial x_1} = 0, \dfrac{\partial V_1(x_1,x_2,x_3,x_4)}{\partial x_2} = -x_2, \\ \dfrac{\partial V_1(x_1,x_2,x_3,x_4)}{\partial x_3} = 0, \dfrac{\partial V_1(x_1,x_2,x_3,x_4)}{\partial x_4} = 0 \\ \dfrac{\partial V_2(x_1,x_2,x_3,x_4)}{\partial x_1} = 0, \dfrac{\partial V_2(x_1,x_2,x_3,x_4)}{\partial x_2} = -a_{22}x_2, \\ \dfrac{\partial V_2(x_1,x_2,x_3,x_4)}{\partial x_3} = -a_{23}x_3, \dfrac{\partial V_2(x_1,x_2,x_3,x_4)}{\partial x_4} = 0 \\ \dfrac{\partial V_3(x_1,x_2,x_3,x_4)}{\partial x_1} = 0, \dfrac{\partial V_3(x_1,x_2,x_3,x_4)}{\partial x_2} = 0, \\ \dfrac{\partial V_3(x_1,x_2,x_3,x_4)}{\partial x_3} = 0, \dfrac{\partial V_3(x_1,x_2,x_3,x_4)}{\partial x_4} = -x_4 \\ \dfrac{\partial V_4(x_1,x_2,x_3,x_4)}{\partial x_1} = (k_px_1^3 \pm 3k_p\sqrt{k_1}x_1^2 + 2k_1k_px_1), \\ \dfrac{\partial V_4(x_1,x_2,x_3,x_4)}{\partial x_2} = -a_{42}x_2 + (k_px_2^3 \pm 3k_p\sqrt{k_2}x_2^2 + 2k_2k_px_2), \\ \dfrac{\partial V_4(x_1,x_2,x_3,x_4)}{\partial x_3} = -a_{43}x_3 + (k_px_3^3 \pm 3k_p\sqrt{k_3}x_3^2 + 2k_3k_px_3), \\ \dfrac{\partial V_4(x_1,x_2,x_3,x_4)}{\partial x_4} = -a_{44}x_4 + (k_px_4^3 \pm 3k_p\sqrt{k_4}x_4^2 + 2k_4k_px_4) \end{cases}$$

The expansion of the components of the velocity vector in coordinates is obtained in the form

$$\begin{cases} \left(\dfrac{dx_1}{dt}\right)_{x_1} = 0, \\ \left(\dfrac{dx_1}{dt}\right)_{x_2} = x_2, \left(\dfrac{dx_1}{dt}\right)_{x_3} = 0_3, \left(\dfrac{dx_1}{dt}\right)_{x_n} = 0 \\ \left(\dfrac{dx_2}{dt}\right)_{x_1} = 0, \left(\dfrac{dx_2}{dt}\right)_{x_2} = a_{22}x_2, \\ \left(\dfrac{dx_2}{dt}\right)_{x_3} = a_{23}x_3, \left(\dfrac{dx_2}{dt}\right)_{x_n} = 0, \\ \left(\dfrac{dx_1}{dt}\right)_{x_1} = 0, \\ \left(\dfrac{dx_1}{dt}\right)_{x_2} = 0, \left(\dfrac{dx_1}{dt}\right)_{x_3} = 0_3, \left(\dfrac{dx_1}{dt}\right)_{x_n} = x_4, \\ \left(\dfrac{dx_4}{dt}\right)_{x_1} = -(k_px_1^3 \pm 3k_p\sqrt{k_1}x_1^2 + 2k_1k_px_1), \\ \left(\dfrac{dx_4}{dt}\right)_{x_2} = a_{42}x_2 - (k_px_2^3 \pm 3k_p\sqrt{k_2}x_2^2 + 2k_2k_px_2), \\ \left(\dfrac{dx_4}{dt}\right)_{x_3} = a_{43}x_3 - (k_px_3^3 \pm 3k_p\sqrt{k_3}x_3^2 + 2k_3k_px_3), \\ \left(\dfrac{dx_4}{dt}\right)_{x_n} = a_{44}x_4 - (k_px_4^3 \pm 3k_p\sqrt{k_4}x_4^2 + 2k_4k_px_4) \end{cases}$$

The total time derivative of the Lyapunov vector-function is found in the form

$$\begin{aligned} \frac{dV(x)}{dt} &= -x_2^2 - a_{22}^2x_2^2 - a_{23}^2x_3^2 - x_4^2 - \\ &- k_p^2(x_1^3 + 3\sqrt{k_1}x_1^2 + 2k_1x_1)^2 - \\ &- a_{42}^2x_2^2 + k_p^2(x_2^3 + 3\sqrt{k_2}x_2^2 + 2k_2x_2)^2 - \\ &- a_{43}^2x_3^2 + k_p^2(x_3^3 + 3\sqrt{k_3}x_3^2 + 2k_3x_3)^2 - \\ &- a_{44}^2x_4^2 + k_p^2(x_4^3 + 3\sqrt{k_4}x_4^2 + 2k_4x_4)^2 \end{aligned} \tag{28}$$

The total derivative (28) of the Lyapunov vector-function is a negative function.

Using the gradient, Lyapunov functions can be created

$$\begin{aligned} V(x) &= \frac{1}{4}k_px_1^4 \mp k_p\sqrt{k_1}x_1^3 + k_pk_1x_1^2 + \\ &+ \frac{1}{4}k_px_2^4 \mp k_p\sqrt{k_1}x_1^3 - \frac{1}{2}(1 + a_{22} + a_{42} - 2k_pk_2)x_2^2 + \\ &+ \frac{1}{4}k_px_3^4 \mp k_p\sqrt{k_1}x_1^3 - \frac{1}{2}(a_{23} + a_{43} - 2k_pk_3)x_3^2 + \\ &+ \frac{1}{4}k_px_4^4 \mp k_p\sqrt{k_4}x_4^3 - \frac{1}{2}(1 + a_{44} - 2k_pk_4)x_4^2 \end{aligned} \tag{29}$$

According to the Morse lemma, the function (29) can be replaced by the quadratic form

$$\begin{aligned} V(x) &= k_pk_1x_1^2 - \frac{1}{2}(1 + a_{22} + a_{42} - 2k_pk_2)x_2^2 - \\ &- \frac{1}{2}(a_{23} + a_{43} - 2k_pk_3)x_3^2 - \frac{1}{2}(1 + a_{44} - 2k_pk_4)x_4^2 \end{aligned} \tag{30}$$

The condition for the positive definiteness of the functions (29) is obtained in the form

$$\begin{cases} k_p k_1 > 0 \\ 1 + a_{22} + a_{42} - 2k_p k_2 < 0 \\ a_{23} + a_{43} - 2k_p k_3 < 0 \\ 1 + a_{44} - 2k_p k_4 < 0 \end{cases} \Rightarrow \begin{cases} k_p < 0, k_1 < 0 \\ k_2 > \dfrac{(1 + a_{22} + a_{42})}{2k_p} \\ k_3 > \dfrac{(a_{23} + a_{43})}{2k_p} \\ k_4 > \dfrac{(1 + a_{44})}{2k_p} \end{cases} \Rightarrow$$

$$\left| \begin{array}{l} a_{22} = -\dfrac{1}{T_0} = -\dfrac{1}{2.5} = -0.4; \\ a_{23} = \dfrac{V}{T_0} = \dfrac{280}{2.5} = 112; \\ a_{42} = \dfrac{1}{VT_0^2} - \dfrac{2\xi\omega_0}{VT_0} + \dfrac{\omega_0^2}{V} \approx 0.003; \\ a_{43} = \dfrac{1}{T_0^2} + \dfrac{2\xi\omega_0}{T_0} - \omega_0^2 = -0.44; \\ a_{44} = \dfrac{1}{T_0} - 2\xi\omega_0 = -0.6; \end{array} \right| \Rightarrow \begin{cases} k_p = -2.375 \\ k_1 < 0 \\ k_2 > -0.13 \\ k_3 > -23.5 \\ k_4 > -0.08 \end{cases} \quad (31)$$

It follows from (31) that the stationary states (23-24) will be asymptotically stable.

Thus, the control system for linear aircraft landing processes, constructed in the class of single-parameter structurally stable maps, the "fold" catastrophe, shows that the control system stable in a limited range of parameters becomes stable, i.e. the system has no restrictions on changing the parameters.

The stationary state (22) is globally asymptotically stable when the conditions (27) are satisfied and unstable if these conditions are violated, and for the stability of the stationary states (23) and (24) under conditions (31). Fork_p, branching occurs and new stable branches appear.

In other words, branches (23), (24) appear as a result of a bifurcation at the moment when the stationary state (22) loses stability, and these branches themselves are stable. The stationary states (22), (23) and (24) do not exist simultaneously. This allows to increase the robust stability potential of the system under conditions of uncertainty of parameters.

Table-1. Parameters of the aircraft landing process control system.

k_1	k_2	k_3	k_4	k_p
-0.9	1	-2	-0.2	-2.375
-0.9	0.5	-5	-1	-2.375

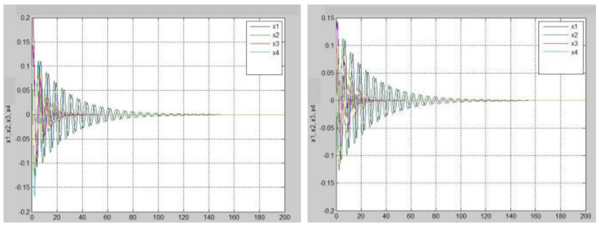

Figure-1. Results of simulations.

Table-2. Parameters of the aircraft landing process control system.

k_1	k_2	k_3	k_4	k_p
0.9	-1	-48	-0.2	-2.375
0.9	-0.5	-47.5	-1	-2.375

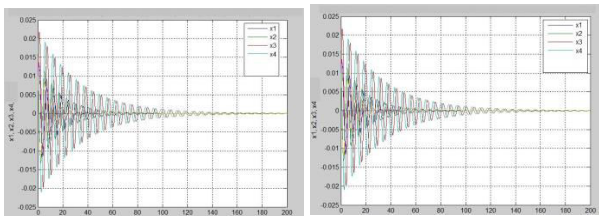

Figure-2. Results of simulations.

The aircraft landing control system, built in the fold catastrophe, is stable over a wide range of changes in the uncertain parameters of the control object. The stationary state (22) exists and is stable when the parameters of the system in the region (27) change, and the stationary states (23-24) appear when the state (22) becomes unstable, and they do not exist simultaneously. Stationary states (23-24) are stable when the system of inequalities (31) is fulfilled, which excludes the regime of deterministic chaos and instability during aircraft landing, which manifest themselves in the form of vibrations, instability and accidents.

CONCLUSIONS

The study presents the analysis and synthesis of a dynamic control system for the landing of an aircraft, a control law is introduced in the form of one-parameter structurally stable mappings. The considered method of constructing the Lyapunov function makes it possible to determine the limits of the stability of the system. Numerical experiments demonstrate the presence of asymptotic stability of the aircraft landing control system.

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