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KINEMATICS AND SINGULARITY ANALYSIS OF 3-PRS PARALLEL KINEMATIC MECHANISM

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ABSTRACT

Parallel manipulators have gained a lot of attraction from the research community. They come in various configurations. This paper mainly addresses the issues regarding kinematics and singularity analysis of prismatic revolute and spherical (3-PRS) parallel mechanism. A comprehensive and simple approach is presented to estimate the detailed kinematics, and singularity model of 3-PRS mechanism. For kinematic analysis first CAD model of the proposed 3 PRS mechanism is made using Autodesk inventor professional software. Starting with the inverse kinematics an analytical model is derived. For forward kinematics both analytical and numerical analysis are performed to ensure the efficiency of the presented methodology and results are compared with the presented CAD model which are closely related to each other. The singularity analysis is also done and presented in this paper.

Keywords: kinematics, parallel manipulators, singularity, robotics.

INTRODUCTION

In recent history, parallel robots have gained a lot of attention from the research community and industrial community as well. One of the main reason for their usage is their ability to provide high load capacity, high stiffness and high dynamics[1], [2]. Their drawbacks are their limited workspace. Parallel manipulators usually having a fixed base plate and a movable top plate. These plates are connected with the each other by kinematic chains. These kinematic chains are formed of joints and actuators. Parallel manipulators were first introduced by Gough and then used for simulation by Stewart [3]. Six degrees of freedom manipulators have many application and advantages, which are discussed well in [4], and other various literature. However, six degree of freedom manipulators are not always required for operations. Parallel manipulators with degrees of freedom less than six have gained attention of research community.

There is a vast range of applications in which parallel manipulators having degrees of freedom less than six are being used [5], [6]. Although, these parallel manipulators have many advantages over serial manipulators but till date the literature lacks in the area of their kinematics study. Solving the inverse kinematics of parallel manipulators is easy in comparison to solve its forward kinematics. Kinematic study of parallel manipulators was firstly introduced by Hunt [7]. Parallel manipulators were also studied in [8]. They introduced a 3-RRR configuration in planner manipulator. Up to now tripod parallel manipulators are developed for many configurations like, PRS, SPS, RPS, UPU. Different researchers have studied parallel manipulators in different configurations.

Despite their advantages in tool cost reduction and other manufacturing operations, some issues related to the position and orientation of parallel manipulators become complicated due to coupling. This difficulty in analysing the orientation and position of the parallel tripod manipulator has lead to difficult kinematics solution. Apart from kinematics, singularity is another issue which arises in such manipulators. Jacobian and singularity analysis of parallel manipulator is also important for optimal kinematic design and velocity analysis. Singularities must be avoided in order to control the manipulator [9].

Position of the 3-PRS manipulator is discussed in this paper. The mechanism of the manipulator is discussed in section 2. Inverse kinematics and forward kinematics are discussed in section 3. Singularity analysis is presented in section 4.

MECHANISM DESCRIPTION

The CAD model of the 3PRS manipulator is shown in Figure-1. The mechanism consists of fixed base, moving platform and the mechanism’s structure. Movable top plate is connected by three kinematic chains with the fixed base plate. Each Kinematic chain consists of Prismatic, Revolute and Spherical Joints in series. The prismatic joint is active by three actuators at each link. Total kinematic structure is based on three identical prismatic, revolute and spherical linkages. The 3-PRS mechanism has three degree of freedoms; one is the vertical translational movement and the other two are of rotations about two axes in horizontal plane.

KINEMATICS

Kinematics is categorized in to two categories; Inverse kinematics and Forward Kinematics. In this research both categories of kinematics are presented. Inverse kinematics is the one in which the final position of the end effector is known and there is a need to find out the respective leg lengths and angles. Forward kinematics is the one in which the position of the end effector is calculated by using the known leg length values and angles.

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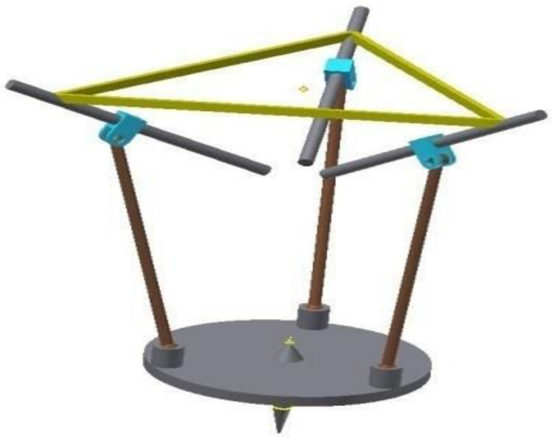

Figure-1. The CAD model of the 3PRS manipulator.

Inverse Kinematics

Inverse kinematic of 3PRS parallel mechanism is to find the actuators position from a given position and orientation of the moving platform. To find the inverse kinematic solution vector loop method is used.

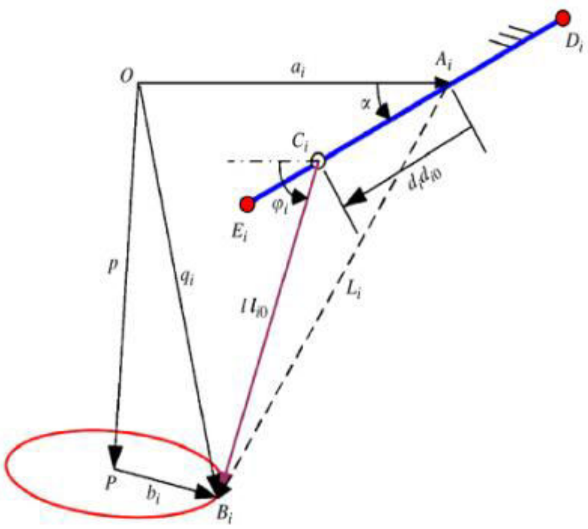

Figure-2. Vector diagram of one kinematic chain.

From Figure-2,

$$q_i = a_i + L_i \tag{1}$$

Loop closure for L_i is,

$$L_i = d_i d_{i0} + l I_{i0} \tag{2}$$

Where

d_{i0} and I_{i0} are the position vector of the ith linear actuator and fixed leg length, respectively. From equation (2)

$$L_i - d_i d_{i0} = l I_{i0} \tag{3}$$

Squaring both sides of equation (3)

$$L_i.L_i + d_i^2 - 2d_i.L_i.d_{i0} = l^2 \tag{4}$$

$$L_i.L_i + d_i^2 - 2d_i.L_i.d_{i0} - l^2 = 0 \tag{5}$$

Applying quadratic formula, where a=1,

b $=2(L_i.d_{i0})$ and c=$L_i.L_i - l^2$ we get

$$d_i = (L_i.d_{i0}) \pm \sqrt{(L_i.d_{i0})^2 - L_i.L_i + l^2} \tag{6}$$

Equation 6 gives two solutions for each linear actuator positive and negative, and for moving platform, position and orientation there are total eight possible solutions. Six of them are for three actuators and two for the final matrix $d = [d_1 d_2 d_3]$. In this study only the negative square root is selected, in which three legs are inclined inward from top to bottom.

Forward Kinematics

Forward kinematic of 3PRS parallel mechanism is to find the position and orientation of the moving platform from a given actuators position. To find the forward kinematic solution vector loop method is used. From Figure-2 we obtain

$$q_i = a_i + d_i d_{i0} + l I_{i0} \tag{7}$$

Where for $i = 1\ to\ 3$, d_{i0} and i_{i0} can be written as

$$d_{10} = [-c\alpha \ \ 0 - s\alpha]^T \tag{8}$$

$$d_{20} = \left[c\alpha/2 - \sqrt{3}c\alpha/2 \ \ \text{-} s\alpha\right]^T \tag{9}$$

$$d_{30} = \left[c\alpha/2 \ \sqrt{3}c\alpha/2 \ \ \text{-} s\alpha\right]^T \tag{10}$$

$$I_{10} = [-c\varphi_1 \ \ 0 - s\varphi_1]^T \tag{11}$$

$$I_{20} = \left[c\varphi_2/2 - c\varphi_2\sqrt{3}/2 \ \ \text{-} s\varphi_2\right]^T \tag{12}$$

$$I_{30} = \left[c\varphi_3/2 \ \ c\varphi_3\sqrt{3}/2 \ \ \text{-} s\varphi_3\right]^T \tag{13}$$

Putting the values of vectors in equation (7), the value of three vectors q_1, q_2 and q_3 can be obtained as:

$$q_1 = \begin{bmatrix} a - d_1 c\alpha - l c\varphi_1 \\ 0 \\ -d_1 s\alpha - l s\varphi_1 \end{bmatrix}$$
$$q_2 = \begin{bmatrix} -(a - d_2 c\alpha - l c\varphi_2)/2 \\ \sqrt{3}(a - d_2 c\alpha - l c\varphi_2)/2 \\ -d_2 s\alpha - l s\varphi_2 \end{bmatrix} \tag{14}$$
$$q_2 = \begin{bmatrix} -(a - d_2 c\alpha - l c\varphi_2)/2 \\ \sqrt{3}(a - d_2 c\alpha - l c\varphi_2)/2 \\ -d_2 s\alpha - l s\varphi_2 \end{bmatrix}$$

From the geometry of the moving platform one can find the geometric distance between the two spherical joints i.e. B_i and B_j, in which $i \neq j$.

$$||\overrightarrow{B_iB_j}|| = \sqrt{3}b \quad (15)$$

Where $\overrightarrow{B_iB_j}$ is the resultant of two vectors PB_i and PB_j so the equation (15) can be written as:

$$||\overrightarrow{PB_i} - \overrightarrow{PB_j}|| = \sqrt{3}b\left[\left[\overrightarrow{PB_i} - \overrightarrow{PB_j}\right]^{T} \cdot \left[\overrightarrow{PB_i} - \overrightarrow{PB_j}\right] = 3b^2 \quad (16)$$

We can write equation(16) in terms of q_i and q_{i+1}as:

$$[q_i - q_{i+1}]^T \cdot [q_i - q_{i+1}] - 3b^2 = 0 \quad (17)$$

Putting the values of vectors q_1, q_2 and q_3 from equation (14) into equation (15) the following equation (1) is obtained.

$$e_{1i}c\varphi_i c\varphi_{i+1} + e_{2i}s\varphi_i s\varphi_{i+1} + e_{3i}c\varphi_i + e_{4i}s\varphi_i + e_{5i}c\varphi_{i+1} + e_{6i}s\varphi_{i+1} + e_{7i} = 0 \quad (18)$$

Where for $i = 1,2$ and 3.

$$\begin{aligned}
e_{1i} &= l^2 \\
e_{2i} &= -2l^2 \\
e_{3i} &= [(2d_i + d_{i+1})c\alpha - 3a]l \\
e_{4i} &= 2l(d_i + d_{i+1})s\alpha \\
e_{5i} &= [(d_i + 2d_{i+1})c\alpha - 3a]l \\
e_{6i} &= -2l(d_i + d_{i+1})s\alpha \\
e_{7i} &= (d_i + d_{i+1})^2 + 3d_i d_{i+1}c^2\alpha - 3a(d_i + 2d_{i+1})\,c\alpha \\
&\quad + 3(a^2 - b^2) + 2l^2
\end{aligned}$$

By putting the trigonometric identities in equation (18)the following Eq(19) and Eq(20) are obtained.

$$s\varphi_i = \left(\frac{2t_i}{1+t_i^2}\right), \varphi_i = \left(\frac{1-t_i^2}{1+t_i^2}\right), s\varphi_{i+1} = \left(\frac{2t_{i+1}}{1+t_{i+1}^2}\right)\text{and } c\varphi_{i+1} = \left(\frac{1-t_{i+1}^2}{1-t_{i+1}^2}\right) \quad (19)$$

$$e_{1i}\left(\frac{1-t_i^2}{1+t_i^2}\right)\left(\frac{1-t_{i+1}^2}{1-t_{i+1}^2}\right) + e_{2i}\left(\frac{2t_i}{1+t_i^2}\right)\left(\frac{2t_{i+1}}{1+t_{i+1}^2}\right) + e_{3i}\left(\frac{1-t_i^2}{1+t_i^2}\right) + e_{4i}\left(\frac{2t_i}{1+t_i^2}\right) + e_{5i}\left(\frac{1-t_{i+1}^2}{1+t_{i+1}^2}\right) + e_{6i}\left(\frac{2t_{i+1}}{1+t_{i+1}^2}\right) + e_{7i} = 0 \quad (20)$$

Multiplying whole equation (20) by $(1 + t_i^2)(1 + t_{i+1}^2)$ and simplifying.

$$\text{€}_{1i}t_i^2t_{i+1}^2 + \text{€}_{2i}t_it_{i+1}^2 + \text{€}_{3i}t_i^2t_{i+1} + \text{€}_{4i}t_i^2 + \text{€}_{5i}t_{i+1}^2 + \text{€}_{6i}t_it_{i+1} + \text{€}_{7i}t_i + \text{€}_{8i}t_{i+1} + \text{€}_{9i} \quad (21)$$

Where for $i = 1$, 2 and 3.

$$\text{€}_{1i} = e_{1i} - e_{3i} - e_{5i} + e_{7i}\,\text{€}_{2i} = 2e_{4i}\text{€}_{3i} = 2e_{6i}\ \text{€}_{4i} = -e_{1i} - e_{3i} + e_{5i} + e_{7i}\text{€}_{5i} = -e_{1i} + e_{3i} - e_{5i} + e_{7i} \quad (22)$$

$$\text{€}_{6i} = 4e_{2\ i}\text{€}_{7i} = 2e_{4i}\ \ \text{€}_{8i}2e_{\text{€}_{9i}=e_{1i}+e_{3i}+e_{5i}+e_{7i}6i} \quad (23)$$

Equation (23) represents three fourth-degree polynomials in t_1, t_2 and t_3 which can be eliminated by Sylvester dialytic method which reduces the equation (23) system of equation into a16th-degree polynomials in one variable as follows. In the first step eliminate t_3 for which equation (23) is written for i =2 and $i + 1 = 3$ to reduce two second-degree polynomials in t_3.

$$\text{€}_{12}t_2^2t_3^2 + \text{€}_{22}t_2t_3^2 + \text{€}_{32}t_2^2t_3 + \text{€}_{42}t_2^2 + \text{€}_{52}t_3^2 + \text{€}_{62}t_2t_3 + \text{€}_{72}t_2 + \text{€}_{82}t_3 + \text{€}_{92} \quad (24)$$

$$\text{€}_{13}t_3^2t_1^2 + \text{€}_{23}t_3t_1^2 + \text{€}_{33}t_3^2t_1 + \text{€}_{43}t_3^2 + \text{€}_{53}t_1^2 + \text{€}_{63}t_3t_1 + \text{€}_{73}t_3 + \text{€}_{83}t_1 + \text{€}_{93} \quad (25)$$

From equation 25,

$$\begin{aligned}
A &= \text{€}_{12}t_2^2 + \text{€}_{22}t_2 + \text{€}_{52} \\
B &= \text{€}_{32}t_2^2 + \text{€}_{62}t_2 + \text{€}_{82} \\
C &= \text{€}_{42}t_2^2 + \text{€}_{72}t_2 + \text{€}_{92}
\end{aligned} \quad (26)$$

and from equation (25);

$$\begin{aligned}
D &= \text{€}_{13}t_1^2 + \text{€}_{33}t_1 + \text{€}_{43} \\
E &= \text{€}_{23}t_1^2 + \text{€}_{63}t_1 + \text{€}_{73} \\
F &= \text{€}_{53}t_1^2 + \text{€}_{83}t_1 + \text{€}_{93}
\end{aligned} \quad (27)$$

Writing these coefficients as

$$At_3^2 + Bt_3 + C = 0 \quad (28)$$

$$Dt_3^2 + Et_3 + F = 0 \quad (29)$$

Taking $(2.58 \times A - 2.57 \times D)$ and $(2.58 \times C - 2.57 \times F)$ yields two equations as follows:

$$(AE - BD)t_3 + AF - CD = 0 \quad (30)$$

$$(CD - AF)t_3 + CE - BF = 0 \quad (31)$$

Writing equation (30) and (31) in Matrix form as

$$\begin{bmatrix} AE - BD & AF - CD \\ CD - AF & CE - BF \end{bmatrix}\begin{bmatrix} t_3 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (32)$$

Taking the determinant of the coefficients matrix from equation (32) to eliminate the t_3.

$$(AE - BD)(CD - AF) + (AF - CD)^2 = 0 \quad (33)$$

Now we have to eliminte t_2 from equation (33) by writing the values of A, B & Cfform equation (26) will gives;

$$Lt_2^4 + Mt_2^3 + Nt_2^2 + Pt_2 + Q = 0 \quad (34)$$

Here we have from equation (34),

$$L = (\text{€}_{12}\text{€}_{42})E^2 - (\text{€}_{12}\text{€}_{32})EF - (\text{€}_{32}\text{€}_{42})DE + (\text{€}_{32})^2\,DF + (\text{€}_{12})^2F^2 + (\text{€}_{42})^2D^2 - 2(\text{€}_{12}\text{€}_{42})FD \quad (35)$$

$M = (€_{12}€_{72} + €_{22}€_{42})E^2 - (€_{12}€_{62} + €_{22}€_{32})EF - (€_{32}€_{72} + €_{62}€_{42})DE - 2(€_{12}€_{72} + €_{22}€_{42})FD$ (36)

$N = (€_{12}€_{92} + €_{22}€_{72} + €_{52}€_{42})E^2 - (€_{12}€_{82} + €_{22}€_{62} + €_{52}€_{32})EF - (€_{32}€_{92} + €_{72} + €_{82}€_{42})DE - 2(€_{12}€_{92} + €_{22}€_{72} + €_{52}€_{42})FD + (€_{62})^2 DF + (€_{22})^2F^2 + (€_{72})^2D^2$ (37)

$P = (€_{52}€_{72} + €_{22}€_{92})E^2 - (€_{72}€_{82} + €_{52}€_{62})EF - (€_{62}€_{92} + €_{82}€_{72})DE - 2(€_{52}€_{72} + €_{22}€_{92})FD$ (38)

$Q = (€_{52}€_{92})E^2 - (€_{52}€_{82})EF - (€_{82}€_{92})DE - 2(€_{52}€_{92})FD$ (39)

Writing equation (25) for i =1 and $i + 1 = 2$;

$€_{11}t_1^2t_2^2 + €_{21}t_1t_2^2 + €_{31}t_1^2t_2 + €_{41}t_1^2 + €_{51}t_2^2 + €_{61}t_1t_2 + €_{71}t_1 + €_{81}t_2 + €_{91}$ (40)

Taking coefficents of t_2 as

$= €_{11}t_1^2 + €_{21}t_1 + €_{51}$
$H = €_{31}t_1^2 + €_{61}t_1 + €_{81}$
$I = €_{41}t_1^2 + €_{71}t_1 + €_{91}$ (41)

Then equation (27) becomes,

$$\begin{bmatrix} \mathrm{LH-MG} & \mathrm{IL-GN} & \mathrm{-PG} & \mathrm{-GQ} \\ \mathrm{GN-LI} & \mathrm{GP+NH-MI} & \mathrm{GQ+PH} & \mathrm{HQ} \\ \mathrm{G} & \mathrm{H} & \mathrm{I} & 0 \\ 0 & \mathrm{G} & \mathrm{H} & \mathrm{I} \end{bmatrix} = 0 \quad (42)$$

We can find the t_1 by expanding equation (42) which gives eight pairs of solutions with eighth-degree polynomials in t_1 . Once, value of φ_i is found we can find the position vector q_1, q_2 and q_3 for the position vector p of the moving platform.

$$p = \frac{1}{3}(q_1 + q_2 + q_3) \quad (43)$$

The unit vector u, v and w can be found by the rotation matrix ${}^{O}R_p$ as

$$u = \frac{q_1 - p}{b}$$
$$v = \frac{q_2 - p_3}{\sqrt{3}b} \quad (44)$$
$$w = v \times u$$

The Euler Angles ϕ, ψ & θ can be found by equations(1) and (4) are as under

$$\psi = \tan^{-1}\left(\frac{-\omega_y}{\sqrt{u_y^2 + v_y^2}}\right)$$
$$\theta = \tan^{-1}\left(\frac{\omega_x}{\omega_z}\right)$$

$$\phi = \tan^{-1}\left(\frac{u_x}{v_y}\right) \quad (45)$$

Above presented approach is the analytical method to find forward kinematics for all possible configurations for moving platform. However, most of the solutions are meaningless and this approach is time consuming. So, to reduce our effort for finding only meaningful solutions, we shift towards a better and time-saving numerical approach. We can calculate the forward kinematics by classical Newton-Raphson Iterative method.

SINGULARITIES ANALYSIS

Singularities of parallel manipulators have been the area of interest of many researchers. These singularities occur, respectively, when

a) Matrix J_q is rank deficient, or
b) Matrix J_x is rank deficient, or
c) The positioning equations degenerate.

For serial manipulators, only the first type of singularity occurs.

Inverse Kinematics Singularity

The inverse kinematic singularity is related with the rank of matrix J_q. When the matrix $J_q$become rank deficient this type of kinematics singularity occurs. For the J_q to be rank deficient following condition must occurs

$$\det(J_q) = 0$$

The occurrence of this type of singularity is because the corresponding configurations occur at the boundaries of workspace. These boundaries can also be internal boundaries between the sub regions of workspace where the number of solutions of inverse kinematics problem is not the same. For these cases, the null space of J_q is not empty and there exist a non-zero q vector which corresponds to a Cartesian Twist vector $\dot{X}$which vanishes. In other words, infinitesimal motion of platform along certain directions is restricted. This results in the loss of one or more degree of freedom of the manipulator. Also at this type of singularity the resistance of forces and moments occurs in some direction with zero actuator forces or torques.

Forward Kinematics Singularity

Same as the case of Inverse kinematics singularity, the forward kinematic singularity occur when the matrix J_x is rank deficient. As the matrix$J_x$is an $m \times n$ matrix with $m \geq n$, this occurs when the condition

$$\det(J_x^T J_x) = 0$$

For redundant actuations,$m = n$ the matrix becomes square and the condition is reduced to
$\det(J_x) = 0$

This type of singularity occurs within the workspace of manipulator corresponding to the set of configuration where two different branches of forward kinematic problem meet. For serial manipulator this type of singularity does not occur because of unique solution of forward kinematics. The null space of matrix of $J_x$is not empty showing the existence of non-zero Cartesian twist vectors$\dot{X}$. This means that infinitesimal motion of actuator is possible even when the actuators are locked. When this type of singularity occurs the manipulator gain one or more degree of freedom in the corresponding direction, also the stiffness of manipulator becomes zero.

Combined Kinematics Singularity

The nature of this type of singularity is of a slightly different from the first two. As the name suggest the combined kinematics singularity occurs when both $J_x$and $J_q$becomes zero. This corresponds to degeneracy of orientation or position equation. This type of singularity occurs when the manipulator is under some special constraints on geometric parameter. Since certain architecture leads to this type of singularity that justify the name architecture singularity usually given to them. Such singularities will lead to configurations where a finite motion of the end effector is possible even if the actuators are locked, or in situations where a finite motion of the actuators produces no motion of the end effector. In both cases, the manipulator cannot be controlled.

RESULTS AND DISCUSSIONS

This section discussed the results obtained for inverse kinematics, forward kinematics and stiffness analysis.

Inverse Kinematics Flow Chart

Flow diagram for inverse kinematics is shown in Figure-4. The length of the actuators d_i is calculated from the given inputs p_z, θ and ψ. First the program calculates the p_x, p_y and ϕ from the constraint equations (3), (4) and (5) by computing the rotation matrix. Then the vectors form origin to spherical joint is calculated to find the unit vectors of actuators. The vector $L_i$is then calculated from point A to B. The calculatedunit vectors of actuators and vector L_i used as an input to find the inverse kinematic solution for 3PRS manipulator. Code is written in MATLAB® to find the solution.

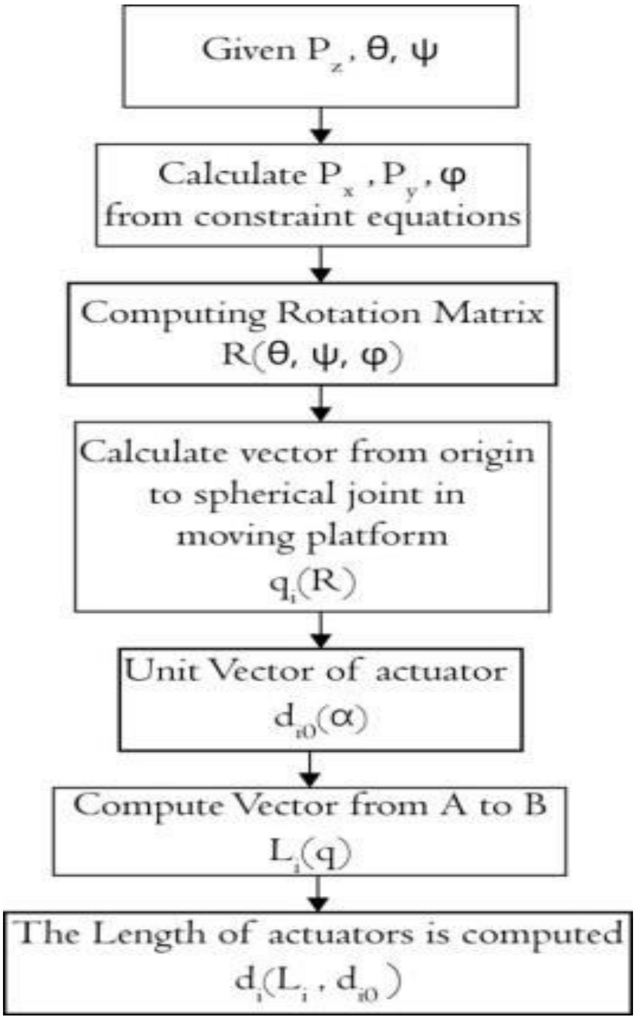

Figure-3. Flow diagram for inverse kinematics of the 3-PRS mechanism.

First the inverse kinematics is calculated from the code writing in MATLAB® then the numerical solutions of inverse kinematics are calculated. sFive configurations are taken randomly from the CAD model and their values are taken to compare the results of computational kinematic. These configurations are shown in Figure-4 and the parameters of the model are presented in Table-1. The values of all configurations are shown in Table-2.

Table-1. Architecture parameters for 3 PRS manipulator.

Parameters	
a	400mm
b	200mm
l	550mm
$\boldsymbol{\alpha}$	30°

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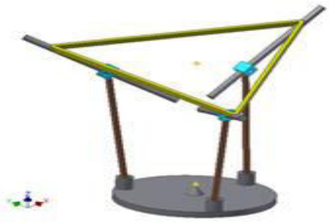
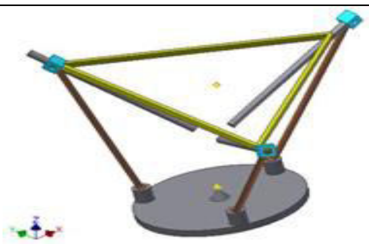
Table-2. Actuated Joint variables and unconstraint variable for 5 configurations from the CAD Model.

Config-uration	d1 (mm)	d2 (mm)	d3 (mm)	p_z (Rad)	Ψ (Rad)	Θ (Rad)
1	80.67	353.5	-5.63	-598.8	-0.75	-0.09
2	242.9	171.5	90.78	-628.3	-0.15	0.22
3	300	150	15	-610.3	-0.33	0.46
4	-89.23	-25.25	-103.69	-444.4	-0.21	-0.065
5	100.5	91.81	-80.56	-521.2	-0.6	0.35


CAD Model of Configuration-1 for 3-PRS manipulator


CAD Model of Configuration-2 for 3-PRS manipulator


CAD Model of Configuration-3 for 3-PRS manipulator


CAD Model of Configuration-4 for 3-PRS manipulator


Model of Configuration-4 for 3-PRS manipulator

Figure-4. CAD Model for 3-PRS manipulator for different configuration presented in Table-2.

The analytical model of 3-PRS manipulator presented is solved by a MATLAB program. The results from the MATLAB program are shown in Table-3. These Results shows small difference between the CAD model and analytical model results. The percentage error between CAD model and analytical Model Results is shown in Table-4.

Table-3. Analytical Model Results for Inverse Kinematic.

Analytical model results (mm)		
d1	**d2**	**d3**
88.77	350.01	-5.68
241.81	169.31	93.04
289.87	154.81	17.31
-88.84	-26.10	-103.38
83.56	105.15	-80.04

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Table-4. Percentage error between CAD model and analytical model results for inverse kinematic.

Inverse Kinematic Results			
% Error			
d1	**d2**	**d3**	**Average**
10.04%	0.99%	0.94%	3.99%
0.43%	1.30%	2.49%	1.41%
3.38%	3.21%	15.40%	7.33%
0.44%	3.37%	0.31%	1.37%
16.85%	14.53%	0.66%	10.68%

Forward Kinematics Numerical Results

To implementation of Newton's method for forward kinematics, let at a certain pose of moving platform of 3PRS parallel manipulator the equation can be represented by the independent variable i-e $f(p_z, \psi, \theta)$. This equation for this function is;

$$f(p_z, \psi, \theta) = d(p_z, \psi, \theta) - d_{given} = 0 \quad (46)$$

where $d(p_z, \psi, \theta)$ are the joint space co-ordinates vector calculated from inverse kinematic analytical solution and d_{given} is known joint space co-ordinate which can be directly measured. Let the set of independent variables (p_z, ψ, θ) be denoted by x.

The tolerance criteria used to end the iterative process is when the maximum absolute value of $\left[d(x^k) - d_{given}\right]$ is less than a specified tolerance. For the initial guess, it is necessary to select a set of values that are as close as possible to the actual pose of the moving platform, since there are multiple forward kinematics solutions. In practice, an initial guess can be chosen as the desired pose calculated from the analytical forward position kinematics analysis or the measured pose of the moving platform at the initial configuration.

Flow chart of Newton's method is given in Figure-5. The given flow chart has been implemented in MATLAB. The known joint space co-ordinate and initial guessed joint space co-ordinate are given to program as input.

The Jacobian is calculated from initial guess. The new joint space co-ordinate is calculated from the Newton's method for forward kinematics. The code then calculates the tolerance criteria and check the given tolerance if the calculated tolerance is less than the given tolerance than the code updates the initial guessed value by replacing it with new calculated co-ordinates. The above presented technique with the initial guess for CAD model, 5 configurations are solved the results deviates a little form desired position. The results with initial guess are shown in Table-5. Percentage Error between Numerical model results and CAD model of unconstrained Variables is shown in Table-6.

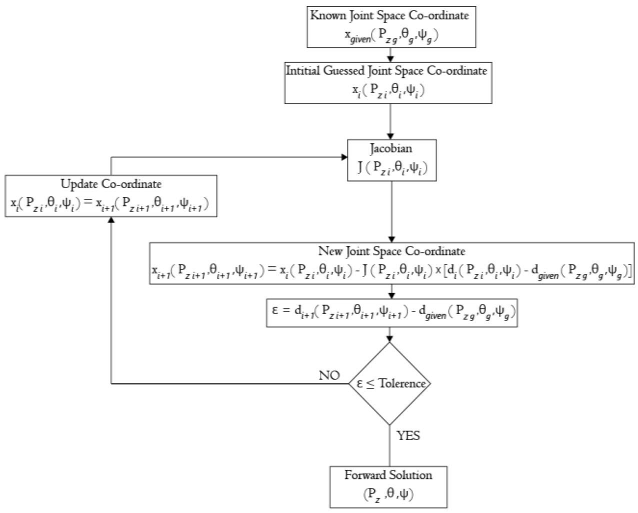

Figure-5. Flow diagram for forward kinematic of 3-PRS parallel manipulator.

Table-5. Comparison of the numerical model results.

Initial guess			Numerical model results			
pz	**ψ**	**Θ**	**Iterations**	**pz**	**ψ**	**Θ**
-600	-0.68	-0.08	23	-598.80	-0.75	-0.089
-626	-0.15	1.21	93	-628.30	-0.15	1.23
-615	-0.4	0.5	17	-610.30	-0.33	0.460
-440	-0.2	-0.06	6	-444.40	-0.21	-0.07
-520	0.55	-0.32	14	-521.20	0.60	-0.350

Table-6. Comparison of the numerical model results

Forward Kinematic Results			
% Error			
pz	ψ	ϴ	Average
0.008%	0.027%	0.224%	0.086%
0.005%	0.273%	466.820%	155.699%
0.006%	0.000%	0.000%	0.002%
0.004%	0.000%	0.000%	0.001%
0.004%	0.000%	0.000%	0.001%

If configuration-4 is taken, in which the convergence is reached after six iterations. The iteration results are shown in Table-7.

Table-7. Iteration results of using Newton method to solved Forward kinematic of 3-PRS manipulator.

Iteration values for Configuration-4			
Iterations	pz	ψ	ϴ
0	-440	-0.2	-0.06
1	-444.56	-0.2126278	-0.0653
2	-444.39	-0.2119722	-0.065
3	-444.4	-0.2120009	-0.065
4	-444.4	-0.2119999	-0.065
5	-444.4	-0.212	-0.065
6	-444.4	-0.212	-0.065

Singularity Analysis

The importance of knowledge of singularities is that it helps in path planning and geometric design of manipulator workspace, which is free from singularities at designing stage. The investigation of singularity analysis of parallel robots gives an insight on the better design and control of manipulator.

For 3-PRS parallel manipulator, singularity analysis is based on the instantaneous kinematics, which is described by equation (47)

$$J_x \dot{X} = J_q \dot{d} \tag{47}$$

Where $\dot{X}$ is the vector of output moving platform and $\dot{d}$ is input accuator joint rates. The three types of singularities are discussed below.

Inverse Kinematics Singularity

The inverse kinematics singularity occur when J_q is non invertible i-e $det(J_q) = 0$. So, when this condition is satisfied then

$$\det(J_q) = \begin{vmatrix} I_{10}.d_{10} & 0 & 0 \\ 0 & I_{20}.d_{20} & 0 \\ 0 & 0 & I_{30}.d_{30} \end{vmatrix} = 0 \qquad (47)$$

This physical interpretation of this equation is that one or more of the leg is perpendicular to their corresponding actuator directions. This causes the manipulator to lose one or more degree of freedom depending upon the number of legs perpendicular to corresponding actuator. This type of singularity is shown in figure6 by the CAD model.

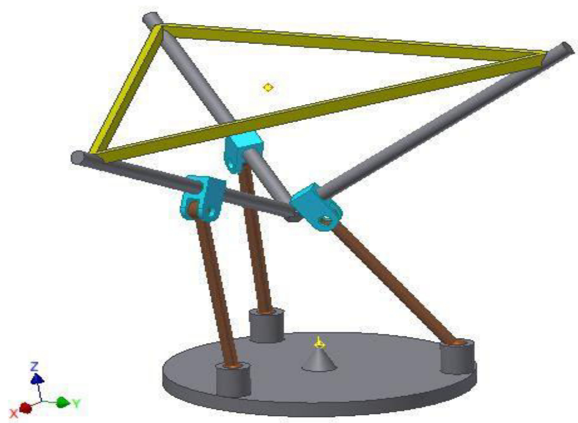

Figure-6. Inverse kinematic singularity of 3-PRS manipulator.

Direct Kinematic Singularity

Now this type of singularity arises when J_x is not invertible. Since, matrix J_x is not a square matrix, this type of singularity will occur when matrix is not of full rank, i-e, its rank is equal to 2 or 1. As J_x is given by.

$$J_x = \begin{bmatrix} I_{10}^T & (b_1 \times I_{10})^T \\ I_{20}^T & (b_2 \times I_{20})^T \\ I_{30}^T & (b_3 \times I_{30})^T \end{bmatrix}_{3\times 6}$$

We can see that $b_i \times I_{i0}$ represent a normal vector n_i, which is perpendicular to the plane containing points P, C_i, B_i therefore the three vectors of n_i are parallel to fixed base platform.

ButJ_x is not rank deficient even if n_i are linearly dependent or only one of three vector have zero components. If two or all the three vectors have zero components,J_x will become singular.

$$b_i \times I_{i0} = 0 \tag{48}$$

This equation physically shows that links are aligned with the moving platform, with the external lines C_iB_i passing through the center point of moving platform denoted by P. This singularity causes the platform to gain one or more degree of freedom even when there is a locking of all actuators. The forward kinematic singularity is shown in Figure-5.8 using a CAD model.

Combined Singularity

As the name suggest, this type of singularity will occurs if both of the above singularity condition is satisfied i-e J_q is not invertible and J_x is not full rank. For this type of singularity to occur the manipulator must have special kinematic architecture.

CONCLUSIONS

The kinematic and singularity analysis of 3PRS manipulator is performed in this paper. In kinematic analysis both inverse and forward kinematic analysis are investigated by analytical method. Since the analytical solution for forward kinematic is very complicated and time consuming so another better and time-saving numerical approach can be implemented. This approach has a better match of result with the virtual prototype CAD model measurements. Singularity analysis of 3PRS mechanism is performed and implemented in the physical CAD model. The better geometric design can be achieved by the knowledge of the type of the singularity.

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NOMENCLATURE

CNC	Computerized Numerical Control
SPS	Spherical Prismatic Spherical
RRR	Revolute Revolute Revolute
RPS	Revolute Prismatic Spherical
PSP	Prismatic Spherical Prismatic
CAD	Computer Aided Design
FEA	Finite Element Analysis
PKM	Parallel Kinematic Manipulator
PRS	Prismatic Revolute Spherical
PRPR	Prismatic Revolute Prismatic Revolute
PUU	Prismatic Universal Universal
UPU	Universal Prismatic universal