VOL. 17, NO. 8, APRIL 2022 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences


www.arpnjournals.com

# HYBRID MIXED BASIS VECTOR BASED DIRECTION OF ARRIVAL ESTIMATION USING SPARSE BAYESIAN LEARNING ALGORITHM

Mamatha M C and H C Sateesh Kumar
Department of ECE, Sapthagiri College of Engineering, Bengaluru (Affiliated to Visvesvaraya Technological University, Belagavi) India
E-Mail: mamatharesearch2021@gmail.com

## ABSTRACT

Antenna with Non-uniform linear array is sophisticated for the reconfigurable antenna setup. This paper deals with the DOA estimation of signals with non-uniform linear array setup adopting the sparse representation based implementation. A Mixed basis vector based sparse representation is adopted in this paper to predict the DOA estimation. MATLAB based implementation is carried out for the DOA estimation for non-uniform linear array setup and estimation accuracy is analyzed for the proposed work. Estimation accuracy is found from different waveforms generated from the results which involve graphs like Signal to Noise Ratio versus Mean Square Error. The results thus obtained are compared with the traditional methods to validate the performance of the results obtained from the proposed method.

**Keywords:** mixed basis vector, direction of arrival, sparse representation, non-uniform linear array.

## INTRODUCTION

The concept of smart antennas inspired by the fact that it can automatically detect the signal direction from the source that falls on the sensor which is usually the Uniform Linear Array (ULA) of antennas. The signal direction also known as Direction of Arrival (DOA) estimation without miscalculating the interference as the signal of interest, and detecting the DOA for the noise and other correlation issues in finding the signal direction. The accurate DOA estimation and novel methods to obtain this higher efficient DOA estimation is an important milestone to be obtained after continuous research on this topic. The signal processing algorithms decide the performance efficiency of the smart antenna in DOA estimation methods. Sparse representation is a new stream of signal processing algorithm that gives accurate DOA estimation implementation in the antenna arrays.

Important term meant for the DOA estimation is the target sparsity which is the ability of the algorithm to approximate the source using the basis vectors. This is also called as basis pursuit. The implementation involves creation of the manifold matrix which is the combination of the basis vector that would create the dictionary of signals that get represented with signal under study for it can be able to find the direction of the signal falling on the antenna.

The manifold matrix also known as the sparse matrix is the main component of the sparse representation implementation since the way this matrix varies depends how better the basis pursuit can occur. The following details would discuss about the basics of sparse representation and its applications in various fields of engineering. Representation of the higher dimensional data into effective and compressed way is a challenge which is a fast-growing research area in signal processing paradigm. The term for the above said representation is called Spare Representation that combines the elementary components called as atoms to develop the signal models that are chosen from the dictionary. The dictionary is the set of all such combinations of vectors. Source Separation, signal denoising, compressed sensing and signal recovery are few applications where Sparse Representation implementation is carried out. A deeper study of finding the over complete dictionaries and the best ‘bases’ for sparse representation is being investigated to adapt it to the signal under study. Compressed Sensing is one such application that got revolutionized due to the sparse representation technique. The sub sampling at very low rates that is evident in compressed sensing is advantageous to the conventional sampling method. The random projections that depict the dimensionality reduction on the signals conveys the sparsity of the algorithm. The assumptions that are meant for sparsity, if properly designed, is enough for the recovery of the signal with lesser error. Both sampling and compression comprises of the compressed sensing implementation. The fullest potential of this compressed sensing algorithm is not yet reached considering the amount of research work carried out with Sparse Representation. Although DOA estimation using sparse representation is researched by many researchers in the recent literatures publications the complete sparsity level is not utilized. Sparse representation is applied in applications including source separation in audio processing applications and other learning-based applications.

## LITERATURE SURVEY

Orthogonal projection of the steering vector on both noise and the source subspace attributes to the knowledge that the noise and the source subspace are orthogonal components. This orthogonal concept is used in MUSIC algorithm that infers that the noise subspace does not contribute to the DOA estimation. The literature involves a novel noise subspace called the proper noise subspace for the MUSIC based DOA estimation algorithm. To attain it the oblique projector is used. The oblique projector projects the steering vector on the proper noise subspace along the signal subspace unlike the traditional MUSIC algorithm. The efficiency of the algorithm is found to be satisfactory for the DOA estimation [1].

The ability of the MIMO radar system to provide higher spatial resolution can be exploited to obtain a better performance for target detection. The ability to estimate the DOA estimation parameter is improved significantly in the MIMO radar paradigm in the literature. Impulsive noise is introduced in the implementation which incorporates robust covariation (ROC). Normalisation using the fractional lower order moment (FLOM), fractional lower order cyclic (FLOC) is adopted. The narrow band mono static MIMO radar signals both at transmitted and received arrays is close to each other. A modified MUSIC algorithm that uses the Eigen structure for DOA estimation is carried out. Among the four methods involved in DOA estimation the concept of similar normalization performed better. The computational complexity of the algorithm in the literature needs improvement in the computational complexity [2].

L shaped antenna array are converted to two sub arrays and two 1D DOA estimation is carried out to obtain the DOA estimation in traditional estimation methods. A pair matching approach is developed in the literature that finds the matched 1D-DOAs from the covariance matrices obtained. Objective function that is defined using the two source covariance matrix is used to obtain the optimal pair matching by permutating matrix. This pair matching algorithm showed satisfactory results for all scenarios including deficient snapshot, coherent and uncorrelated signal, signals with lesser SNR. The obtained angle pairs from the cost function minimization exploit the signal covariance matrices [3].

The multisensory array setup with non-stationary signals is sampled sparsely in the literature. Time-frequency signal representation (TFSR) using Sparse representation is used for DOA estimation. The kernelled processing in the sparse representation implementation makes the signal DOA estimation possible since traditional time frequency analysis creates artifacts due to data that are missing. This is a group sparse reconstruction problem since multiple signals are denoted in the same time frequency region. The spatial time frequency distribution (STFD) matrix formed using both the auto and cross sensor TFSR to obtain the advanced MUSIC algorithm. DOA estimation from the STF MUSIC algorithm exhibited good discrimination capability [4].

The literature incorporates discrete Fourier transform (DFT) - (ESPRIT) for TOA and DOA estimation. The computational complexity involved in the MUSIC and ESPRIT algorithm in vehicle frequency modulated continuous-wave (FMCW) radars reckons an alternate algorithm that can detect TOA and DOA in real time. The location tasks that is involved in finding the targets from the vehicle radar needs high speed detection algorithms. The DFT-ESPRIT algorithm for TOA and DOA estimation with multitarget environment is compared with the Monte-carlo simulation with AWGN channels [5].

Advanced Signal ratio estimate from radar with unknown input signal SNR is carried out in this literature. The two unresolved targets detection is carried out using the maximum likelihood angle extractor for DOA estimation. The simulation implementation of the method is implemented using the prior information on SNR [6].

Space Alternating Generalized Expectation (SAGE) Maximization algorithms for DOA and Angular Spread (AS) is utilized in the literature. The angular spread of the wave which is incident due to scattering, reflection and diffraction needs to be considered while estimating DOA. DOA Matrix method and SAGE method id combined to obtain the DOA estimation and the AS respectively. The estimation performance is improved using the Stochastic Local Search (SLS) algorithms [7].

DOAs obtained from the sparse recovery implementation does not need the sampling grid. Instead a basis is required which is a off grid problem. But the issue faced in the off grid estimation is that it develops a large basis. A sparse matrix or a offset matrix is developed by remodeling the basis. The DOA estimation from the offset matrix which compensates the off grid problem is developed in the literature. This method shows better effectiveness in DOA estimation since the in spatial domain each spectrum of the input signal is sparse by itself. The joint estimation of both sparse matrix and the DOA is found to be performing efficiently [8].

The spatial observation matrix incorporated in the sparse representation based DOA estimation algorithms are larger while larger array scale and wide angular range is used. This increases the computational complexity in the sparse representation based implementation. Literature introduces the separable sparse representation (SSR) based DOA estimation to reduce the computational complexity. SSR (Separable sparse representation) DOA algorithm is said to be working efficiently for the high resolution implementation [9].

Exploiting the sparsity of the signals in spatial domain DOA estimation a novel method is used. The method works while the dynamics of sensor's gain and uncertainties in phase of the signal is considered. One existing finding on uncertainty matrix is it is a sparse matrix since it only is diagonal. Phase and gain uncertainties are estimated by using the sparse property of the uncertainty matrix. Iterative process with one step estimating the DOA and the second step estimating the gain and phase uncertainties is applied and find that the effectiveness to be satisfactory.

Although the algorithm is off grid in nature and is able to estimate DOA by solving the two step iterative process, improvement of robustness is still a challenge [10].

Antenna arrays arranged in circular or concentric circular arrangement is considered for DOA estimation with MUSIC algorithm in the literature considered. The probabilities of occurrence in spatial spectrum false peaks in various geometries of the array and with different noise environments are simulated. It is observed that greater the aperture lesser the probability of false peak occurrence [11].

A real valued signal that is received is split into real and imaginary array and are combined to form a sampling vector with twice the number of elements. The extended dimension MUSIC(ED-MUSIC) algorithm developed in the literature improved the estimation accuracy. The algorithm is capable of estimating DOA for 2M-1 sources if M array elements are used. ED-MUSIC algorithm is equivalent to applying MUSIC with double the number of elements and also the Degree of Freedom [12].

AS of the incoming signal due to the scattering, diffraction and reflection of the signal needs to be considered while DOA estimation is carried out. The linear array for DOA estimation is converted to planar array which combine both the AS and DOA. Estimation errors due to the overlapping error needs to be removed by implementing SLS algorithm [13].

Literature applies DOA on the Phase Monopulse antenna used for radar communication with RF multiplier integrated planar topology. DOA estimation is carried out using the ratio of sum and difference of the signals from two antennas. Unlike the traditional methods where half angle space can be estimated, the proposed method increases the range of estimation employing RF multiplier [14].

Decoupled matrix for DOA estimation using the Rank-reduction (RARE) methods reduces its performance as the number of elements starts reducing in the UCA. The approach that revises the beam space data while rank reduction implementation using the MUSIC algorithm in the Uniform Circular Array (UCA) is developed in the literature. The approach estimates with better performance for both equal and unequal power signals with both the directional antennas and omni-directional antennas in the UCAs [15].

DOA estimation in Unmanned Aerial Vehicle (UAV) by generating a suitable steering vector is developed in the literature. Estimation algorithm is approached while the input signals experience equal power and uncorrelated and with different angles. First the steering vector suitable for UAV is developed and theoretical and practical DOA estimation while mutual coupling occurs is incorporated. Results obtained from the simulation is satisfactory by developing a mutual coupling matrix (MCM) to estimate DOA [16].

Modified ESPRIT algorithm that exploits the Multi Invariance (MI) property of the signal is proposed for the DOA estimation of the signal. Received signal is represented as the time-frequency data model which facilitates the MI property from the signal. Frequency and temporal variations from the ULA sub arrays are extracted to obtain the spatial distribution matrix. MI-ESPRIT algorithm thus developed from the chirp signal input estimates the DOA with different SNR and snapshot numbers. As a future improvement in the algorithm fast algorithms are applied that reduces the computational burden [17].

Support Vector Regression (SVR) based DOA estimation is simulated on signals with tailed noise modeled using the Laplacian Distribution in the literature. SVR based approach outperformed both MUSIC and ESPRIT based algorithm with improved estimation accuracy and better computational time [18].

Literature introduces the Continuous Compressed Sensing based combined estimation of Doppler frequency and DOA. This method adopts the atomic norm minimization technique to generate the sparse virtual aperture along with the spatial-temporal co-array to recover the signals. The sparse recovery from the coprime arrays and the coprime samplers which recover the full virtual array aperture is utilized. The virtual aperture is fully recovered without discretization which indirectly increases the computation due to dense grid. Thus a search free method is adopted with unitary ESPRIT for the joint estimation in the literature [19].

Multi Task -Bayesian Compressive Sensing (MT-BCS) is utilized to find the DOA for multiple signals received on the linear array. Spectral correlation from the signal acquired with different frequency helps in estimating signal bandwidths. Bayesian Compressed Sensing method proposed in the literature has given better results in the estimation accuracy point of view [20].

DOA estimation introduces new challenges while implemented on the ULA while end-fire direction targets are to be estimated and underwater target detection is carried out. To engage the challenge, the literature introduces the A-shaped antenna array for the wideband signal. Minimum Variance Distortionless Response (MVDR) based algorithm is developed using two ULAs which is distributed with specific angle. Spatial Spectrum for the sub bands are obtained from the two sub arrays. Location of spectral peak is followed by the spatial spectrum development. The A-shaped array thus developed outperforms the traditional ULAs in efficient DOA estimation. Lower beam width, robustness to lower SNR are few advantages of adopting the A shaped array [21].

DOA and Direction of Departure (DOD) estimation for the non-stationary signals is achieved using ambiguity function based algorithm in the literature for the radar signals. Joint DOA and DOD estimation starts with generation of the spatial time-frequency distribution (STFD) matrix. Multidimensional spectrum peak searching algorithm is avoided by using the ESPRIT-Root MUSIC algorithm. Extraction of the auto term and nullifying the noise term is efficient in the ambiguity algorithm compared to the pseudo Wigner-Ville distribution [22].

Wave velocity variation in the received signal affects the efficiency of the DOA estimation. A velocity independent DOA estimation is carried out using the modified MUSIC algorithm with L-shaped antenna array. The imprecise wave velocity issue is removed by using the L shaped antenna array [23].

Algorithm proposed in the literature reduces the two dimensional search space to the single dimensional search the reduced dimension MUSIC algorithm is introduced for DOA estimation that reduces the computational cost to larger extent. Although the computational cost is less the performance of the algorithm is near to the 2D MUSIC algorithms [24].

Spatial Smoothing technique is used to adopt the MUSIC algorithm for coherent signal DOA estimation in linear arrays. Smoothed signal after decorrelation is applied with MUSIC algorithm. The comparison with the MUSIC and ESPRIT algorithm found that ESPRIT algorithm required lesser computational time [25].

Joint DOD and DOA estimation is carried out using the MIMO MUSIC algorithm in the referred publication. Gibbs sampling is applied on the signal to facilitate joint DOA DOD estimation. Markov Monte Carlo algorithm is combined with the MIMO MUSIC algorithm to estimate the DOA. Although the joint DOA DOD estimation provided the same DOA estimation as the MIMO MUSIC, the computational complexity is low [26].

Underwater acoustic sources will suffer from low SNR ratio, and lesser data snapshot. The algorithm involved created a unit circle which is mapped with the response beamformer formed by minimum variance distortion within the selected angular sector. This circularly mapped beamformer is then calculated for pseudo spatial spectrum. Even with lower SNR and few number of snapshots the algorithm delivered sharper DOA estimation in the long-distance underwater acoustic sources environment [27].

Both DOA and polarization is tracked by the Electromagnetic (EM) vector sensor arrays. The capability of crossed-dipole linear arrays, to track only two polarization parameters and one DOA parameter reckons newer algorithms. The literature adopts a algorithm where dimensionality reduction is applied after reconfiguring This problem could be solved by extending the geometry to a two dimensional (2-D) rectangular array so that both the azimuth and elevation angles figuring the crossed dipole linear arrays to a linear tri pole array. Instead of the 4D estimation problem it is converter to two 2-D estimation problem. Maintaining the effectiveness of estimation, computational complexity is also reduced [28].

In case more sources than the array elements need to be adopted in the implementation, a subspace based DOA estimation is introduced in the literature. Coprime array is developed by generating the Toeplitz matrix which does not need the number of sources for spatial spectrum calculation. The problem of computational cost needs to be solved even though the accuracy is found to be satisfactory [29]

DOA estimation for coherent signals with MUSIC algorithm developed in the literature. An antenna array is adopted and reconfigured into two non-coherent sub arrays. The DOA estimation with this configuration of sub array failed for two coherent signals. Thus the 8 element ULA (two 4-element sub arrays) based DOA estimation is used to solve the incoherency problem. Computational complexity is higher [30].

DOA estimation is considered as a Basis Pursuit De-Noisisng (BPDN) problem in the literature. Interior Point method (IPM) usually adopted in the BPDN method is replaced with the Alternating Direction Method of Multipliers (ADMM). The purpose is to eliminate the higher computational complexity that is inherent in the IPM method. Considering the real time implementation of the DOA, ADMM is introduced that splits the iterative procedure to smaller pieces, thus reducing the computational complexity [31].

This paper adopts the Mixedbasis vector based DOA estimation and results are detailed.

Since the research is based on Basis Pursuit DeNoising (BPDN) based sparse representation DOA estimation methods an introduction about this method is introduced. The advancement in the existing DOA estimation algorithm in Sparse representation paradigm is applied and compared with the traditional method discussed in the previous chapter. The BPDN based DOA estimation algorithm thus developed in the existing literature is advanced with the Hybrid basis vector and Mixed Basis vector based implementation.

**Signal Model**

Omnidirectional antennas with M elements are placed in a non-uniform linear array which is located at different distances $[0,d_1,\ldots d_{M-1}]$ , which denotes distance between the reference location and different antennas. This distance is the integral multiples of half the wavelength. Improvement of convergence in any sparse representation problem is improved by increasing the Degree of Freedom (DOF). DOF considered in the omnidirectional antenna array is the difference co-array as discussed in [30]. The difference co-array is defined as

$$\Omega = \{d_{m1} - d_{m2}\}_{m_1=0,1,\ldots\ldots M-1;m_2=0,1,\ldots.M-1}$$

For M antennas $\Omega$ provides more DOFs.Considering that N far-filed sources uncorrelated in nature is falling on M antennas. The narrow band sources is defined by $S_n(t), k = 1,2,\ldots,N$ which impinges on antenna arrays. The proposed implementation calculates the DOA estimation with spatially white Gaussian noises as the channel for all the M antennas denoted by $n_m(t), m = 0,1,\ldots.,M-1$. T snapshots of the signal with noise is defined as,

$$x(t)=As(t)+n(t) \quad (1)$$

Array received vector **x**(t), signal from the transmitting source **s**(t) and the noise in the channel **n**(t) for the $t^{th}$snapshot is denoted in equation (1). The steering vectors of all the N sources are consolidated in the manifold matrix A.

$A=[a(\theta_1), a(\theta_2),\cdot] \cdots, a(\theta_N)]$

Where the steering vector $a(\theta_n)$, n=1,2,⋯,N, corresponding to the $n^{th}$ incident signal is defined as $a(\theta_n) = [1, v(d_{1,}\theta_n),\cdots, v(d_{M-1,}\theta_n)]^T$, phase component $v(d,\theta)$ is defined as $v(d_m, \theta) = exp[-j2\pi\left(\frac{d_m}{\lambda}\right)sin\theta]$, and $\{\cdot\}^T$ denotes the transpose. It is considered that the signal and the noise are uncorrelated and thus the covariance matrix is formulated as defined in equation (2).

$$R_x=E\{x(t)x^H(t)\}=A\mathrm{diag}(\sigma_1^2, \sigma_2^2,\cdots, \sigma_N^2)A^H+ \sigma_n^2 I_M, \quad (2)$$

The uncorrelation between the source and the noise is denoted in equation (2) by introducing multiple variances $\sigma_1^2$, $\sigma_2^2$,….,$\sigma_N^2$ corresponding to N sources. Expectation E{·} for the component $x(t)x^H(t)$ defines the covariance matrix. The identity matrix $I_M$ with size M X M. Vectorizing the equation (2) as described in [91] creates the virtual array from the covariance matrix. The vectorization involves Khatri Roa (KR) product in the equation (3).

$$Y=vec(R_x)=vec(AR_sA^H)+ \sigma_n^2 vec(I)=(A^* \circledast A)g+ \sigma_n^2 I_M \quad (3)$$

In equation (3) KR product ⊛, conjugate transpose$\{\cdot\}^H$ $g=[\sigma_1^2, \sigma_2^2,\cdots, \sigma_N^2]^T$ denotes source variance vector, $1_m=[e_1^1,e_2^T,\cdots,e_M^T]^T$ with $e_m$vector being zeros excluding the $m^{th}$ entry which is 1. $\underline{A}=(A^* \circledast A)$, is the virtual array manifold matrix. The virtual manifold matrix $\underline{A}$ with N virtual steering vectors $\underline{a}(\theta_n) = a^*(\theta_n)\otimes a(\theta_n)$, n=1,2,⋯,N, where ⊗ indicates the Kronecker product. Distinct entries of $a^*(\theta)\otimes a(\theta)$ increases the DOF of the DOA estimation problem. The provided data is the sample covariance matrix while in reality defined as $\hat{R}_x = \sum_{t=1}^{T} X(t)X^H(t)/T$. As the incident signals are defined as circularly symmetric Gaussian distribution a asymptotic complex gaussion distribution results as the residual error of covariance matrix as defined in [92]. The residual error is defined in equation (4),

$$\hat{Y} - Y = vec(\hat{R}_x) - vec(R_x) \sim CN\left(0, \frac{1}{T} R_x^T \otimes R_x\right). \quad (4)$$

Let $\tilde{R}_x = R_x^T \otimes R_x/T$, and by using(3) equation (4) is transformed to be

$$\hat{Y} \sim CN(\underline{A}g + \sigma_n^2 1_M, \tilde{R}_x) \quad (5)$$

Equation (5) defines the BPDN formulation for DOA estimation. The sparse solution space is identified in the sample grid represented as $\Theta = \{\theta_1, \theta_2, \cdots, \theta_N\}$. The sample grid or the sparse solutions space spans range of all possible incident directions of the signal. Thus the equation (5) is converted to the following equation,

$$\hat{Y} \sim CN(\Phi w + \sigma_n^2 1_M, \tilde{R}_x) \quad (6)$$

Matrix $\Phi$ acts as the overcomplete dictionary of the DOA estimation problem. All the direction in the grid $\Theta$ is utilized in $\Phi$ and acts as the basis vector for matrix$\underline{A}$. The non negative sparse matrix w contains ones where actual DOA is present and zeros in all other positions. $N$being the non-negative Gaussian distribution defined in [93].

## The Basis Vector

Since the components of the $\Phi$ matrix [please explain some example matrix ]is similar to the basis vector as discussed in the term basis vector is introduced in the research and also the variations in the reference basis vector is created in the Sparse Bayesian learning technique discussed .The performance evaluation while applying DOA estimation using the NNSBL methods with Mixed basisvector and while multiple basis vectors are used is carried out .

Similar to the eigen decomposition utilized in the traditional algorithms a symmetric positive definite basis vector can act as the infinite Eigen function. The basis vector is defined as $k(x,x') = \sum_{i=1}^{\infty} \lambda_i e_i(x) e_i(x')$ with $< e_{i(x)}, e_i(x) >_{Hx} = \delta_{i-i'}$, where $\delta_i$ denotes the kronecker's delta using the weighted Eigen functions $\phi_i(x) := \sqrt{\lambda_i} e_i(x)$, $i \epsilon R^\infty$ such that $\phi_i = \phi_i(x), i \epsilon N$. The mapping of the basis vector in the search space in $R^\infty$, since for two points $x, x' \epsilon X$,

$$k(x,x') = \sum_{i=1}^{\infty} \phi_i(x)\phi_i(x') := \phi^T(x)\phi(x'). \quad (7)$$

This inner product interpolation is the basis for the basis vector.

## SPARSE BAYESIAN MODELLING

In non-negative Sparse Bayesian Learning (NNSBL) discussed in w needs to be considered as nonnegative. Due to that consideration even the BPDN problem is considered as the real valued problem incorporating the positive source variance. It is discussed in that if the incident signals follow a circular-symmetric Gaussian pattern the positive source variance is converted to Gaussian distribution with real values. Thus equation (6) is rewritten as equation (8).

$$P(\hat{y}|w, \sigma_n^2) = N(\tilde{\Phi}w, \underline{R}), \quad (8)$$

Where $\tilde{y}(\sigma_n^2) = [Re(\hat{y}^T) - \sigma_n^2 \cdot 1_M^T, Im(\hat{y}^T)]^T$, $\tilde{\Phi} = [Re(\Phi)^T, Im(\Phi)^T]^T$ and
$$\underline{R} = \frac{1}{2}[Re(\tilde{R}_x) - Im(\tilde{R}_x); Im(\tilde{R}_x) Re(\tilde{R}_x)]$$

Traditional SBL uses $l_1$-norm as the objective for sparse learning problem. NNSBL uses the Laplacian prior distribution in place of $l_1$-norm.The prior distribution is defined in equation (9)

$$P(w|\lambda) = \frac{\lambda^N}{2^N} exp(-\lambda \parallel w \parallel_1) \quad (9)$$

Equation (9) is rewritten considering that w is a nonnegative vector and given in (10).

$$p(\lambda) = \lambda^N exp(-\lambda \textstyle\sum_{i=1}^{N} w_i), w_i \geq 0, \text{ i=1,2,}\cdots\text{,N} \quad (10)$$

Bayesian framework starts with the prior distribution. The solutions for the sparse problem starts with this prior and develops a posterior distribution. As discussed in if the prior distribution that is defined in equation (10) does not appear to be conjugate of the conditional distribution of the observed data the hierarchical nonnegative Laplace prior is developed. The hierarchical prior model with first stage is as given in the equation (11).

VOL. 17, NO. 8, APRIL 2022 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences



$p(\gamma) = \prod_{i=1}^{N} N_+(0,\gamma),$ (11)

Gaussian Probability Density Function (PDF) with zero mean is defined as $N_+(0,\gamma) = 2\,N(0,\gamma)$, $w_i \geq 0$ .

Allowing the hyper parameter $\gamma_i$ in equation (11) to be the one which creates the sparsity in the solution is the idea in SBL. A hyperprior is built such that the w is ensured with a heavy tailed distribution in the equation (12).

$$p(\lambda) = \prod_{i=1}^{N} \frac{\lambda}{2}\, exp\, exp\left(-\frac{\lambda\gamma_i}{2}\right)\gamma_i \geq 0, \lambda \geq 0, i = 1,2,\dots.,N \quad (12)$$

A generalized inverse Gaussian (GIG) PDF function with its integrable property and modified second kind Bessel Function $K_{1/2}(z) = \sqrt{\pi/2e^{-z}}z^{-1/2}$,z>0, the prior function is indicated as follows:

$$p(\lambda) = \int p(\gamma)p(\lambda)d\lambda = \prod_{i=1}^{N}\int_0^\infty 2(2\pi\gamma_i)^{-1/2}\, e^{-w_i^2/(2\gamma_i)}\,.\,(\lambda/2)\;\; e^{-\lambda\gamma_i/2}\, d\gamma_i = \lambda^{N/2}\, exp\, exp\left(-\lambda^{N/2}\sum_{i=1}^{N} w_i\right), w_i \geq 0,1,2,\dots,N, \quad (13)$$

The marginal prior of the w is defined as a laplace distribution while the hyper prior for the hyper parameter $\lambda$ is given as a gamma Distribution.

$$P(\lambda; v) = \Gamma(\lambda|v,v) \quad (14)$$

Definition of the Gamma Probability Distribution Function is given as $\Gamma(a,b) = b^a\lambda^{a-1}\, exp\, exp(-b\lambda)/\Gamma(a)$. Where V is the hyperparameter V which defines the set of constant values v 0, called as the Jeffrey's hyper prior. Another prior for the variance value $\sigma_n^2$, is considered as a noninformative distribution to complete the Bayesian Model.

$$p(\sigma_n^2) \propto 1, \quad \sigma_n^2 > 0 \quad (15)$$

All the distributions defined for different variables and hyper parameters are combined to obtain a joint PDF to form the Bayesian model combining equation (7),(10),(11),(13) and (14) as defined in equation (16).

$$\mathrm{p}(\mathrm{w},\gamma,\lambda,\sigma_\mathrm{n}^2,\hat{y}) = \mathrm{p}(\mathrm{w},\sigma_\mathrm{n}^2)\mathrm{P}(\mathrm{w}|\gamma)\mathrm{p}(\gamma|\lambda)\mathrm{p}(\lambda)\mathrm{p}(\sigma_\mathrm{n}^2). \quad (16)$$

With the developed Bayesian model the Bayesian inference and the solutions are obtained to estimate the DOA of the given signal.

## BAYESIAN INFERENCE

Once the Bayesian model is ready with all the priors combined the posterior has to be obtained in order to infer from the signal. The posterior PDF for the Bayesian model is defined in (16) is as defined in equation (17).

$$p(\hat{y}) = p(w,\gamma,\lambda,\sigma_n^2,\hat{y})/p(\hat{y}) \quad (17)$$

The equation (17) has the denominator that can't be calculated analytically a approximation is applied to find the solution.

Thus a Expectation Maximization algorithm is adopted to find the solution. Variable w is unknown and the objective is to maximize the expectation defined $E\{log\, logp(w,\gamma,\lambda,\sigma_n^2,\hat{y})\}$ with variation in the posterior of w. It is found that the posterior distribution of w is nonnegative in nature by combining (8) and (11).

$$p(\hat{y},\gamma,\sigma_i^2) = N_+(\mu,\Sigma) = \frac{N(w|\mu,\Sigma)}{\int_{w\succcurlyeq 0} N(w|\mu,\Sigma)dw}\, w \succcurlyeq 0 \quad (18)$$

with parameters

$$\mu = \Sigma\tilde{\Phi}^T\underline{R}^{-1}\tilde{y}(\sigma_n^2), \Sigma\; = \left(\tilde{\Phi}^T\underline{R}^{-1}\tilde{\Phi} + \Gamma^{-1}\right)^{-1}, \quad (19)$$

Element wise greater than or equal is denoted as $\succcurlyeq$ and $\Gamma = diag\{\gamma_1,\gamma_2,\cdots,\gamma_N\}$

The terms that are independent to $\gamma_i$,i=1,2,⋯,N is ignored to update the value of hyperparameter $\gamma_i$. The complete posterior prior in$log\, logp(w,\gamma,\lambda,\sigma_n^2,\hat{y})$ will ignore $\gamma_i$ and thus the maximizing problem is converted to $E_{p(w|\hat{y},\gamma,\sigma_n^2)}\{log\, logp(\gamma) + log\, logp(\lambda)\}$, which leads to

$$\gamma_i = -1/2(2\lambda) + \sqrt{1/(4\lambda\text{^}2) + \langle w_i^2\rangle/\lambda}, \quad (20)$$

Where $\langle w_i^2\rangle = E_{p(w|\hat{y},\gamma,\sigma_n^2)}\{w_i^2\}$ denotes the second moment of $w_i$by$P(w|\hat{y},\gamma,\sigma_n^2)$

The hyperparameter $\lambda$ update is obtained by maximizing the equation $E_{p(w|\hat{y},\gamma,\sigma_n^2)}\{log\, log\, p(\lambda) + log\, logp(\lambda)\}$ with respect to $\lambda$,which gives

$$\lambda = (N - 1 + v)/(\textstyle\sum_{i=1}^{N}\gamma_i/2 + v) \quad (21)$$

Similar to the hyper parameter update the non negative variance $\sigma_n^2$ is updated ny maximizing $E_{p(w|\hat{y},\gamma,\sigma_n^2)}\{log\, logp(\hat{y}|w,\sigma_n^2)\}$with respect to $\sigma_n^2$which result in

$$\sigma_n^2 = \{\alpha_0 = 1_M^T(v_1 - V_2)/(1_M^T Re(\tilde{R}_x^{-1})1_M), \quad when\ \alpha\_0 > 0 \quad unchanged, \quad When\ \alpha_0 \leq 0, \quad (22)$$

Where $v_1 = Re(\tilde{R}_x^{-1})(Re(\hat{y}) - Re(\Phi\langle w\rangle)), v_2 = Im(\hat{R}_x^{-1})(Im(\hat{y}) - Im(\Phi)\langle w\rangle), \langle w_i\rangle = E_{p(\hat{y},\gamma,\sigma_n^2)}\{w_i\}$and$\tilde{R}_x^{-1}$ can be simplified $\tilde{R}_x^{-1} = T(R_x^{-1} \otimes R_x^{-1})$.

As the solution procedure involves the variable $w_i$and $w_i^2$, the different proposition discussed in the following:

Proposition 1: first and the second moments of the posterior distribution defined in (18) is as shown in equation (23), as discussed in [94]

$$\langle w_i \rangle = \mu_i + \frac{\sigma_{ii} e^{\frac{\mu_i^2}{2\sigma_{ii}^2}}}{\sqrt{\frac{\pi}{2}} \mathrm{erfc}\left(-\frac{\mu_i}{\sqrt{2\sigma_{ii}}}\right)}, \qquad \langle w_i^2 \rangle = \sigma_{ii}^2 + \mu_i^2 + \frac{\mu_i \sigma_{ii} e^{-\frac{\mu_i^2}{2\sigma_{ii}^2}}}{\sqrt{\frac{\pi}{2}} \mathrm{erfc}\left(-\frac{\mu_i}{\sqrt{2\sigma_{ii}}}\right)}, \tag{23}$$

Parameter $\mu_i = \mu[i], \sigma_{ii} = \sqrt{\Sigma\ [i,i]}$ , and the complementary error function $erfc(x) = 2\int_x^\infty e^{-t^2}\, dt/\sqrt{\pi}$ . The concept is that when the complementary error function tends to zero $erfc(\mu_i/(\sqrt{2}\sigma_{ii})) \rightarrow 0$, then $\langle w_i \rangle \rightarrow 0$ and $\langle w_i^2 \rangle \rightarrow \sigma_{ii}^2$. Values of$\langle w_i \rangle$ and $\langle w_i^2 \rangle$ varies that fits inside the limiting condition, $-\frac{\mu_i}{\sqrt{2}\sigma_{ii}} \geq 10$ . Considering $\langle w_i \rangle = 0$ and $\langle w_i^2 \rangle = \sigma_{ii}^2$ in the algorithm the sparse learning problem is carried out. The algorithm thus used is relevant for both the uniform linear array and the sparse arrays. But the algorithm is better suited for the sparse array.The consideration that the incident signals must be uncorrelated makes it unsuitable for coherent signals. And for uniform array many algorithms are available that solves the DOA estimation. Since it is a sparse array even if more sources are there compared to the number of antenna elements it can resolve; thus is particularly suitable for this underdetermined condition. It can be observed in the simulation and results chapter that the algorithm can handle partly correlated source but with degradation in performance. Higher computational burden lies while matrix inversion in equation (19) is determined . While $M^2 < N$ , the Woodbury matrix identity reduce the computational burden:$\sum = \Gamma - \Gamma\widetilde{\Phi}^T\left(\underline{R} + \widetilde{\Phi}\Gamma\widetilde{\Phi}^T\right)^{-1}$, and therefore in each iteration the computational complexity is calculated as $O(\{M^6, N^3\})$ . The convergence pattern in the algorithm makes sure that the peaks of the spatial spectrum is kept intact even after multiple iteration once it reaches convergence (i.e., $\langle w_i^2 \rangle, i = 1,2,\cdots,N$). The algorithm shows the same time cost that of BPDN-based methods spends.

**Multi Basis Vector**

The NNSBL based DOA estimation algorithm uses the Matrix $\Phi$ acts as the overcomplete dictionary. This overcomplete matrix is generated using usually a Gaussian basis vector. This basis vector is advanced in the proposed algorithm to make it a Hybrid basis vector implementation.

In NNSBL DOA estimation during the search a or to generate manifold matrix processing time to using multi basis vector using more than one basis vector using is a multi-basis vector

Using multi-basis vector to finding manifold matrix its taking less time to achieve near to zero of the signal its help dual basis vector using this is basis vector is Gaussian basis vector using to finding manifold matrix
$\Sigma_{i=1}^{\infty}\phi^T(x)\phi(x')$
$\phi(x)$its manifold matrix

**Why Mentioned Only About Basis Vector**

**Mix basis vector**

In this thesis using a Mixed basis vector means using two different basis vector using one is a Gaussian and hyperbolic tangent basis vector to find manifold basis vector.

**Gaussian basis vector**

In order to give a proper introduction to Gaussian basis vectors, this week's post is going to start out a little bit more abstract than usual. This level of abstraction isn't strictly necessary to understand how Gaussian basis vectors work, but the abstract perspective can be extremely useful as a source of intuition when trying to understand probability distributions in general. and in order to get back to the computational world, we can recover our original five-dimensional basis vector by just forgetting all but the first five of the entries. In fact, the original five-dimensional space is contained in this infinite dimensional space.

The Gaussian basis vector transforms the dot product in the infinite dimensional space into the Gaussian function of the distance between points in the data space: If two points in the data space are nearby then the angle between the vectors that represent them in the basis vector space will be small.

$$k(x,y) = exp\ exp\left(-\gamma\left||x-y|\right|^2\right)$$

**Hyperbolic tangent**

Basis vector hyperbolic tangent basis vectors owe their popularity to neural networks, which traditionally used the hyperbolic tangent activation function

A provides a basis vector based on the hyperbolic tangent of a dot product with fixed linear scaling. Hyperbolic tangent basis vectors are popular as neural network activation functions.

$$f(x) = tanh\ tanh(\alpha(x.x') + c)$$

**Adjusting parameter $\alpha$equilibrium constraint$c$ intercept constant**

The over complete basis vector that is used in the NNSBL algorithm is changed with the Hybrid basis vector and the Mixed basis vector basis vector and the performance is checked and compared.

**RESULTS**

Matlab based simulation is developed for the proposed algorithm with the deviation Mixed Hybrid basis vector based Sparse learning based algorithm. The results obtained from the proposed algorithm is compared with the other advanced algorithms and the traditional algorithms. The proposed Mixed Basis vector based algorithm is compared with the MUSIC, SBL and NNSBL algorithms. The parameter on which the simulation is carried out is as given in Table-1.

VOL. 17, NO. 8, APRIL 2022 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

**Table-1.** Parameters considered for proposed algorithm.

| Details | Configuration |
|---|---|
| Number of Antennas | 6 |
| Antenna Array type | Non-uniform |
| Angle Range | $-\frac{\pi}{3}\ to\ \frac{\pi}{3}$ |
| Min to Max degrees | -40 to 40 |
| Carrier frequency | 280Hz |
| Propagation velocity | 360 |
| Interval of angle Searching | 1 |
| Angles of source signals | -54.8, -28.6 -9.2, 10.5 31.4, 56.7 |

The manifold matrix which defines the basis vectors of the DOA estimation algorithm needs to be generated for the number of source signals impinging on the antennas.

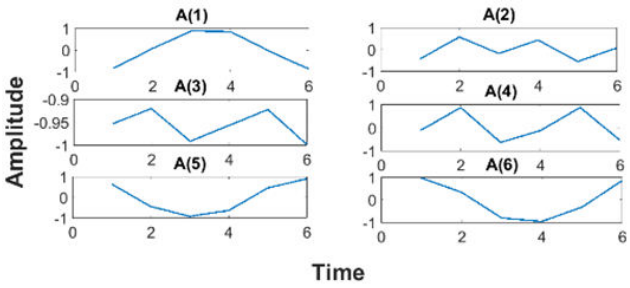


**Figure-1.** Manifold matrix.

The source matrix that is incident on the 4 antennas that is arranged with different distance between them for receiving the signals is depicted in Figure-2.

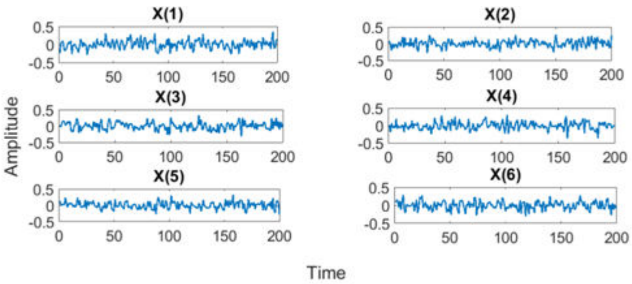


**Figure-2.** Source signals.

The channel used in the DOA estimation for the proposed and the traditional algorithms is the AWGN channel. The AWGN noise added to each source signal is as given in Figure-3.

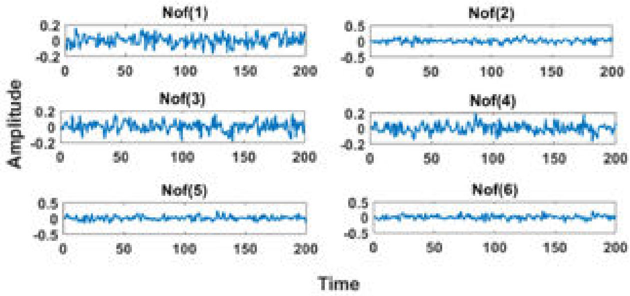


**Figure-3.** AWGN noise signal.

The AWGN noise is added with the source signal to obtain the complete incident signal that fall on the antennas. Incident signal is as depicted in Figure-4.

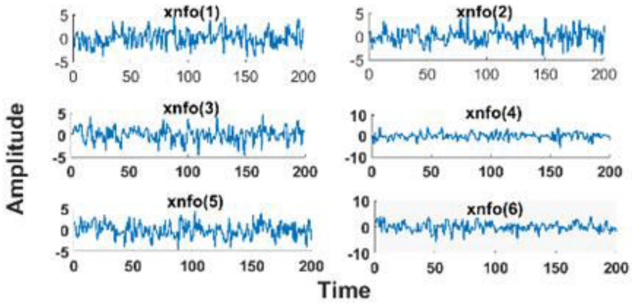


**Figure-4.** Source signal with noise.

In order to obtain the DOA estimation using the Sparse Bayesian learning algorithm manifold matrix is developed and combined with the source signal as discussed in the previous chapter. The manifold matrix with different angles and combination of basis vector is used to develop the manifold matrix in the implementation. The product of a manifold matrix with the incident source signal is as defined in Figure-5.

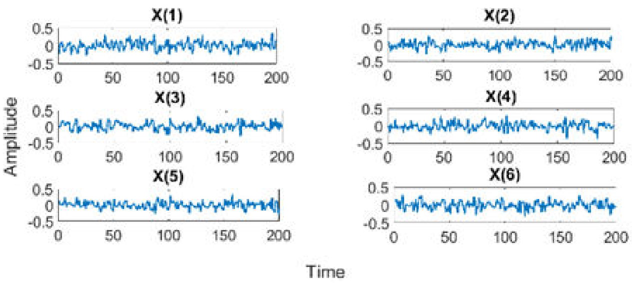


**Figure-5.** Source signal and manifold product.

Proposed implementation exploits the stochastic nature of the manifold matrix by introducing different basis vectors that can acquire better basis pursuit while DOA estimation using any sparse learning algorithms. While NNSBL is recent algorithm with better robustness it is adopted in the proposed implementation. Hybrid basis vector manifold matrix is developed by combining the gaussianbasis vectors multiple times to obtain the Hybrid basis vector manifold matrix. Similarly, different basis vectors are combined to obtain the Mixed Hybrid basis

ARPN Journal of Engineering and Applied Sciences



vector manifold matrix to obtain the DOA estimation. The SNR, mean and variance of the noise added to the signal is as given in the Table-2.

**Table-2.** SNR, mean and variance of noise added.

| S. No | SNR | mean | variance |
|---|---|---|---|
| 1 | -10 | -100.0000 | 10.0000 |
| 2 | -8 | -50.4766 | 6.3096 |
| 3 | -6 | -23.8864 | 3.9811 |
| 4 | -4 | -10.0475 | 2.5119 |
| 5 | -2 | -3.1698 | 1.5849 |
| 6 | 0 | 0 | 1.0000 |
| 7 | 2 | 1.2619 | 0.6310 |
| 8 | 4 | 1.5924 | 0.3981 |
| 9 | 6 | 1.5071 | 0.2512 |
| 10 | 8 | 1.2679 | 0.1585 |
| 11 | 10 | 1.0000 | 0.1000 |
| 12 | 12 | 0.7571 | 0.0631 |
| 13 | 14 | 0.5574 | 0.0398 |
| 14 | 16 | 0.4019 | 0.0251 |

The proposed algorithm is compared with the traditional and the recent algorithm to validate the performance of the method. The DOA estimation of the same setup with six incident source signals on six antennas for MUSIC algorithm is as given in Figure-6.

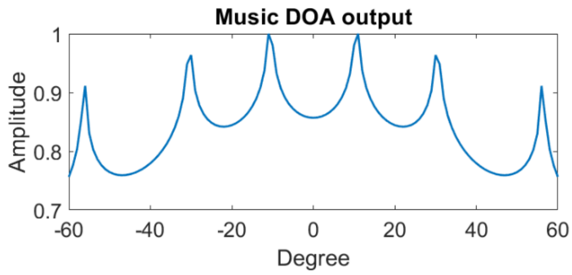


**Figure-6.** DOA estimation -MUSIC algorithm.

Traditional MUSIC algorithm exhibits lesser sharpness in the estimated DOA beam forming. NNSBL algorithm DOA estimation as shown in Figure-7 shows a sharper DOA estimation compared to the MUSIC algorithm.

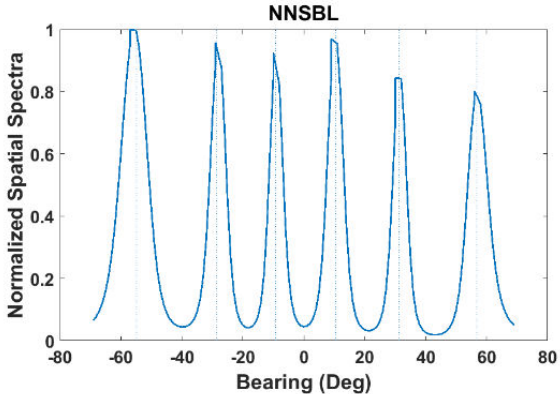


**Figure-7.** DOA estimation using NNSBL estimation.

Mixed Hybrid basis vector DOA estimation to obtain a better and sharper DOA estimation compared to all the implementation considered.

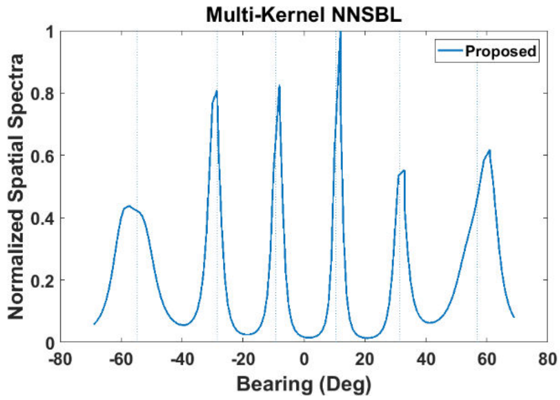


**Figure-8.** Mixed Multi basis vector NNSBL DOA estimation.

Performance validation of DOA estimation algorithms are carried out using the SNR vs Root Mean Square Error (RMSE) graph. The SNR is varied between -10 to 20db and for different possible db between this range the RMSE is estimated. SNR versus RMSE graph for NNSBL DOA estimation is as defined in Figure-9.

VOL. 17, NO. 8, APRIL 2022 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences



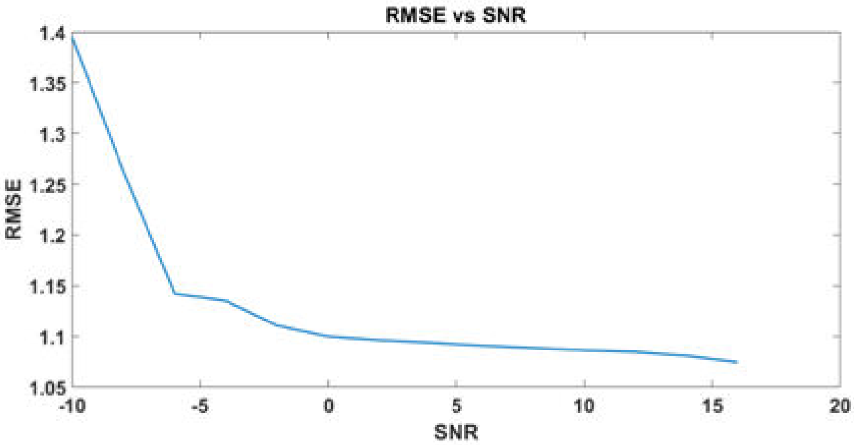


**Figure-9.** NNSBL SNR vs RMSE graph.

The SNR vs RMSE graph for Mixed Hybrid basis vector based NNSBL algorithm is as given in Figure-10. It can be observed from the SNR-RMSE graph of the Mixed Hybrid basis vector based SNR-RMSE graph that it is better all the other methods considered.

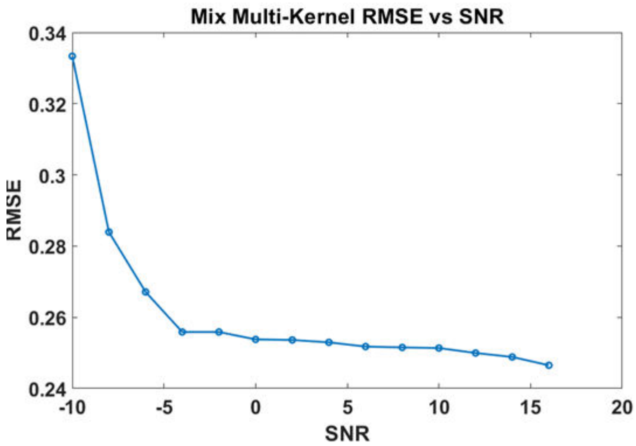


**Figure-10.** Mixed hybrid basis vector SNR-RMSE graph.

For validation of SNR-RMSE values obtained from the traditional MUSIC algorithm with the proposed algorithm.

Considering Different snapshot window from the signal the RMSE is found for a range is SNR. The results are obtained as follows. Figures 11 and 12 are the graphs that are drawn with different snapshots.

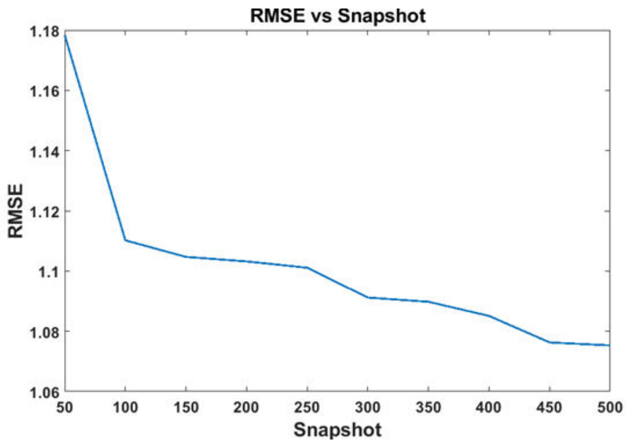


**Figure-11.** RMSE versus Snapshot NNSBL.

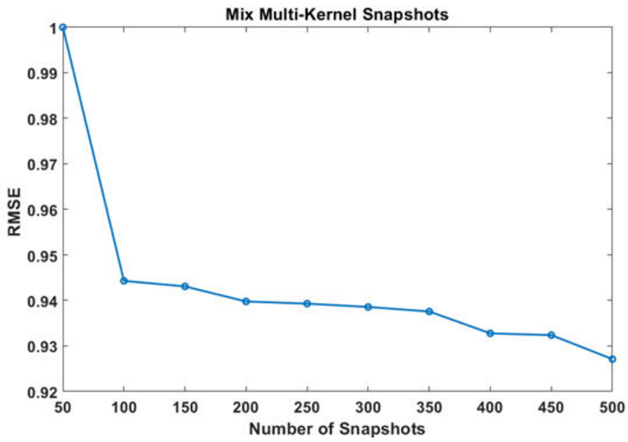


**Figure-12.** RMSE versus snapshot mixed hybrid basis vector NNSBL.

**Table-3.** Execution time comparison for algorithm considered.

| Comparison between Multi-basis vector NNSBL and NNSBL | | |
|---|---|---|
| **S. No** | **Algorithm type** | **Compilation time** |
| 01 | NNSBL | 0.448159 seconds |
| 02 | Mix Hybrid basis vector | 0.375230 seconds |

The above table depicts that the execution time of the proposed algorithm is found to be improved thus the computational complexity is improved. The stochastic nature of the adaptive basis vectors has performed better.

## CONCLUSIONS

MATLAB based implementation is carried out for the DOA estimation for non-uniform linear array setup and estimation accuracy is analyzed for the proposed work. Estimation accuracy is found from different waveforms generated from the results which involve graphs like Signal to Noise Ratio versus Mean Square Error. The results thus obtained are compared with the traditional methods to validate the performance of the results obtained from the proposed method. The overall performance of the MixedHybrid basis vector algorithm on DOA estimation is found to be satisfactory.

## REFERENCE

[1] Akira Tanaka and Hideyuki. 2014. Music - Based DOA Estimation By Oblique Projection Along The Signal Subspace. imaidivision of Computer Science, Graduate School of Information Science and Technology, Hokkaido University.

[2] Ming Diao, Li Li, hongyuan Gao. 2014. DOA Tracking Based On Mimo Radar In Impulsive Noise Background. College of Information and Communication Engineering, Harbin Engineering University, Harbin, China.

[3] Yinsheng Wei. 2014. Pair-Matching Method by Signal Covariance Matrices for 2D-DOA Estimation. Member, IEEE, and Xiaojiang Guo.

[4] Liang Guo, Yimin D. Zhang, Qisong Wu and Moeness G. Amin. 2015. DOA Estimation of Sparsely Sampled Nonstationary Signals. School of Physics and Optoelectronic Engineering, Xidian University, Xi'an 710071, China Center for Advanced Communications, Villanova University, Villanova, PA 19085, USA.

[5] Sangdong Kim, Daegun Oh and Jonghun Lee. 2015. Joint DFT-ESPRIT estimation for TOA and DOA in vehicle FMCW radars. Member, IEEE.

[6] WANG Xuan, WANG Young. 2015. Monopulse DOA Estimation of thwo unresolved Targets wirh SNR Estimation. lihao, DENG Jia-hao, mechanical Engineering, Beijiing institute of Technology Beijing China LUO Guo-hua Air Defense force Academy Zhengzgou, China.

[7] Japan Kunio Sakakibara., *et al*. 2015. Simultaneous Estimation of DOA and Angular Spread of Incident Radio Waves by DOA-Matrix Method with SLS and SAGE Algorithms. Dept. Of Computer Science and Engineering Nagoya Institute of Technology Nagoya 466-8555, Japan.

[8] Liu Luming Zhao, Yong Li. 2015. Off-grid DOA Estimation Hongqing. Chongqing Key Lab of Mobile Communications Technology Chongqing University of Posts and Telecommunications Chongqing, China.

[9] Guanghui Zhao, Guangming Shi. 2015. A Sparse Representation based DOA Estimation Algorithm with Separable Observation Model. Senior Member, IEEE, Fangfang Shen, Xi Luo, Yi Niu.

[10] Luming Zhao, Hongqing Liu and Yong Li. 2015. DOA Estimation under Sensor Gain and Phase Uncertainties. Chongqing Key Lab of Mobile Communications Technology Chongqing University of Posts and Telecommunications Chongqing, China.

[11] Nechaev Yuri. 2016. Probability of False Peaks Occuring via Circular and Concentric Antenna Arrays DOA Estimation Information Systems dept. Voronezh State University Voronezh, Russia nechaev_ub@mail.ru Peshkov Ilia Radio electronics and Computer Science dept. Yelets State University Yelets, Russia.

[12] Li LIU, Jia XU, Guan WANG, Xiang-Gen XIA, Yang GAO, Teng LONG. 2016. An Extended Dimension MUSIC Method for DOA Estimation of Multiple Real-Valued Sources. China Shen Zhen University, Shen Zhen, China, and Dept. Of ECE, University of Delaware, Newark, USA.

[13] Makoto Jomoto, Nobuyoshi Kikuma and KunioSakakibara. 2016. Simultaneous Estimation of Azimuth DOA and Angular Spread of Incident Radio Waves by DOA-Matrix Method Using Planar Array. Dept. Of Computer Science and Engineering Dept. Of Electrical and Mechanical Engineering Nagoya Institute of Technology, Nagoya 466-8555, Japan.

[14] Rimi Rashid, Daiki Hattori, Eisuke Nishiyama and Ichihiko. 2016. An RF Multiplier Integrated Planar Antenna for DOA Estimation. Toyoda Graduate School of Science and Engineering Saga University, Japan.

[15] Guo-Jun Jiang, Xing-Peng Mao, Yong-Tan Liu. 2016. Decoupled 2D DOA estimation with a small number

of elements in ucas. School of Electronics and Information Engineering, Harbin Institute of Technology, Harbin 150001, China.

[16] 2016. Performance Analysis of DOA Estimation in the Presence of Mutual Coupling for UAV. Junhyuk Kim1, Jaehoon Lee1, Hyowon Kim1, Young Mi Park2, Jaehoon Choi1, KeumCheol Hwang 3, Sunwoo Kim1 2016

[17] 2016. Time-Frequency Multi-Invariance ESPRIT for DOA Estimation. Jincheng Lin, Xiaochuan Ma, Shefeng Yan, seniormember, IEEE, and Chengpeng Hao, Member, IEEE.

[18] Ashok C and N. Venkateswaran. 2016. Support Vector Regression based DOA Estimation in Heavy Tailed Noise Environment. Department of Electronics and Communication Engineering SSN College of Engineering Chennai, India.

[19] Jie Pan, Changling Zhou, Bo Liu, Kai Jiang. 2016. Joint DOA and Doppler Frequency estimation for Coprime Arrays and Samplers Based on Continuous Compressed Sensing. College of Information Engineering, Yangzhou University Yangzhou, Jiangsu Province, China PR.

[20] Mohammad Abdul Hannan1, Nicola Anselmi1, Giacomo Oliveri1. 2016. Joint doa and Bandwidth Estimation of Unknown Signals through Single Snapshot Data and MT-BCS Approach", Research Center (ELEDIA@unitn - University of Trento), Trento, Italy 2 ELEDIA Research Center (ELEDIA@L2S - UMR 8506), Gif-sur-Yvette, France.

[21] Juan Shi, Yue Wang, Qunfei Zhang. 2017. Wideband DOA Estimation based on A-Shaped Array. Electrical and Computer Engineering Department George Mason University Fairfax, VA, 22030, USA.

[22] Tingting Fan, Hong Jiang, Jing Sun. 2017. Ambiguity function-based ESPRIT-rootmusic algorithm for DOD-DOA estimation in MIMO radar. College of Communication Engineering, Jilin University, Changchun, 130012, China.

[23] Gengxin Ning, Bowen Wang, Changku Zhou, Yizhi Feng. 2017. A velocity independent MUSIC algorithm for DOA estimation. School of Electronic and Information Engineering South China University of Technology.

[24] Hongwei. Zhao, Muren. Cai, Heng. Liu. 2017. Two-dimensional DOA Estimation with Reduced dimension MUSIC Algorithm. Academy of Space Electronic Information Technology, Xi'an, China 710100.

[25] B.VIKAS, Dr. D. Vakula. 2017. Performance Comparision of MUSIC and ESPRIT Algorithms in Presence of Coherent Signals for doa Estimation. Department of ECE NIT Warangal Warangal, India.

[26] Wentao Shi, Chengbing He, Jianguo Huang, Qunfei Zhang. 2017. Fast MUSIC algorithm for joint DOD-DOA estimation based on Gibbs Sampling in MIMO Array. School of Marine and Technology Northwestern Polytechnical University Xi'an, 710072, P.R. China.

[27] Yong Chen, Fang Wang, Jianwei Wan and Ke Xu. 2017. DOA Estimation for Long-distance Underwater Acoustic Sources based on Signal Self-nulling. College of Electronic Science and Engineering, National University of Defense Technology, China. 330022.

[28] Xiang Lan and Wei Liu. 2017. Joint 4-D DOA and Polarization Estimation Based on Linear Tripole Arrays. Department of Electronic and Electrical Engineering University of Sheffield Sheffield, Hong Kong.

[29] Anh-Tuan Nguyen, Takashi Matsubara and TakakazuKurokawa. 2017. High-Performance DOA Estimation for Coprime Arrays with Unknown Number of Sources. Department of Computer Science National Defense Academy, Yokosuka City, Japan.

[30] Xuan Luong Tran, JiříVeselý, Filip Dvořák, Thanh Bang Le. 2017. Subarray Incoherency Compensation for DOA Estimation of 8-element Antenna Array. University of Defence, Kounicova 65, 662 10 Brno, Czech Republic.

[31] Qianli Wang, Z, *et al*. 2017. Fast Compressive Sensing DOA Estimation Via ADMM Solver. Science and Technology on Electronic Information Control Laboratory, China.