



# MODELLING AND NAVIGATION SYSTEM FOR AN USV USING INTERPRETABLE INVERSE FUZZY CONTROLLER

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## ABSTRACT

This paper describes the design and implementation of a fuzzy autopilot for a scale ship model using an Interpretable Inverse Fuzzy Controller. Initially, first and second order Nomoto models, neural network and a fuzzy mathematical maneuvering model are obtained of an unmanned surface vehicle from experimental input and output data gathered from the turning circle manoeuvre and zig-zag tests. Then, an adaptive fuzzy controller is generated from the inverse fuzzy maneuvering model and results of the implementation of the adaptive ship autopilot are showed.

**Keywords:** unmanned autonomous vehicles, course correction, fuzzy control, marine navigation, system identification.

## INTRODUCTION

Autopilots have become a key factor in improving maritime safety and reducing operating costs. The objective is the stabilization of the vessel within a predetermined range and under safe maneuvering conditions, leading to a decrease in travel time and, therefore, a decrease in fuel consumption. It is estimated that the reduction in sailing time that can be achieved with the use of an adequate autopilot steering control system can be in the range of 3 to 4%, while the reduction in fuel consumption, in the range of 3 to 6% [1].

The IMO (International Maritime Organization) through its Resolution A. 757(18), which establishes provisional rules on ship maneuverability, recommends the use of simplified mathematical models of ship models, being first and second order Nomoto models, one of the most used to describe, in a simplified way, the ship dynamics [2], [1], and for the analysis and design of ship autopilots [3], [4].

The dynamics of a ship can be described by six nonlinear differential equations based on the equations of moments and forces acting on a ship in three dimensions. To obtain a simplified model, couplings between the oscillating motions of the ship around the axes must initially be omitted. Subsequently, a linearization is performed around a selected working point and, after the elimination of the roll velocity, a simplified linear differential equation is obtained from which the equation of the second order Nomoto model is derived, which contains three time constants  $T_1$ ,  $T_2$ ,  $T_3$  and  $K$  is the gain constant, which depend on the derivatives of the hydrodynamic forces and moments with respect to the oscillation and wave velocity and the yaw rate [5]. Some authors use even more simplified versions, such as the first-order Nomoto model, although the latter is used in cases where the ship's heading and speed are kept constant [6].

Fuzzy models represent a recently widely used technique for modeling complex processes. Complex nonlinear systems can be approximated by the so-called Takagi-Sugeno (T-S) fuzzy model, in which the dynamics of the system can be captured by combining fuzzy sets of linear local dynamic models [7], [8].

Regarding the control strategy employed in autonomous surface vehicles, there is a wide range of proposals depending on the type of vessel, navigation conditions, vessel dynamics, course, speed, among others. Because the dynamic properties of a ship are not constant, the set points of the classical PID controller must also change, whether the autopilot is to serve to keep the ship on the predefined course, or for course changes, so techniques that allow such adjustments online, such as adaptive controllers, have been resorted to. In the case of small scale unmanned sailboats, it has been detected that the dynamics in course change is not the same at different speeds, so control strategies that allow good performance at different operating points (different speeds) have been used, such as fuzzy PID [9]. In this case, a first-order Nomoto model is used for the controller design. Recently, adaptive fuzzy controllers for heading control of maritime vessels have had a great boom [10], [11], some of which include artificial neural networks in the training process [12].

However, switching between different operating points can generate bounce problems, due to inconsistencies between the internal states of the controller and the control input to the process, as well as saturation in the actuator [13]. Recent proposed solutions to this problem incorporate known methods, such as sliding mode control, combined with artificial intelligence techniques, especially fuzzy logic, to perform better gain management to avoid the bouncing effect of the controller [14]. Other proposals combine command filtering technology and the MLP (Minimal Learning Parameter) approach, which greatly reduces the complexity of the controller, and includes fuzzy logic to approximate unknown nonlinear functions [15].

Some researchers have made comparisons of different types of controllers, such as PID, Linear Quadratic Regulator (LQR) and Model Reference Adaptive Control with Genetic Optimization Algorithm (MRAC-GA), for heading control of a ship, using first and second order Nomoto models. The results presented show that the MRAC-GA controller provides one of the best results in terms of maximum overshoot, response speed and settling time ( $T_s$ ) to reach steady state [16].



Another problem that arises in the design of autopilots is the nonlinear behaviors of the vessels. Some strategies have focused on the application of nonlinear control theory [17], [18], [19], while others make use of artificial intelligence techniques and expert systems [3].

This paper is organized as follows: Section II presents a general description of the ship model and the procedure for obtaining several models that describe the vehicle dynamics in the horizontal plane: the first and second order Nomoto models, and the fuzzy model. Section III describes the design of the fuzzy controller by reference model, using the inverse fuzzy control technique. Section IV presents the results of the implementation of the fuzzy controller in the vehicle for the tracking of previously determined courses. Section V presents the conclusions.

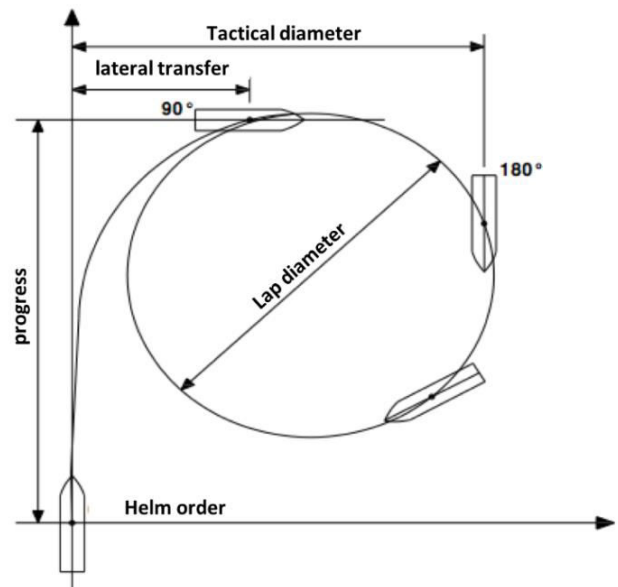
**DESCRIPTION OF THE MODEL**

The surface vehicle employed has a length overall of 2.46 meters, a maximum beam (width of the vessel) of 0.896 meters, a weight of 117 kilograms and a maximum speed of 9 knots. The vessel's draft (vertical distance from the waterline and the vessel's baseline) is approximately 0.15 meters.

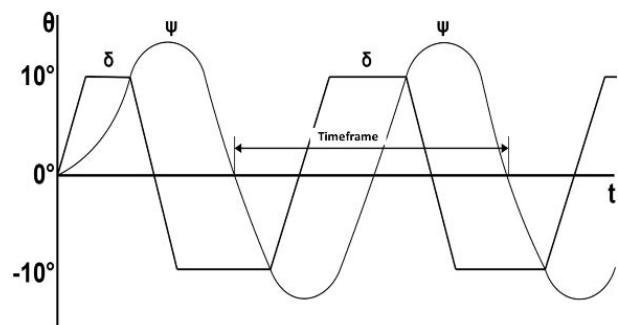


**Figure-1.** Surface vehicle employed.

Initially, a second order Nomoto model was obtained, since it is one of the most used models for the design of ship autopilots. For the model identification process, standard tests defined by the IMO were used: the turning circle test, which consists of keeping the ship on a fixed course, then moving the rudder a certain angle and monitoring the dynamics of the ship's course to the change in rudder, and the zig-zag test, also known as Kempf maneuver, which consists of keeping the ship at a constant speed, in a straight line, for a certain time and, after reaching the equilibrium condition, the rudder is actuated by setting it at  $\theta^\circ$  and kept constant until the ship changes course to  $\theta^\circ$ . Then the rudder angle is changed to  $-\theta^\circ$ , and so on. Due to the restrictions for the use of the vehicle, the model identification and validation experiments used the data from the evolutionary circle test, since there were enough tests performed with this test.



**Figure-2.** Turning circle test.



**Figure-3.** 10°/-10° Zigzag maneuver.

The second order Nomoto model is given by the expression:

$$G(s) = \frac{\psi(s)}{\delta(s)} = \frac{K(T_3s+1)}{(T_1s+1)(T_2s+1)} = \frac{\frac{KT_3}{T_1T_2}(s+\frac{1}{T_3})}{(s+\frac{1}{T_1})(s+\frac{1}{T_2})} \tag{1}$$

where  $\Psi$  represents the yaw or  $\delta$  heading angle and represents the rudder angle. Decomposing into partial fractions and solving, we have that

$$\psi(t) = \frac{K(T_1-T_3)}{T_1(T_1-T_2)} * e^{-\frac{1}{T_1}t} + \frac{K(T_2-T_3)}{T_2(T_2-T_1)} * e^{-\frac{1}{T_2}t} \tag{2}$$

In the Z domain we have

$$G(z) = \frac{A_1(1-e^{-\frac{1}{T_2}T}z^{-1})+A_2(1-e^{-\frac{1}{T_1}T}z^{-1})}{(1-e^{-\frac{1}{T_2}T}z^{-1})(1-e^{-\frac{1}{T_1}T}z^{-1})} \tag{3}$$

$$G(z) = \frac{A_1+A_2-\left(\left(A_1*e^{-\frac{1}{T_2}T}z^{-1}\right)+\left(A_2*e^{-\frac{1}{T_1}T}z^{-1}\right)\right)z^{-1}}{1-\left(e^{-\frac{1}{T_1}T}+e^{-\frac{1}{T_2}T}\right)z^{-1}+e^{-\left(\frac{1}{T_1}+\frac{1}{T_2}\right)T}z^{-2}} \tag{4}$$



Where T is the sampling time, and

$$A_1 = \frac{K(T_1 - T_3)}{T_1(T_1 - T_2)}; \quad A_2 = \frac{K(T_2 - T_3)}{T_2(T_2 - T_1)}$$

Analogizing to a discrete second order model expressed by the following equation:

$$G(z) = \frac{b_0 + b_1 z^{-1}}{1 + a_1 z^{-1} + a_2 z^{-2}} \quad (5)$$

We have:

$$\begin{aligned} b_0 &= A_1 + A_2 \\ b_1 &= -\left(A_1 * e^{-\frac{1}{T_2}T} + A_2 * e^{-\frac{1}{T_1}T}\right) \\ a_1 &= -\left(e^{-\frac{1}{T_1}T} + e^{-\frac{1}{T_2}T}\right) \\ a_2 &= e^{-\left(\frac{1}{T_1} + \frac{1}{T_2}\right)T} \end{aligned}$$

From which the coefficients of the second order Nomoto model, K, T<sub>1</sub>, T<sub>2</sub> y T<sub>3</sub>, can be obtained as follows:

$$T_2 = \frac{T}{\frac{T}{T_1} - \ln(a_2)} \quad (6)$$

If we do

$$a_1 * e^{-\frac{1}{T_1}T} = -\left(e^{-\frac{1}{T_1}T} + e^{-\frac{1}{T_2}T}\right) * e^{-\frac{1}{T_1}T} \quad (7)$$

We will have

$$a_2 * e^{\frac{2T}{T_1}} + a_1 * e^{\frac{T}{T_1}} + 1 = 0 \quad (8)$$

and

$$T_1 = \frac{T}{\ln(x_{1,2})} \quad (9)$$

where

$$x_{1,2} = \frac{-a_1 \pm \sqrt{a_1^2 - 4(a_2)(1)}}{2 * a_2} \quad (10)$$

The coefficient K is calculated from the discrete-time model gain (z = 1), as follows:

$$K = \frac{b_0 + b_1}{1 + a_1 + a_2} \quad (11)$$

And the T<sub>3</sub> coefficient by

$$T_3 = \frac{b_0 T_1 T_2}{K} \quad (12)$$

With the results of the zigzag tests, we proceeded to identify the coefficients of the second order discrete model b<sub>0</sub>, b<sub>1</sub>, a<sub>1</sub>, a<sub>2</sub>, using the method of least squares, from which we obtained the coefficients of the Nomoto

model K, T<sub>1</sub>, T<sub>2</sub> y T<sub>3</sub>. The result obtained, in the Laplace domain, was:

$$G(s) = \frac{s\psi(s)}{\delta(s)} = \frac{0.004864s + 0.4364}{0.04719s^2 + 1.614s + 1}$$

Where K = 0.4364, T<sub>1</sub> = 1.5845, T<sub>2</sub> = 0.0298 and T<sub>3</sub> = 0.0111. The normalized root mean square error was 0.024136. Figure-4 shows the comparison of the actual heading and the model heading at a rudder angle change of -13°.

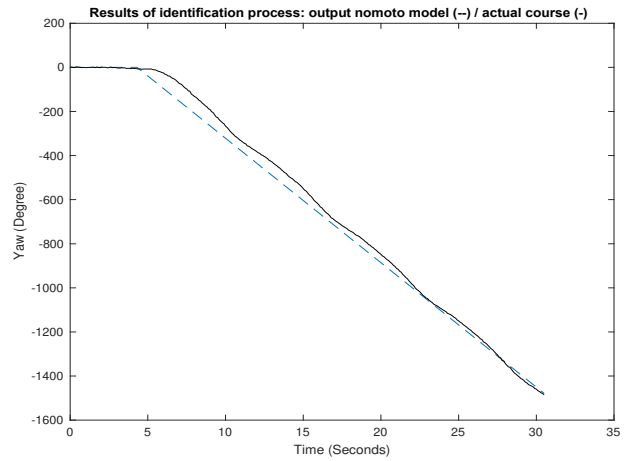


Figure-4. Result of the Nomoto model (second order) identification process.

Figure-5 shows the results of one of the validations performed, comparing the actual course change with a rudder change from 0° to +13°. The normalized root mean square error was 0.067182.

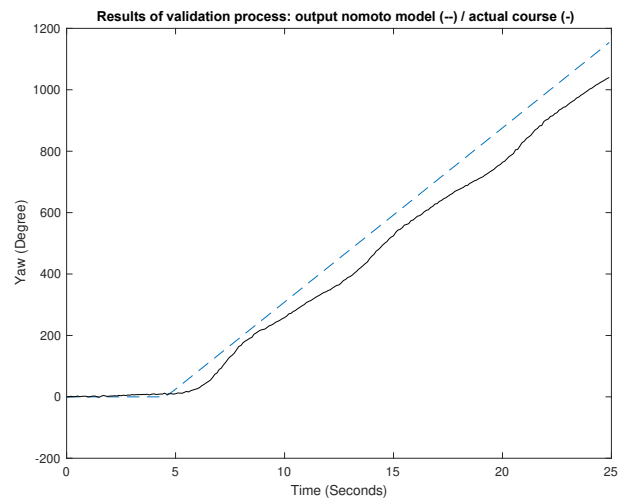


Figure-5. Result of the Nomoto model (second order) validation process.

With the same data, the first-order Nomoto model was also obtained expressed as follows:



$$G(s) = \frac{s\psi(s)}{\delta(s)} = \frac{0.432}{0.03719s + 1}$$

Where  $K = 0.432$  and  $T = 0.03719$ . The normalized mean square error was 0.027324 (identification process) and 0.071608 (validation process).

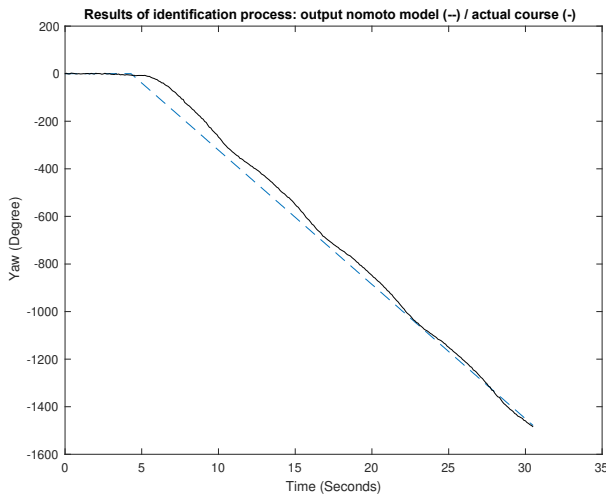


Figure-6. Result of the Nomoto model (first order) identification process.

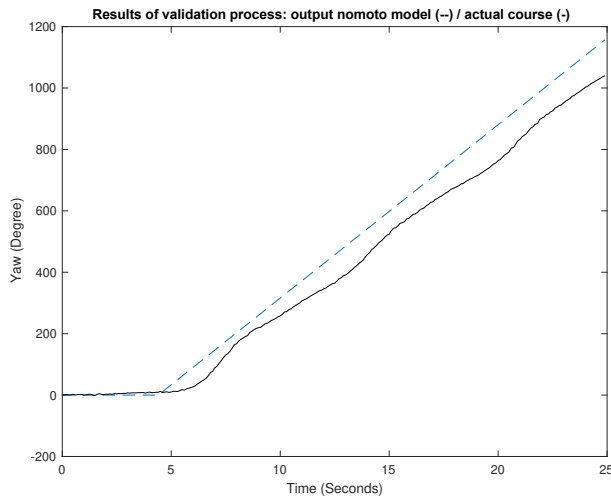


Figure-7. Result of the Nomoto model (first order) validation process.

A third model was obtained using neural network and the same variables used in the fuzzy model: three input variables, the values of the past outputs  $y(k - 1)$ ,  $y(k - 2)$  and the current input  $u(k)$  to predict the output  $y(k)$ . The structure of the best neural network model obtained is shown in Figure-8.

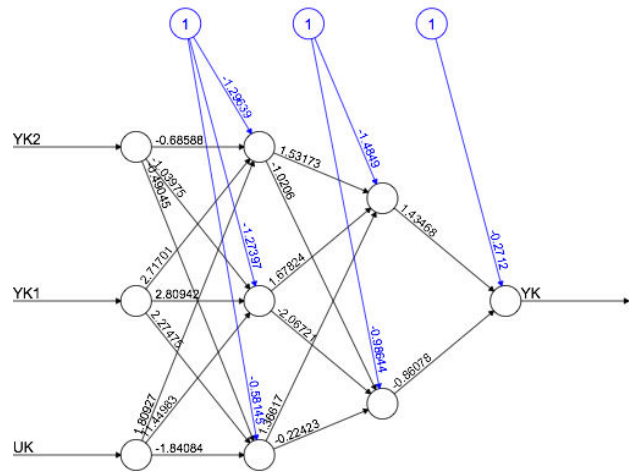


Figure-8. Structure of the best neural network model obtained.

The normalized mean square error obtained was 0.0009899 in the identification process and less than 0.00191 in all validation experiments.

Subsequently, we proceeded to obtain a fuzzy model with singleton consequents and triangular partitions with overlap at 0.5 and their modal values located, respectively, at the minimum and maximum of the universe of discourse [20], [21], fulfilling the characteristics of a Strong Fuzzy Partition (SFP) that satisfies the following semantic constraints [22]: distinguish ability; overlap at 0.5; coverage; normality; convexity and the number of fuzzy sets is no more than 9. For each triangular membership function, defined by each input variable, a singleton consequent is generated, which are estimated from the experiment data and using recursive least squares techniques. A rule is generated for each singleton consequent, as follows:

$$IF u_i \text{ is } A_i^j \text{ THEN } y_{ij} \text{ IS } \theta_{ij} \tag{13}$$

Where  $A_i^j$  represents the  $j$ -th linguistic value of the linguistic variable  $u_i$  defined over the universe of discourse  $U_i$ , while  $\theta_{ij}$  is the singleton associated to the linguistic value  $A_i^j$ .

The output of the fuzzy system is given by

$$Y = W\theta_{ij} \tag{14}$$

Where  $W$  is defined by the following vector

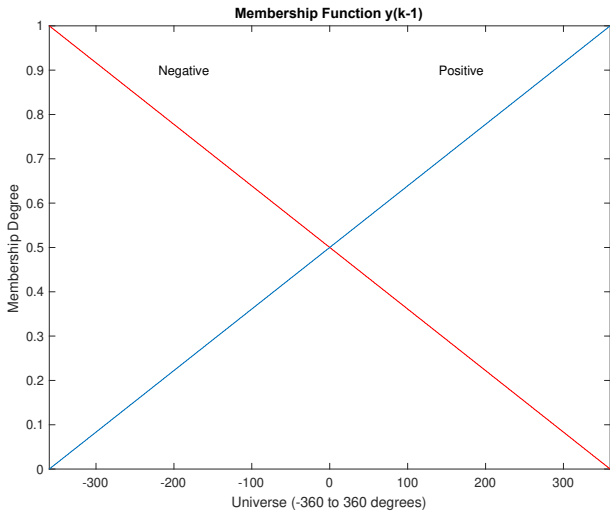
$$W = [u_{A_1^1}(x_1^k) u_{A_1^2}(x_1^k) \dots u_{A_1^j}(x_1^k) \dots u_{A_n^k}(x_n^k)] \tag{15}$$

Where  $n$  represents the  $n$ -th input variable and  $j$  represents the number of the membership function for each input variable.

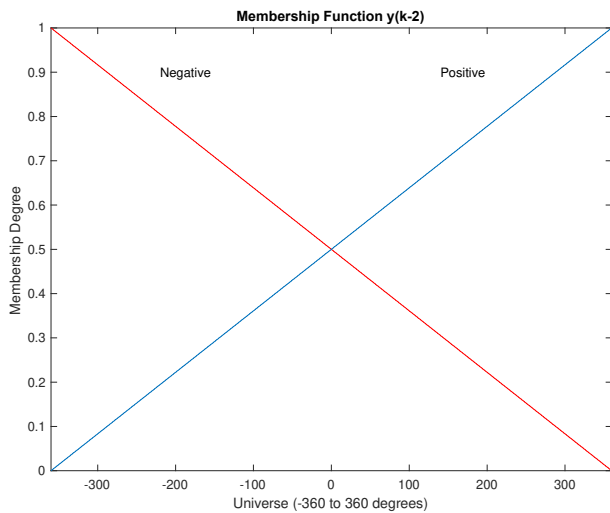
The fuzzy model obtained uses three input variables: the values of the past outputs  $y(k - 1)$ ,  $y(k - 2)$  and the current input  $u(k)$  to predict the output  $y(k)$ . For the variables  $y(k - 1)$ ,  $y(k - 2)$ , two triangular fuzzy sets



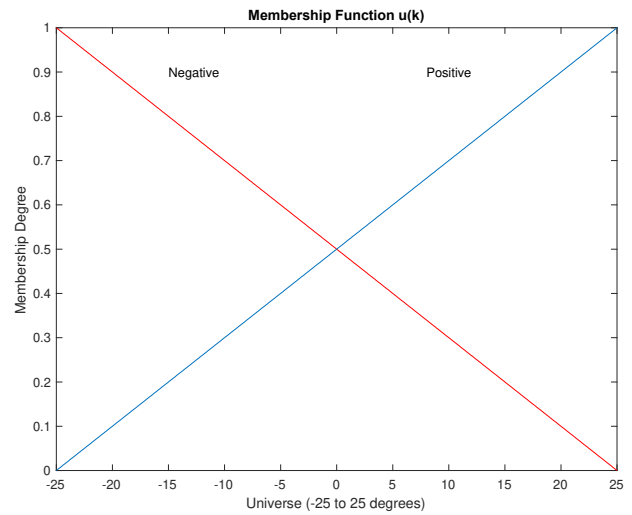
are defined in the range  $-360^\circ$  to  $+360^\circ$ . For the variable  $u(k)$ , two triangular sets are also defined in the range  $-25^\circ$  to  $25^\circ$ , corresponding to the operating range of the rudder angle used in the tests. The membership functions for each variable are presented in Figure-9.



(a)



(b)



(c)

**Figure-9.** Fuzzy partitions of the input variables (a)  $y(k-1)$ ; (b)  $y(k-2)$ ; (c)  $u(k)$ .

For the model obtained we have

$$W = [u_1(y(k-1)) u_2(y(k-1)) u_1(y(k-2)) u_2(y(k-2)) u_1(x(k)) u_2(x(k))]$$

And

$$\theta = [\delta_1 \delta_2 \delta_3 \delta_4 \delta_5 \delta_6]^T$$

By employing the recursive least squares method, the following vector of consequents was obtained:

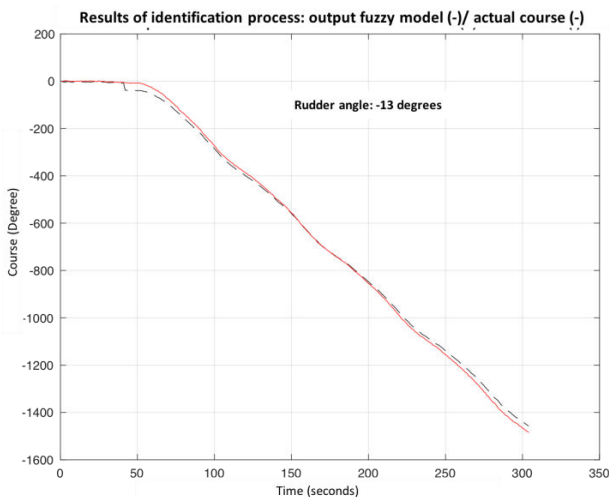
$$\theta = [-721.867 \ 710.537 \ -720.464 \ 709.135 \ -49.267 \ 37.937]$$

The rule base of the fuzzy model can be expressed as follows:

- IF  $u(y(k-1))$  is Negative THEN  $\theta = -721.867$
- IF  $u(y(k-1))$  is Positive THEN  $\theta = 710.537$
- IF  $u(y(k-2))$  is Negative THEN  $\theta = -720.464$
- IF  $u(y(k-2))$  is Positive THEN  $\theta = 709.135$
- IF  $u(x(k))$  is Negative THEN  $\theta = -49.26$
- IF  $u(x(k))$  is Positive THEN  $\theta = 39.937$

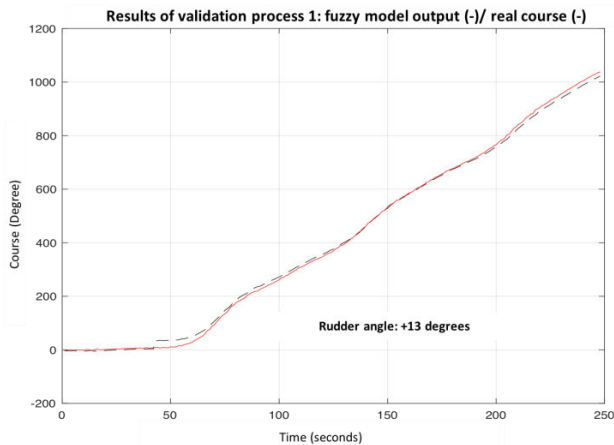
When the fuzzy model receives the input data, the rule base is evaluated using equation (14).

The normalized mean square error obtained was 0.000069 in the identification process and less than 0.00013 in all validation experiments. Figure-10 shows the results of the identification process, showing the actual course change at a rudder change from  $0^\circ$  to  $-13^\circ$  and the course change estimated by the fuzzy model obtained. The root mean square error was 15.8216.



**Figure-10.** Result of the fuzzy model identification process.

Figure-11 shows the results of one of the validations performed, comparing the actual course change with a rudder change from 0° to +13° with the course change estimated by the fuzzy model obtained. The root mean square error was 11.4774.



**Figure-11.** Result of the fuzzy model validation process.

A comparison of each model's accuracy is shown in Table-1.

**Table-1.** Normalized Mean Square Error.

Model	Identification	Validation
Nomoto model (first order)	0.027324	0.071608
Nomoto model (second order)	0.024136	0.067182
Neural Network	0.0009899	0.00191
Our fuzzy model	0.000069	0.00013

**CONTROLLER DESIGN**

Our approach (Fuzzy model with singleton consequents) has the most accuracy among the models.

Because that this model has been used for control system design.

A model reference signal  $Y_r$  was included to propose a smooth change of the heading angle, thus

$$Y_r = W_1\theta_1 + W_2\theta_2 + W_3\theta_3 \tag{16}$$

where

$$W_1 = [u_1(y(k-1)) \ u_2(y(k-1))]; \ \theta_1 = [\delta_1 \ \delta_2]^T$$

$$W_2 = [u_1(y(k-2)) \ u_2(y(k-2))]; \ \theta_2 = [\delta_3 \ \delta_4]^T$$

$$W_3 = [u_1(x(k)) \ u_2(x(k))]; \ \theta_3 = [\delta_5 \ \delta_6]^T$$

The vectors  $u_1(y(k-1))$ ,  $u_2(y(k-1))$ ,  $u_1(y(k-2))$ ,  $u_2(y(k-2))$ ,  $u_1(x(k))$  y  $u_2(x(k))$  represent the membership functions of each variable:  $y(k-1)$ ,  $y(k-2)$  y  $u(k)$  respectively. The first four vectors are known, as well as the reference signal  $Y_r$ . The objective is to find the control signal that generates the desired change in rudder with the form of  $Y_r$ , for which we will employ the fuzzy inverse control technique, as follows:

$$W_1\theta_1 = Y_r - W_2\theta_2 - W_3\theta_3 \tag{17}$$

Proceeding to calculate

$$u_1(x(k)) = \frac{Y_r}{\frac{\delta_5 - \delta_6}{\delta_6}} = \frac{[u_1(y(k-1)) \ u_2(y(k-1)) \ u_1(y(k-2)) \ u_2(y(k-2))] \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \end{bmatrix}}{\delta_5 - \delta_6} \tag{18}$$

Since we have a partition with triangular membership functions and overlap at 0.5, we have that

$$u_2(x(k)) = 1 - u_1(x(k))$$

Knowing the membership function  $u_1(x(k))$ , the value  $x(k)$ , which is the rudder angle to be applied at time  $k$ , is determined.

**RESULTS**

The results of the response of the controlled system to different values of desired heading are presented below, with a reference model given by  $Y_r(1 - e^{-8t})$

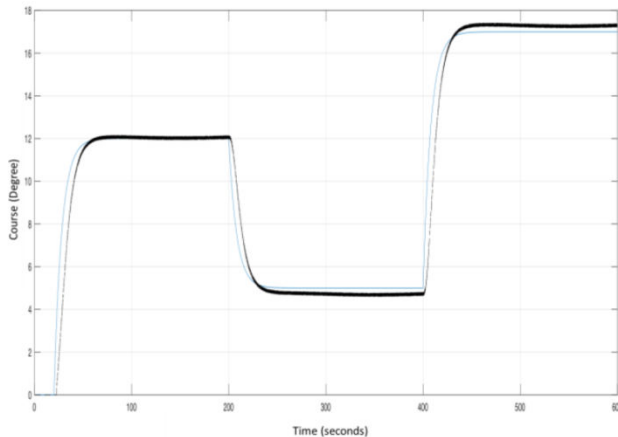
$Y_r$  being the desired course. It starts with a heading of 0° (referenced to the ship's axis, stern to bow, where it is initially located), followed by an initial order to change course to +12° (port turn). Subsequently a turn to starboard is ordered to reach +5°, followed by a further turn to port to reach +17°.

The inverse fuzzy controller has the same distributions as the fuzzy model obtained, for the variables  $y(k-1)$ ,  $y(k-2)$  and  $u(k)$ , shown in Figure-4. The rudder



angle, or control signal, is calculated using equation (18), from which the value  $x(k)$  is derived.

Figure-12 shows the results achieved by the system with the fuzzy controller under the ordered course changes. The normalized mean square error was used as a metric, reaching in this experiment a value of 0.0323.



**Figure-12.** Vessel response with fuzzy controller to course change requests.

## CONCLUSIONS

This paper initially presents the modeling of the dynamics of a surface vehicle in the horizontal plane using identification techniques to generate first and second order Nomoto models, widely used for the design of ship autopilots. The data was taken from one of the standard maneuverability tests defined by the International Maritime Organization, such as the evolving circle test, using the normalized mean square error as a metric to determine the accuracy. Both models showed high accuracy, both in identification and validation.

Subsequently, the fuzzy modeling of the system was presented, with a triangular partition for each of the input variables and an overlap of 0.5. This in order to guarantee the interpretability of the system and the subsequent application of the fuzzy inverse control strategy. The output sets of the model are of the singleton type.

Finally, we showed how to obtain the inverse fuzzy control, with reference model, which shows a good result for tracking a previously defined trajectory, with course changes to port and starboard.

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