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RAYLEIGH WAVES IN A RESERVOIR ROCKS LYING ON THE BED OF SANDY ROCKS

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ABSTRACT

An analysis of the propagation behaviour of Rayleigh waves has been carried out in anisotropic layer like reservoir rocks overlying a sandy medium. The most general case of anisotropic material i.e., of triclinic crystal has been taken. An analytical approach is incorporated in a state to obtain the solutions for layer and half-space. A closed relation of phase velocity and wave number is obtained as determinant form with weld contact boundary at the interface of lower half and the adjacent layer with the upper boundary condition as a rigid called dispersion relation. For the numerical interpretation of results, two different reservoir rocks of triclinic type have been taken. The outcomes of phase velocity and wave number have been illustrated by graphs obtained with the aid of Mathematica-7.

Keywords: rayleigh wave, reservoir rocks, anisotropic, sandiness parameter, shielded.

1. INTRODUCTION

The crust of earth is a layered structure with varieties of rocks like igneous, metamorphic and sedimentary rocks. The nature of rocks is unique and consists of different types of minerals and possesses different anisotropy. Rasolofosaon and Zinszner [2002] made analysis of reservoir rocks and given the data for elastic coefficients for triclinic symmetry. Pal *et al*. [2015] focussed about the propagation nature of Rayleigh waves in a layer and half-space consists of anisotropic and sandy medium. Mandal *et al*. [2014] predicted the disturbance caused due to propagation of SH-type waves due moving stress discontinuity. The medium they considered are anisotropic soil layer and inhomogeneous elastic half-space. Pal *et al*. [2014] and Kumar and Pal [2014] studied the effect of in homogeneity and two thermal relaxation times for plane waves. Kumar *et al*. [2015a, 2015b] also focuses on study of plane SH-waves in anisotropic magneto-elastic medium.

Wilson [1942], Biot [1965], Newlands [1950] and Stonely [1934] gave their contribution in the study of Rayleigh waves propagation that cannot be considered as base to do further investigation. Dutta [1963] devoted their time for study of Rayleigh waves propagation in a geometry of one layer and half-space and nature of medium as inhomogeneous isotropic for layer and isotopic for half-space while Abd-Alla [1999] discussed propagation of Rayleigh waves in an elastic half-space of orthotropic material. Vishwakarma and Gupta [2014] have studied the influence of rigid boundary on the Rayleigh waves in a structure of layer and half-space. The explicit formula for velocity of Rayleigh wave in compressible medium of type orthotropic has been found by Vinh and Ogden [2004]. Singh and Kumar [2013] analyzed a finite rigid past a shallow ocean for Rayleigh waves propagation. Vinh *et al*. [2014] considered the Rayleigh waves in an elastic half-space which is surrounded by a thin elastic layer. From the study Rayleigh wave velocity formula obtained approximately. Cerv and Plesek [2013] considered two-dimensional anisotropic media and in order to analyze the Rayleigh wave propagation. They have obtained the secular equations both implicitly and explicitly.

Considering the perception that the earth surface is shielded that is the displacement at the surface of earth are null; we are indenting to analyze the propagation of Rayleigh waves in the layer of reservoir rocks and semi-infinite sandy rocks. The most general case of anisotropic material has been considered. The analytical results have been given for both layer and half-space. The dispersion equation is attained as determinant form subject to weld contact boundary at the interface and the upper contact with layer with vacuum as a rigid surface. For the numerical interpretation of results two different reservoir rocks of triclinic type has been taken. The physical results for phase velocity against wave number using Mathematica-7 are shown via graphs.

2. FORMULATION OF THE PROBLEM AND BASIC EQUATIONS

A layer of anisotropic reservoir rocks with thickness h past a semi-infinite sandy medium is examined. The reservoir rocks of triclinic symmetry have been considered for formulation of problem and sandy medium is of isotropic type. The interface of layer and half-space is taken at $z=0$ whereas the upper contact of layer at $z=-h$. Here, z axis is pointed vertically downward and the direction of the propagation of wave with velocity c taken along x axis. The physical description is shown in Figure-1.

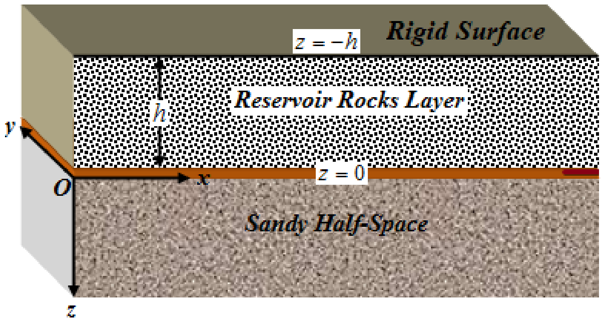

Figure-1. Physical description.

The dynamical equation of motion is given by

$$\mathbf{DS}=\rho\ddot{\mathbf{U}} \tag{1}$$

where, ρ is the material density of the layer, $\mathbf{S}$ is the symmetric stress matrix, $\mathbf{D}$ is row matrix of order 3 and $\mathbf{U}$ is row matrix of order 3 and defined as

$$\mathbf{D}=\begin{pmatrix}\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\end{pmatrix}, \mathbf{S}=\begin{pmatrix}\tau_{xx} & \tau_{xy} & \tau_{xz}\\ \tau_{yx} & \tau_{yy} & \tau_{yz}\\ \tau_{zx} & \tau_{zy} & \tau_{zz}\end{pmatrix} \text{ and } \mathbf{U}=\begin{pmatrix}u & v & w\end{pmatrix}$$

.

The dot in the equation (1) represents partial differentiation with respect to t.

For Rayleigh waves propagation in xz-plane, we have

$$\mathbf{U}=\begin{pmatrix}u & 0 & w\end{pmatrix} \text{ and } \frac{\partial}{\partial y}\equiv 0. \tag{2}$$

The relations for stress-strain in anisotropic layer of triclinic symmetry are taken as

$$\sigma=\mathbf{C}\mathbf{E}^{\mathrm{T}} \tag{3}$$

where σ is stress column matrix, $\mathbf{C}$ is coefficient matrix of elastic constants and $\mathbf{E}$ is the strain row matrix and defined as below:

$$\sigma=\begin{pmatrix}\tau_{xx} & \tau_{yy} & \tau_{zz} & \tau_{yz} & \tau_{zx} & \tau_{xy}\end{pmatrix}^T$$

$$\mathbf{C}=\begin{pmatrix}C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16}\\ & C_{22} & C_{23} & C_{24} & C_{25} & C_{26}\\ & & C_{33} & C_{34} & C_{35} & C_{36}\\ & Sym. & & C_{44} & C_{45} & C_{46}\\ & & & & C_{55} & C_{56}\\ & & & & & C_{66}\end{pmatrix}$$

$$\mathbf{E}=\begin{pmatrix}e_{xx} & e_{yy} & e_{zz} & 2e_{yz} & 2e_{xz} & 2e_{xy}\end{pmatrix}$$

For half-space sandy medium, the stress-strain relations are

$$\sigma=\eta\mathbf{C}'\mathbf{E}^{\mathrm{T}} \tag{4}$$

where, $\mathbf{C}'=\begin{pmatrix}\lambda_2+2\mu_2 & \lambda_2 & \lambda_2 & 0 & 0 & 0\\ & \lambda_2+2\mu_2 & \lambda_2 & 0 & 0 & 0\\ & & \lambda_2+2\mu_2 & 0 & 0 & 0\\ & Sym. & & \mu_2 & 0 & 0\\ & & & & \mu_2 & 0\\ & & & & & \mu_2\end{pmatrix}$

and λ_2, μ_2 are Lame constants and η is sandiness parameter given by $\frac{E}{\mu_2}=2\eta\left(1+\nu_2\right)$ (Weiskopf [1945]), where and E and ν_2 are the Young's modulus and Poisson's ratio respectively.

3. SOLUTION OF THE PROBLEM

3.1 Solution for Layer

Motions equation for propagation of Rayleigh waves in the layer can be obtained by using equations (1) - (2) and (3) we have

$$\mathbf{C_1 X}=\rho_1\ddot{\mathbf{U}}_1 \tag{5}$$

where,

$$\mathbf{C_1}=\begin{pmatrix}C_{11} & C_{15} & C_{55} & C_{35} & 2C_{15} & C_{13}+C_{55}\\ C_{15} & C_{55} & C_{35} & C_{33} & C_{13}+C_{55} & 2C_{35}\end{pmatrix}$$

$$\mathbf{X}=\begin{pmatrix}\frac{\partial^2 u_1}{\partial x^2} & \frac{\partial^2 w_1}{\partial x^2} & \frac{\partial^2 u_1}{\partial z^2} & \frac{\partial^2 w_1}{\partial z^2} & \frac{\partial^2 u_1}{\partial x\partial z} & \frac{\partial^2 w_1}{\partial x\partial z}\end{pmatrix}^T \text{ and}$$

$$\mathbf{U_1}=\begin{pmatrix}u_1\\ w_1\end{pmatrix}.$$

The assumption of $\mathbf{U_1}=\Theta e^{ik(x-ct)}$ made equation (4) to reach the vector-matrix differential equation of form as

$$\mathbf{P}\frac{d^2\Theta}{dz^2}+ik\mathbf{Q}\frac{d\Theta}{dz}-k^2\mathbf{R}\Theta=\mathbf{0} \tag{6}$$

where,

$$\mathbf{P}=\begin{pmatrix}C_{55} & C_{35}\\ C_{35} & C_{33}\end{pmatrix}, \mathbf{Q}=\begin{pmatrix}2C_{15} & C_{13}+C_{55}\\ C_{13}+C_{55} & 2C_{35}\end{pmatrix},$$

$$\mathbf{R}=\begin{pmatrix}C_{11}-\rho_1 c^2 & C_{15}\\ C_{15} & C_{55}-\rho_1 c^2\end{pmatrix}, \Theta=\begin{pmatrix}U_1\\ W_1\end{pmatrix}$$

and k is the wave number.

Now, considering the solution of the above equation as $\Theta=\Psi e^{-k\xi z}$ and substituting in equation (6) we have

$$\begin{aligned}&\left[C_{55}\xi^2-2iC_{15}\xi+\left(\rho_1c^2-C_{11}\right)\right]\Psi_1+\\&\left[C_{35}\xi^2-i\left(C_{13}+C_{55}\right)\xi-C_{15}\right]\Psi_2=0\end{aligned}\tag{7}$$

$$\begin{aligned}&\left[C_{35}\xi^2-i\left(C_{13}+C_{55}\right)\xi-C_{15}\right]\Psi_1+\\&\left[C_{33}\xi^2-2iC_{35}\xi+\left(\rho_1c^2-C_{55}\right)\right]\Psi_2=0\end{aligned}\tag{8}$$

In order to obtain the non trivial solution of equations (7) and (8) we have

$$A_0\xi^4+A_1\xi^3+A_2\xi^2+A_3\xi+A_4=0\tag{9}$$

where, A_0, A_1, A_2, A_3 and A_4 have been defined in Appendix.

Let $\xi_j\left(j=1,...,4\right)$ be the roots of equation (9) and displacement components ratio of U_{1j}, W_{1j} from equation (7) corresponding

to $\xi=\xi_j$ is

$$\frac{W_{1j}}{U_{1j}}=\frac{\Psi_{2j}}{\Psi_{1j}}=\frac{-\left[C_{55}\xi_j^2-2iC_{15}\xi_j+\left(\rho_1c^2-C_{11}\right)\right]}{\left[C_{35}\xi_j^2-i\left(C_{13}+C_{55}\right)\xi_j-C_{15}\right]}=\delta_j\tag{10}$$

Thus, the solution of equation (5) is given by

$$\mathbf{U_1}=\Psi'\mathbf{Z}e^{ik(x-ct)}\tag{11}$$

where, $\Psi'=\begin{pmatrix}\Psi_{11}&\Psi_{12}&\Psi_{13}&\Psi_{14}\\\delta_1\Psi_{11}&\delta_2\Psi_{12}&\delta_3\Psi_{13}&\delta_4\Psi_{14}\end{pmatrix}$ and $\mathbf{Z}=\begin{pmatrix}e^{-k\xi_1z}&e^{-k\xi_2z}&e^{-k\xi_3z}&e^{-k\xi_4z}\end{pmatrix}$.

3.2 Solution for half-space

Motions equation for Rayleigh waves propagation in the half-space can be obtained by using equations (1) - (2) and (4) we have

$$\mathbf{C_2X}=\rho_2\ddot{\mathbf{U}}_2\tag{12}$$

where,

$\mathbf{C_2}=\eta\begin{pmatrix}\lambda_2+2\mu_2&0&\mu_2&0&0&\lambda_2+\mu_2\\0&\mu_2&0&\lambda_2+2\mu_2&\lambda_2+\mu_2&0\end{pmatrix}$ and $\mathbf{U_2}=\begin{pmatrix}u_2\\w_2\end{pmatrix}$.

The solution of the above equation can be assumed as $\mathbf{U_2}=\Theta'e^{ik(x-ct)}$ and substituting in (12), we have vector-matrix differential equation form as

$$\mathbf{P}'\frac{d^2\Theta'}{dz^2}+ik\mathbf{Q}'\frac{d\Theta'}{dz}-k^2\mathbf{R}\Theta'=\mathbf{0}\tag{13}$$

where,

$\mathbf{P}'=\eta\begin{pmatrix}\mu_2&0\\0&\lambda_2+2\mu_2\end{pmatrix},\mathbf{Q}'=\eta\begin{pmatrix}0&\lambda_2+\mu_2\\\lambda_2+\mu_2&0\end{pmatrix},$

$\mathbf{R}'=\begin{pmatrix}\eta\left(\lambda_2+2\mu_2\right)-\rho_2c^2&0\\0&\eta\mu_2-\rho_2c^2\end{pmatrix}$ and $\Theta'=\begin{pmatrix}U_2\\W_2\end{pmatrix}$.

Now, considering the solution of above equation as $\Theta'=\Phi e^{-k\zeta z}$ and putting in equation (13) we have

$$\left[\rho_2c^2-\eta\left(\lambda_2+2\mu_2\right)+\eta\mu_2\zeta^2\right]\Phi_1-i\eta\left(\lambda_2+\mu_2\right)\zeta\Phi_2=0\tag{14}$$

$$-i\eta\left(\lambda_2+\mu_2\right)\zeta\Phi_1+\left[\rho_2c^2-\eta\mu_2+\eta\left(\lambda_2+2\mu_2\right)\zeta^2\right]\Phi_2=0\tag{15}$$

For non-trivial solution of equations (14) and (15) we get a biquadratic equation in ζ in the form

$$B_0\zeta^4+B_1\zeta^2+B_2=0\tag{16}$$

where, B_0, B_1 and B_2 are defined in Appendix. Let $\pm\zeta_1,\pm\zeta_2$ be the roots of equation (16) then

$\zeta_1^2=\dfrac{-B_1-\sqrt{B_1^2-4B_0B_2}}{2B_0}$ and $\zeta_2^2=\dfrac{-B_1+\sqrt{B_1^2-4B_0B_2}}{2B_0}$.

The displacement components ratio of U_{2j}, W_{2j} from equation (14) corresponding to $\zeta=\zeta_j$ is

$$\frac{W_{2j}}{U_{2j}}=\frac{\Phi_{2j}}{\Phi_{1j}}=\frac{\left[\eta\mu_2\zeta_j^2+\left\{\rho_2c^2-\eta\left(\lambda_2+2\mu_2\right)\right\}\right]}{i\eta\left(\lambda_2+\mu_2\right)\zeta_j}=\varepsilon_j.\tag{17}$$

Thus, the solution of equation (12) is given as

$$\mathbf{U_2}=\Phi'\mathbf{Z}'e^{ik(x-ct)}\tag{18}$$

where, $\Phi'=\begin{pmatrix}\Phi_{11}&\Phi_{12}&\Phi_{13}&\Phi_{14}\\\varepsilon_1\Phi_{11}&\varepsilon_2\Phi_{12}&\varepsilon_1\Phi_{13}&\varepsilon_2\Phi_{14}\end{pmatrix}$ and $\mathbf{Z}'=\begin{pmatrix}e^{-k\zeta_1z}&e^{-k\zeta_2z}&e^{k\zeta_1z}&e^{k\zeta_2z}\end{pmatrix}$.

displacement component should be bounded, i.e., as $z\to\infty$, $\mathbf{U_2}\equiv0$. Hence, the approximate solution of the equation equation (18) is given as

$$\mathbf{U_2}=\Phi''\mathbf{Z}''e^{ik(x-ct)}\tag{19}$$

where, $\Phi''=\begin{pmatrix}\Phi_{11}&\Phi_{12}\\\varepsilon_1\Phi_{11}&\varepsilon_2\Phi_{12}\end{pmatrix}$ an $\mathbf{Z}''=\begin{pmatrix}e^{-k\zeta_1z}&e^{-k\zeta_2z}\end{pmatrix}$.

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4. BOUNDARY CONDITIONS

(1). The interface of the layer and half-space is considered as weld contact so that stresses and displacements are continuous at $z=0$ i.e., $\left(\tau_{xz}\right)_I=\left(\tau_{xz}\right)_{II}$, $\left(\tau_{zz}\right)_I=\left(\tau_{zz}\right)_{II}$ and $\mathbf{U_1=U_2}$.

(2). The upper contact of layer is considered as rigid surface so displacement vanishes at $z=-h$ i.e., $\mathbf{U_1}\equiv 0$.

Using the above boundary conditions and equations (11) and (19) respectively, we have six homogeneous simultaneous equations as

$$\mathbf{AY=0} \tag{20}$$

where, $\mathbf{A}$ is the coefficient matrix and $\mathbf{Y}$ is column matrix of order 6 of constants and are defined in Appendix.

determinant of the coefficient matrix of equation (20) gives the frequency equation of Rayleigh waves in anisotropic layer lying over sandy medium.

$$|\mathbf{A}|=0 \tag{21}$$

The equation (21) can be separated in the form of real and imaginary part as below

$$\mathbf{A_R}+i\mathbf{A_I}=0 \tag{22}$$

The dispersion of Rayleigh will be given as the real part ($\mathbf{A_R}$) of equation (22).

5. NUMERICAL RESULTS AND DISCUSSIONS

For the numerical interpretation we have taken two data sets for reservoir rocks of triclinic symmetry as obtained by Rasolofosaon and Zinszner [2002].

Data I:

$$\mathbf{C}=\begin{pmatrix} 106.8 & 27.10 & 9.68 & -0.03 & 0.28 & 0.12 \\ & 99.00 & 18.22 & 1.49 & 0.13 & -0.58 \\ & & 54.57 & 2.44 & -1.69 & -0.75 \\ & Sym. & & 25.97 & 1.98 & 0.43 \\ & & & & 25.05 & 1.44 \\ & & & & & 37.82 \end{pmatrix}\mathbf{GPa},$$

$\rho_1=2727\,\mathbf{kg/m^3}$

Data II:

$$\mathbf{C}=\begin{pmatrix} 17.77 & 3.78 & 3.76 & 0.24 & -0.28 & 0.03 \\ & 19.45 & 4.13 & -0.41 & 0.07 & 1.13 \\ & & 21.79 & -0.12 & -0.01 & 0.38 \\ & Sym. & & 8.30 & 0.66 & 0.06 \\ & & & & 7.62 & 0.52 \\ & & & & & 7.77 \end{pmatrix}\mathbf{GPa},$$

$\rho_1=2216\,\mathbf{kg/m^3}$

For sandy half-space, the data is taken as (Babuska and Cara [1991])
$\lambda_2=105\,\text{Gpa}$ $\mu_2=91.6\text{Gpa}$, $\rho_2=3582\,\text{kg/m}^3$

The graphs are drawn for ($\mathbf{A_R}$) between the phase velocity and wave number for the different thickness of the layer and different values of sandiness parameter. In Figure-2, the graph has been plotted for Data I of layer for different values of thickness of the layer. From the figure it can be clearly observed that the phase velocity initially decreases rapidly with wave number and then slowly increases and get stable. As the thickness of layer increases, the rate at which phase velocity decreases increases very fast. In Figure-3, the graph has been plotted for Data I of layer for different values of sandiness parameter. It can be seen from the figure that the curves follow same trends as in Figure-2. The effect of sandiness parameter is marginal. In Figure-4 and Figure-5, the graphs are plotted for Data II of layer for different values of thickness of the layer and sandiness parameter respectively. From the figure it can be clearly noticed that the curves of these graphs follow same nature but the magnitude of phase velocity differ with respect to wave number. From the data set I and II, we can say that the anisotropy of materials drastically affects the phase velocity.

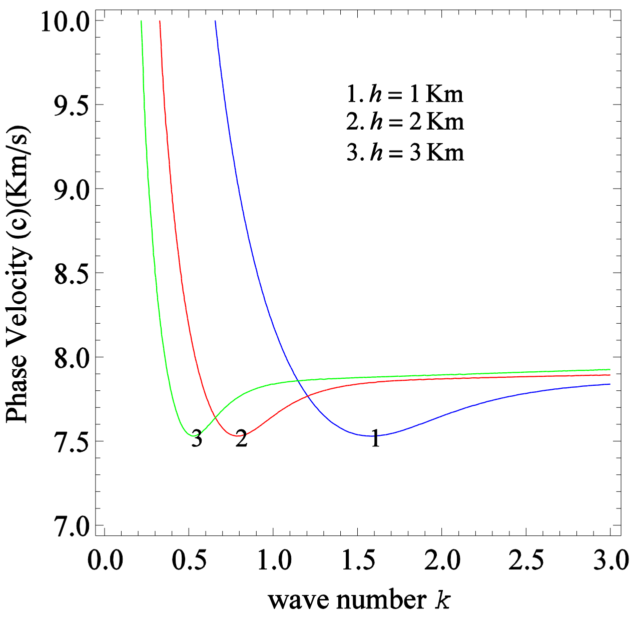

Figure-2. Phase velocity against wave number for various thickness of layer for Data I.

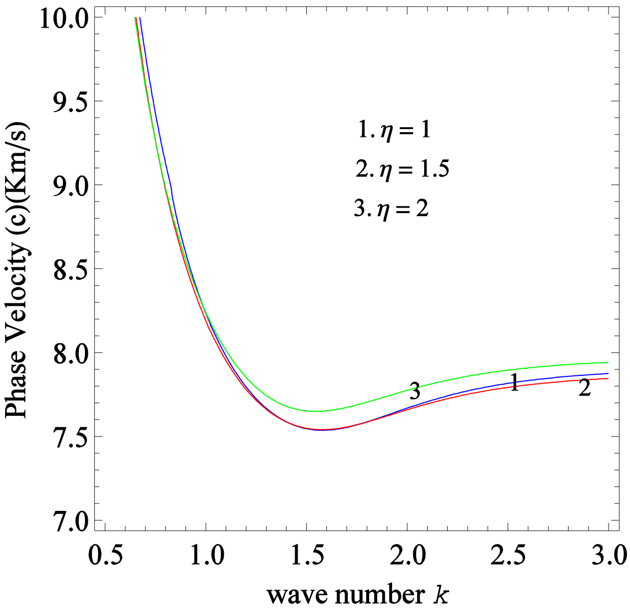

Figure-3. Phase velocity against wave number for various sandiness parameter for Data I.

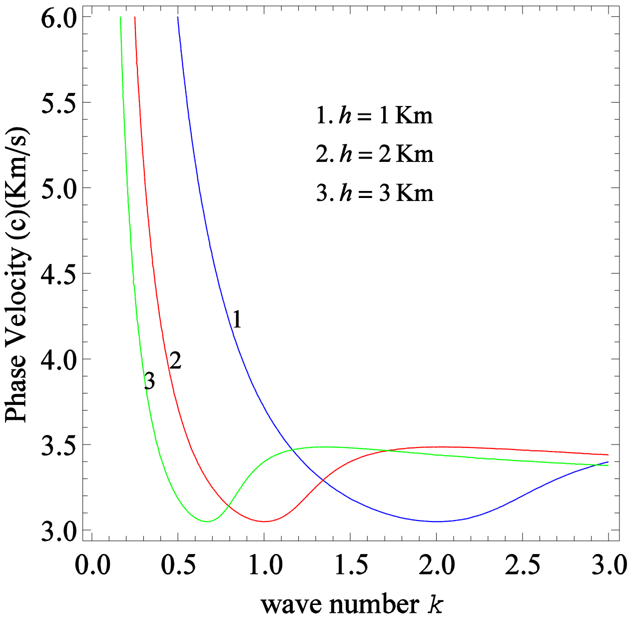

Figure-4. Phase velocity against wave number for various thickness of layer for Data II.

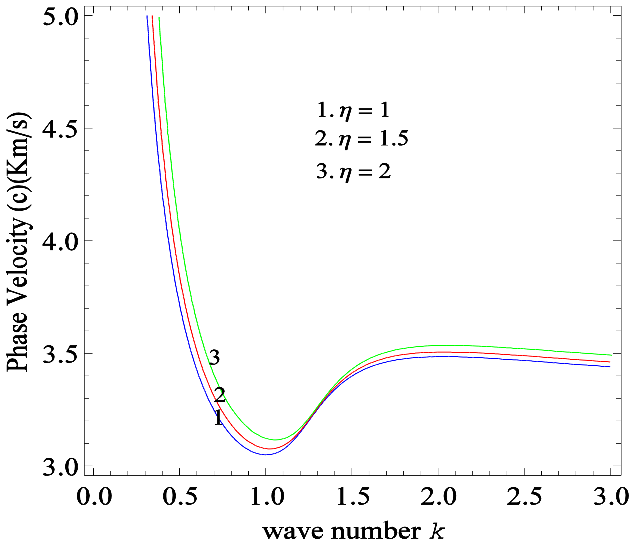

Figure-5. Phase velocity against wave number for various values of sandiness parameter for Data II.

6. CONCLUSIONS

The propagation behaviour of Rayleigh waves in reservoir rocks has been studied on the bed of sandy medium. The layer upper boundary has been assumed as rigid and continuous interface at the contact of layer and half-space. The layer is considered as medium of triclinic symmetry which has minimum number of elastic constants to define an anisotropic material. The half-space is of isotropic material with a sandiness parameter. The solutions of layer and half-space are obtained on the basis of variable separable method. The frequency equation is obtained in determinant form of order 6 with rigid and continuous boundary conditions. The real part of frequency equation gives the dispersion of Rayleigh waves. For connotation of the numerical results two different reservoir rocks of triclinic symmetry have been considered. The plots of phase velocity against the wave number have been presented. From the plots, we can conclude that in general the phase velocity of Rayleigh waves always decreases with increasing wave number. The thickness of layer also plays a key role for the propagation of Rayleigh waves; it can be observed that as the thickness of layer increases the magnitude of phase velocity decreases rapidly. The sandiness parameter has marginal effect on the phase velocity. These results may helpful in predicting the hidden materials inside the earth with the seismograph produced by the seismic waves and predict about their physical properties.

APPENDIX

$$A_0 = C_{33}C_{35} - C_{35}{}^2, A_1 = -2i\left(C_{15}C_{33} - C_{13}C_{35}\right)$$

$$A_2 = \left(C_{33} + C_{55}\right)\rho_0 c^2 - 4C_{15}C_{35} - C_{11}C_{33} + C_{13}{}^2 + 2C_{13}C_{15} + 2C_{15}C_{35}$$

$$A_3 = -2i\left\{\left(C_{15} + C_{35}\right)\rho_0 c^2 - C_{11}C_{35} + C_{13}C_{15}\right\},$$

$$A_4 = \rho_0{}^2 c^4 - \left(C_{55} + C_{11}\right)\rho_0 c^2 + C_{11}C_{55} - C_{15}{}^2$$

$$B_0 = \eta^2 \mu(\lambda + 2\mu),$$

$$B_1 = \eta\mu(\rho_2 c^2 - \mu) + \eta(\lambda + 2\mu)\{\rho_2 c^2 - (\lambda + 2\mu)\} + \eta^2(\lambda + \mu)^2$$

$$B_2 = (\rho_2 c^2 - \eta\mu)\{\rho_2 c^2 - \eta(\lambda_\lambda + 2\mu)\}$$

$$\mathbf{A}=\begin{pmatrix} 1 & 1 & 1 & 1 & -1 & -1 \\ \delta_1 & \delta_2 & \delta_3 & \delta_4 & -\varepsilon_1 & -\varepsilon_2 \\ K_1 & K_2 & K_3 & K_4 & -\eta\mu_2(i\varepsilon_1 - \zeta_1) & -\eta\mu_2(i\varepsilon_2 - \zeta_2) \\ K_5 & K_6 & K_7 & K_8 & -\eta\{i\lambda_2 - (\lambda_2 + 2\mu_2)\varepsilon_1\zeta_1\} & -\eta\{i\lambda_2 - (\lambda_2 + 2\mu_2)\varepsilon_2\zeta_2\} \\ 1 & 1 & 1 & 1 & 0 & 0 \\ \delta_1 & \delta_2 & \delta_3 & \delta_4 & 0 & 0 \end{pmatrix}$$

$$\mathbf{Y}=(\Psi_{11} \quad \Psi_{12} \quad \Psi_{13} \quad \Psi_{14} \quad \Phi_{11} \quad \Phi_{12})^T$$

$$K_1 = iC_{15} - \delta_1\xi_1 C_{35} + (i\delta_1 - \xi_1)C_{55},$$

$$K_2 = iC_{15} - \delta_2\xi_2 C_{35} + (i\delta_2 - \xi_2)C_{55}$$

$$K_3 = iC_{15} - \delta_3\xi_3 C_{35} + (i\delta_3 - \xi_3)C_{55},$$

$$K_4 = iC_{15} - \delta_4\xi_4 C_{35} + (i\delta_4 - \xi_4)C_{55}$$

$$K_5 = iC_{13} - \delta_1\xi_1 C_{33} + (i\delta_1 - \xi_1)C_{35},$$

$$K_6 = iC_{13} - \delta_2\xi_2 C_{33} + (i\delta_2 - \xi_2)C_{35}$$

$$K_7 = iC_{13} - \delta_3\xi_3 C_{33} + (i\delta_3 - \xi_3)C_{35},$$

$$K_8 = iC_{13} - \delta_4\xi_4 C_{33} + (i\delta_4 - \xi_4)C_{35}$$

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