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SYNTHESIZING AN ADAPTIVE CONTROL SINGLEINPUT-SINGLEOUTPUT SYSTEM FOR UNSTABLE AND DETERMINISTIC CHAOTIC PROCESSES

M. A. Beisenbi[1], A. A. Adilbayev[1], S. O. Akhmetova[1], M. S. Tulenbayev[2] and A. B. Kabanbayev[2]
[1]Eurasian National University after L. N. Gumilyov, Kazakhstan
[2]Taraz Regional University after M. Kh. Dulaty, Kazakhstan
E-Mail: jogitama@gmail.com

ABSTRACT

The paper proposes a solution to the problem of synthesizing an adaptive control system from a reference model with the desired dynamics of unstable and deterministic chaotic processes in the class of “dovetail” catastrophes for single input-single output objects. The task of selecting a reference model with the desired dynamics, the synthesis of the main circuit, and the adaptation circuit are solved by the gradient-speed method of the Lyapunov vector functions. From the conditions of aperiodic robust stability of the reference model with the desired dynamics, the main control loop, and the generalized tunable object, tunable coefficients are calculated, and the control goal is achieved.

Keywords: adaptive, deterministic chaos, swallow, synthesis contour, constant stability, invariant, gradient system, scalar work, catastrophe theories, static accuracy, system of inequalities, dynamic systems.

INTRODUCTION

The studies of the last century revealed a wide variety of dynamics of nonlinear systems and led to one of the most important discoveries of the 20th century in nonlinear dynamical systems: deterministic chaos and a strange attractor [1, 2].

It is now generally accepted that real control objects are nonlinear, and deterministic chaos with the generation of a "strange attractor" is an intrinsic property of any nonlinear deterministic dynamical system [2, 3, 4]. In linearized dynamic systems, this is manifested as a loss of robust stability [5, 6].

Deterministic chaos is manifested in mechanical systems in the form of vibration, and in technical and technological systems in the form of "runaway", which leads to accidents in economic, environmental, biological, medical, social, and other systems of fluctuations that provoke a "crisis".

One of the most promising ways of solving the problem of managing unstable and deterministic chaotic processes is synthesizing a control system in the class of “dovetail” disasters and the use of adaptation methods [7, 8, 9].

In conditions of uncertainty of the control object parameters and external impacts, in adaptive control systems external impacts are compensated, and selecting a reference model with the desired dynamics, synthesizing the main control loop and the adaptation loop in the class of “dovetail” catastrophes [10] is a type of protection against instability and deterministic chaotic process. Studying and synthesizing all the subsystems of adaptive control by the gradient-speed method of the Lyapunov vector functions [11, 12, 13, 15] guarantees aperiodic robust stability of systems, and compensation for the lack of a priori information in the system allows achieving a given control quality.

Methods based on the use of Lyapunov functions [14, 15] are the main ones in studying adaptive control systems. But at present, these methods serve only as a tool for theoretical research [15, 16, and 17] and cannot provide answers to the main questions of synthesizing adaptive control systems in real conditions. The main obstacle, in this case, is the absence of a universal approach to the building of Lyapunov functions.

PROBLEM DEFINITION

Synthesizing an adaptive control single input-single output system for unstable and deterministic chaotic processes in the class of “dovetail” catastrophs from the reference model by the gradient-speed method of the Lyapunov vector function [11, 18, 19].

Let the control system be described by the equation of state

$$\dot{x} = Ax + Bu, \tag{1}$$

where $x(t) \in R^n$is the state vector of the controlled object, $u(t) \in R^m$is the control function vector.

The controlled object matrix and the constant control matrix have the form, respectively:

$$A = \begin{Vmatrix} 0 & 1 & 0 & \dots 0 \\ 0 & 0 & 1 & \dots 0 \\ \dots & \dots & \dots & \dots .. \\ 0 & 0 & 0 & \dots 1 \\ -a_n & -a_{n-1} & -a_{n-2} & \dots -a_1 \end{Vmatrix}, B = \begin{Vmatrix} 0 \\ 0 \\ 0 \\ \dots \\ b_n \end{Vmatrix}.$$

The law of control is set in the form of “dovetail” catastrophs:

$$u_i = -\frac{1}{5}x_i^5 - \frac{1}{3}k_i^{\,1}x_i^3 - \frac{1}{2}k_i^{\,2}x_i^2 + k_i^3 x_i, i = 1,\dots,n \quad . \tag{2}$$

System (1) taking into account the law of control (2) has the expanded form:

$$\begin{cases} \dfrac{dx_1}{dt} = x_2 \\ \dfrac{dx_2}{dt} = x_3 \\ \vdots \\ \dfrac{dx_n}{dt} = -\dfrac{1}{5}b_n x_1^5 - \dfrac{1}{3}b_n k_1^1 x_1^3 - \dfrac{1}{2}b_n k_1^2 x_1^2 + (k_1^3 - \dfrac{a_n}{в_n})\cdot x_1 - \dfrac{1}{5}b_n x_2^5 - \dfrac{1}{3}b_n k_2^1 x_2^3 - \\ -\dfrac{1}{2}b_n k_2^2 x_2^2 + (k_2^3 - \dfrac{a_{n-1}}{b_n})x_2 -,\ldots,-\dfrac{1}{5}b_n x_n^5 - \dfrac{1}{3}b_n k_n^1 x_n^3 - \dfrac{1}{2}b_n k_n^2 x_n^2 + (k_n^3 - \dfrac{a_1}{b_n})x_n + \\ + b_n r_1 + b_n r_2 + \ldots + b_n r_n. \end{cases} \quad (3)$$

Let the reference model be selected in the form:

$$\begin{cases} \dfrac{dx_{м1}}{dt} = x_{м2} \\ \dfrac{dx_{м2}}{dt} = x_{м3} \\ \ldots\ldots \\ \dfrac{dx_{мn-1}}{dt} = x_{мn} \\ \dfrac{dx_{мn}}{dt} = -\dfrac{1}{5}x_{м1}^5 - \dfrac{1}{3}d_1^1 x_{м1}^3 - \dfrac{1}{2}d_1^2 x_{м1}^2 + d_1^3 x_{м1} - \dfrac{1}{5}x_{м2}^5 - \dfrac{1}{3}d_2^1 x_{м2}^3 - \\ -\dfrac{1}{2}d_2^2 x_{м2}^2 + d_2^3 x_{м2} -,\ldots,-\dfrac{1}{5}x_{мn}^5 - \dfrac{1}{3}d_n^1 x_{мn}^3 - \dfrac{1}{2}d_n^2 x_{мn}^2 + d_n^3 x_{мn} + b_n^м r_1 + b_n^м r_2 + \ldots + b_n^м r_n. \end{cases} \quad (4)$$

where $x_м(t) \in R^n$ is the state vector of the reference model; $r(t) \in R^m$ is the setting (external) impact; d^1_i, d^2_i and d^3_i(i=1,…, n) are the set parameters of the reference model determined in accordance with imitation experiment on the reference model.

Selecting the reference model depends on the requirements for the closed system (stability, robustness, the desired type of transient process, the time of the transient process, oscillation, overregulation, static error, etc.). In this case, the reference model should naturally be stable, robust, and aperiodic.

Let's formalize the goal of control by requiring that

$$\lim_{t\to\infty} \varepsilon(t) = 0 \quad (5)$$

where $\varepsilon(t) = x(t) - x_м(t)$ is the error of systems (3) and (4).

Thus, the task is to build a self-adjusting control system (SAS) with a reference model. In accordance with the two-level structure of the SAS, there will be solved the problem in three stages: selecting and studying the reference model of the desired quality of the transient process, synthesizing the main circuit, and synthesizing the adaptation circuit.

STUDYING THE REFERENCE MODEL

Adaptive control systems arose in connection with the need to solve a wide class of applied problems for which traditional methods that require exact knowledge of the parameters of the mathematical model of the control object and the conditions of its functioning are unacceptable. In practice, it is rather difficult to provide an accurate mathematical description of the control object. Moreover, the characteristics of the object in the process of functioning can change significantly. The conditions of the control object functioning, i.e. the system can be under the impact of various perturbations.

In this regard, it seems very effective to select a reference model of an adaptive control system for unstable and deterministic chaotic processes in the class of "dovetail" catastrophes.

Let there be some single input-single output system (4) with the desired transient characteristics obtained on the basis of the simulation experiment on the model of this system (4).

ARPN Journal of Engineering and Applied Sciences

Stationary states of the system are determined as follows [11, 18]:

$$x_{м1}^1 = 0, x_{м2}^1 = 0, x_{м3}^1 = 0,..., x_{мn}^1 = 0 \quad (6)$$

$$x_{мi}^2 = \sqrt[4]{\frac{d_i^3}{4}}, d_i^2 = 6\left(\sqrt[4]{\frac{1}{4}d_i^3}\right)^3, d_i^1 = \sqrt{\frac{1}{4}d_i^3}, i = 1,...,n \quad (7)$$

Studying aperiodic robust stability of stationary state (6) and (7) is carried out on the basis of the gradient-speed method of Lyapunov vector functions [11, 12, 13].

From (4) there are determined the components of the gradient vector from the Lyapunov vector functions $V(x) = (V_1(x), V_2(x), ..., V_n(x))$:

$$\begin{cases} \frac{\partial V_1(x_м)}{\partial x_2} = -x_2, \frac{\partial V_2(x_м)}{\partial x_3} = -x_3, ..., \frac{\partial V_{n-1}(x_м)}{\partial x_n} = -x_n \\ \frac{\partial V_n(x_м)}{\partial x_{м1}} = \frac{1}{5}x_{м1}^5 + \frac{1}{3}d_1^1 x_{м1}^3 + \frac{1}{2}d_1^2 x_{м1}^2 - d_1^3 x_{м1}, \\ \frac{\partial V_n(x_м)}{\partial x_{м2}} = \frac{1}{5}x_{м2}^5 + \frac{1}{3}d_2^1 x_{м2}^3 + \frac{1}{2}d_2^2 x_{м2}^2 - d_2^3 x_{м2}, \\ \frac{\partial V_n(x_м)}{\partial x_{м3}} = \frac{1}{5}x_{м3}^5 + \frac{1}{3}d_3^1 x_{м3}^3 + \frac{1}{2}d_3^2 x_{м3}^2 - d_3^3 x_{м3}, \\ \dots\dots\dots, \\ \frac{\partial V_n(x_м)}{\partial x_{мn}} = \frac{1}{5}x_{мn}^5 + \frac{1}{3}d_n^1 x_{мn}^3 + \frac{1}{2}d_n^2 x_{мn}^2 - d_n^3 x_{мn} \end{cases} \quad (8)$$

Based on the Lyapunov vector functions there are built vector functions in the scalar form:

$$V(x_м) = -\frac{1}{2}x_{м2}^2 - \frac{1}{2}x_{м3}^2 -,...,-\frac{1}{2}x^2{}_{мn} + \frac{1}{30}x_{м1}^6 + \frac{1}{12}d_1^1 x_{м1}^4 + \frac{1}{6}d_1^2 x_{м1}^3 - \frac{1}{2}d_1^3 x_{м1}^2 + $$
$$+\frac{1}{30}x_{м2}^6 + \frac{1}{12}d_2^1 x_{м2}^4 + \frac{1}{6}d_2^2 x_{м2}^3 - \frac{1}{2}d_2^3 x_{м2}^2 +,...,+ \frac{1}{30}x_{мn}^6 + \frac{1}{12}d_n^1 x_{мn}^4 + \frac{1}{6}d_n^2 x_{мn}^3 - \frac{1}{2}d_n^3 x^2{}_{мn} \quad (9)$$

The conditions for the positive definiteness of functions (9) are not obvious; therefore, there should be used the Morse lemma from catastrophe theory [10] to functions (9), and obtained the quadratic form:

$$V(x) \approx -\frac{1}{2}d_1^3 x_{м1}^2 - \frac{1}{2}(d_2^3+1)x_{м2}^2 - \frac{1}{2}(d_3^3+1)x_{м3}^2 -,...,-\frac{1}{2}(d_n^3+1)x_{мn}^2, \quad (10)$$

Positive definiteness of the Lyapunov vector functions (9) is determined by the inequalities:

$$d_1^3 > 0, -(d_2^3+1) > 0, -(d_3^3+1) > 0,..., -(d_n^3+1) > 0, \quad (11)$$

System (4) is represented in deviations relative to a stationary state (7) and aperiodic robust stability of the system in deviations is studied by the gradient-speed method of the Lyapunov vector functions [11,12,13]. Moreover, the constructed Lyapunov vector function satisfies all the conditions of the Morse lemma from catastrophe theories [10] and therefore, the function can be represented in the quadratic form:

$$V(x) \approx \frac{1}{2}d_1^3 x_{м1}^2 + \frac{1}{2}(d_2^3-1)x_{м2}^2 +,...,+ \frac{1}{2}(d_n^3-1)x_{мn}^2, \quad (12)$$

From (12) positive definiteness of the Lyapunov vector functions is set by the condition:

$$d_1^3 > 0, (d_2^3-1) > 0,..., (d_n^3-1) > 0, \quad (13)$$

Thus, system (4) under conditions (11) is robustly stable and there will be a stationary state (6). If the system of inequalities (11) is violated, a new stationary state (7) appears and will actually exist, and will also be stable when the system of inequalities (13) is satisfied. These stationary states do not exist at the same time and are not stable at the same time. This property makes it possible to build a self-organizing robustly stable system.

SYNTHESIZING THE MAIN CONTROL CIRCUIT

The problem is solved under the assumption that the parameters of the control object are known.

A control system with a single input and output is described by the equation of state (3). The Stationary states of the system (3) are determined as follows:

$$x_{1s}^1 = 0, x_{2s}^1 = 0,..., x_{ns}^1 = 0; \quad (14)$$

$$x_{iS}^2 = \sqrt[4]{\frac{k_i^3 - a_{n-i+1}}{4}}, k_i^2 = 6\left(\sqrt[4]{\frac{1}{4}(k_i^3 - a_{n-i+1})}\right)^3, k_i^1 = \sqrt{\frac{1}{4}(k_i^3 - a_{n-i+1})}, i = 1,...,n. \quad (15)$$

ARPN Journal of Engineering and Applied Sciences

Studying the aperiodic robust stability of these stationary states (14) and (15) is carried out on the basis of the gradient-speed method of the Lyapunov vector function [11, 12]. Aperiodic robust stability of stationary states guarantees the fulfillment of the control goal (5).

First of all, there will be studied stationary state (14). The components of the gradient vector of the Lyapunov vector function $V(x) = (V_1(x), V_2(x), \dots, V_n(x))$ are determined from (3).

$$\begin{cases} \frac{\partial V_1(x)}{\partial x_2} = -x_2, \frac{\partial V_2(x)}{\partial x_3} = -x_3, \dots, \frac{\partial V_{n-1}(x)}{\partial x_n} = -x_n \\ \frac{\partial V_n(x)}{\partial x_1} = \frac{1}{5}x_1^5 + \frac{1}{3}k_1^1 x_1^3 + \frac{1}{2}k_1^2 x_1^2 - (k_1^3 - a_n)x_1, \\ \frac{\partial V_n(x)}{\partial x_2} = \frac{1}{5}x_2^5 + \frac{1}{3}k_2^1 x_2^3 + \frac{1}{2}k_2^2 x_2^2 - (k_2^3 - a_{n-1})x_2, \\ \frac{\partial V_n(x)}{\partial x_3} = \frac{1}{5}x_3^5 + \frac{1}{3}k_3^1 x_3^3 + \frac{1}{2}k_3^2 x_3^2 - (k_3^3 - a_{n-2})x_3, \\ ,\dots, \\ \frac{\partial V_n(x)}{\partial x_n} = \frac{1}{5}x_n^5 + \frac{1}{3}k_n^1 x_n^3 + \frac{1}{2}k_n^2 x_n^2 - (k_n^3 - a_1)x_n \end{cases} \tag{16}$$

From (16) there are obtained the Lyapunov vector functions in the scalar form:

$$V(x) = \frac{1}{30}x_1^6 + \frac{1}{12}k_1^1 x_1^4 + \frac{1}{6}k_1^2 x_1^3 - \frac{1}{2}(k_1^3 - a_n)x_1^2 + \\ + \frac{1}{30}x_2^6 + \frac{1}{12}k_2^1 x_2^4 + \frac{1}{6}k_2^2 x_2^3 - \frac{1}{2}(k_2^3 - a_{n-1} + 1)x^2{}_2 + \\ + \frac{1}{30}x_3^6 + \frac{1}{12}k_3^1 x_3^4 + \frac{1}{6}k_3^2 x_3^3 - \frac{1}{2}(k_3^3 - a_{n-2} - 2)x^2{}_3 +, \dots, + \\ + \frac{1}{30}x_n^6 + \frac{1}{12}k_n^1 x_n^4 + \frac{1}{6}k_n^2 x_n^3 - \frac{1}{2}(k_n^3 - a_1 + 1)x^2{}_n \tag{17}$$

The conditions for the positive or negative definiteness of function (17) are imprecise, therefore, there is used the Morse lemma from catastrophe theory [10], and function (17) is represented in the quadratic form:

$$V(x) = -(k_1^3 - a_n)x_1^2 - (k_2^3 - a_{n-1} + 1)x_2^2 - (k_3^3 - a_{n-2} + 1)x_3^2 -, \dots, -(k_n^3 - a_1 + 1)x_n^2, \tag{18}$$

Positive definiteness of quadratic form (18) is determined by the system of inequalities:

$$\begin{cases} k_1^3 - a_n < 0 \\ k_2^3 - a_{n-1} + 1 < 0 \\ k_3^3 - a_{n-2} + 1 < 0 \\ ,\cdots, \\ k_n^3 - a_1 + 1 < 0 \end{cases} \tag{19}$$

Aperiodic robust stability of stationary state (15) is studied by the gradient-speed method of the Lyapunov vector function [11, 12, 13]. In this case, system (3) is represented in deviations relative to a stationary state (15), and studying aperiodic robust stability allows representing the Lyapunov vector function in the quadratic form [11, 12, 13]:

$$V(x) = \frac{1}{2}(k_1^3 - a_n)x_1^2 + \frac{1}{2}(k_2^3 - a_{n-1} - 1)x_2^2 + \frac{1}{2}(k_3^3 - a_{n-2} - 1)x_3^2 +, \dots, + \frac{1}{2}(k_n^3 - a_1 - 1)x_n^2 . \tag{20}$$

From (20) positive definiteness of the Lyapunov vector function is set by ineqaulities:

$$\begin{cases} k_1^3 - a_n > 0 \\ k_2^3 - a_{n-1} + 1 > 0 \\ k_3^3 - a_{n-2} + 1 > 0 \\ \cdots\cdots\cdots \\ k_n^3 - a_1 + 1 > 0 \end{cases} \tag{21}$$

Thus, control system (3) provides aperiodic robust stability for any changes in the undefined parameters. When inequality (19) is satisfied, there exists and will be an aperiodic stable stationary state (14). If the system of inequalities (19) is violated, a new stationary state (15) appears and this stationary state exists and is also stable when the system of inequalities (21) is satisfied. Consequently, self-organization takes place in the system.

From the conditions of aperiodic robust stability of the reference model (11), (13) and the main control loop (19), (21), the feedback coefficients are calculated for the known values of the control object parameters:

$$\begin{cases} k_1^3 = a_n - d_1^3 \\ k_2^3 = a_{n-1} - d_2^3 \\ k_3^3 = a_{n-2} - d_3^3 \\ \cdots\cdots\cdots \\ k_n^3 = a_1 - d_n^3 \end{cases} \tag{22}$$

and

$$\begin{cases} k_1^3 = d_1^3 + a_n \\ k_2^3 = d_2^3 + a_{n-1} \\ k_3^3 = d_3^3 + a_{n-2} \\ \cdots\cdots\cdots \\ k_n^3 = d_n^3 + a_1 \end{cases} \tag{23}$$

Thus, systems (3) and (4) are aperiodically stable and, by virtue of (22) and (23), the goal of control (5) $\varepsilon(t) \longrightarrow 0$at $t \longrightarrow \infty$ for known values of the control object parameters.

SYNTHESIZING THE ADAPTATION CIRCUIT

When developing adaptive control systems, a generalized tunable object is usually developed [7,8]. From the condition of asymptotic robust stability of the generalized tunable object with the reference model of the desired dynamics, there is obtained an adaptation algorithm for the control system of unstable and deterministic choatic processes in the "dovetail" catastrophe class for a single input-single output system.

It should be noted that when $b_n=b_n^M$, the structure of the generalized tunable object fully compensates for external impacts (disturbing impacts), i.e. $(b_n-b_n^M)r_1=0$, $(b_n-b_n^M)r_2=0,...,(b_n-b_n^M)r_n=0$.

The generalized tunable object with respect to the error *ε(t)* is described by the system of equations:

$$\begin{cases} \dfrac{d\varepsilon_1}{dt} = \varepsilon_2 \\ \dfrac{d\varepsilon_2}{dt} = \varepsilon_3 \\ \cdots\cdots \\ \dfrac{d\varepsilon_{n-1}}{dt} = \varepsilon_n \\ \dfrac{d\varepsilon_n}{dt} = -\dfrac{1}{5}(b_n^M - 1)\varepsilon_1^5 - \dfrac{1}{3}(b_n^M k_1^1 - d_1^1)\varepsilon_1^3 - \dfrac{1}{2}(b_n^M k_1^2 - d_1^2)\varepsilon_1^2 + (k_1^3 - \dfrac{a_n}{b_n^M} + \dfrac{d_1^3}{b_n^M})\varepsilon_1 - \\ -\dfrac{1}{5}(b_n^M - 1)\varepsilon_2^5 - \dfrac{1}{3}(b_n^M k_2^1 - d_2^1)\varepsilon_2^3 - \dfrac{1}{2}(b_n^M k_2^2 - d_2^2)\varepsilon_2^2 + (k_2^3 - \dfrac{a_{n-1}}{b_n^M} + \dfrac{d^3{}_2}{b_n^M})\varepsilon_2 -,...,- \\ -\dfrac{1}{5}(b_n^M - 1)\varepsilon_n^5 - \dfrac{1}{3}(b_n^M k_n^1 - d_n^1)\varepsilon_n^3 - \dfrac{1}{2}(b_n^M k_n^2 - d_n^2)\varepsilon_n^2 + (k_n^3 - \dfrac{a_1}{b_n^M} + \dfrac{d^3{}_n}{b_n^M})\varepsilon_n \end{cases} \tag{24}$$

System (24) has a stationary state [18, 19].

$$\varepsilon_{1s}^1 = 0, \varepsilon_{2s}^1 = 0, \varepsilon_{3s}^1 = 0,..., \varepsilon_{ns}^1 = 0 \tag{25}$$

$$\varepsilon_{is}^2 = \sqrt{\frac{1}{4}(k_i^3 - \frac{a_{n-i+1}}{b_n^M} + \frac{d^3{}_i}{b_n^M})}, i = 1,...,n \tag{26}$$

ARPN Journal of Engineering and Applied Sciences

There is studied aperiodic robust stability of the stationary state (25) by the gradient-speed method of the Lyapunov vector function [11,12,13].

From (24) there are defined the components of the Lyapunov vector function gradient $V(x) = (V_1(\varepsilon), \ldots, V_n(\varepsilon))$:

$$\begin{cases} \dfrac{\partial V_1(\varepsilon_1)}{\partial \varepsilon_2} = -\varepsilon_2 \\ \dfrac{\partial V_2(\varepsilon_2)}{\partial \varepsilon_3} = -\varepsilon_3 \\ \ldots\ldots \\ \dfrac{\partial V_{n-1}(\varepsilon_{n-1})}{\partial \varepsilon_n} = -\varepsilon_n \\ \dfrac{\partial V_n(\varepsilon)}{\partial \varepsilon_1} = -\dfrac{1}{5}(b_n^M - 1)\varepsilon_1^5 - \dfrac{1}{3}(b_n^M k_1^1 - d_1^1)\varepsilon_1^3 - \dfrac{1}{2}(b_n^M k_1^2 - d_1^2)\varepsilon_1^2 + (k_1^3 - \dfrac{a_n}{b_n^M} + \dfrac{d_1^3}{b_n^M})\varepsilon_1, \\ \dfrac{\partial V_n(\varepsilon)}{\partial \varepsilon_2} = -\dfrac{1}{5}(b_n^M - 1)\varepsilon_2^5 - \dfrac{1}{3}(b_n^M k_2^1 - d_2^1)\varepsilon_2^3 - \dfrac{1}{2}(b_n^M k_2^2 - d_2^2)\varepsilon_2^2 + (k_2^3 - \dfrac{a_{n-1}}{b_n^M} + \dfrac{d^3{}_2}{b_n^M})\varepsilon_2, \\ \dfrac{\partial V_n(\varepsilon)}{\partial \varepsilon_n} = -\dfrac{1}{5}(b_n^M - 1)\varepsilon_n^5 - \dfrac{1}{3}(b_n^M k_n^1 - d_n^1)\varepsilon_n^3 - \dfrac{1}{2}(b_n^M k_n^2 - d_n^2)\varepsilon_n^2 + (k_n^3 - \dfrac{a_1}{b_n^M} + \dfrac{d^3{}_n}{b_n^M})\varepsilon_n \end{cases} \tag{27}$$

Based on the components of the gradient vector there is built the Lyapunov vector function in the scalar form [11,12,13]:

$$\begin{aligned} V(\varepsilon) = {} & \frac{1}{30}(b_n^M - 1)\varepsilon_1^6 + \frac{1}{12}(b_n^M k_1^1 - d_1^1)\varepsilon_1^4 + \frac{1}{6}(b_n^M k_1^2 - d_1^2)\varepsilon_1^3 - \frac{1}{2}(k_1^3 - \frac{a_n}{b_n^M} + \frac{d_1^3}{b_n^M})\varepsilon^2{}_1 + \\ & + \frac{1}{30}(b_n^M - 1)\varepsilon_2^6 + \frac{1}{12}(b_n^M k_2^1 - d_2^1)\varepsilon_2^4 + \frac{1}{6}(b_n^M k_2^2 - d_2^2)\varepsilon_2^3 - \frac{1}{2}(k_2^3 - \frac{a_{n-1}}{b_n^M} + \frac{d^3{}_2}{b_n^M})\varepsilon_2{}^2 +, \ldots, + \\ & + \frac{1}{30}(b_n^M - 1)\varepsilon_n^6 + \frac{1}{12}(b_n^M k_n^1 - d_n^1)\varepsilon_n^4 + \frac{1}{6}(b_n^M k_n^2 - d_n^2)\varepsilon_n^3 - \frac{1}{2}(k_n^3 - \frac{a_1}{b_n^M} + \frac{d^3{}_n}{b_n^M})\varepsilon_n{}^2 \end{aligned} \tag{28}$$

From (28), the positive definiteness condition is not obvious but this function satisfies all the conditions of the Morse lemma from the theory of catastrophes [10] and therefore, function (28) can be represented in the quadratic form:

$$V(\varepsilon) = -\frac{1}{2}(k_1^3 - \frac{a_n}{b_n^M} + \frac{d_1^3}{b_n^M})\varepsilon^2{}_1 - \frac{1}{2}(k_2^3 - \frac{a_{n-1}}{b_n^M} + \frac{d^3{}_2}{b_n^M})\varepsilon_2{}^2 -, \ldots, - \frac{1}{2}(k_n^3 - \frac{a_1}{b_n^M} + \frac{d^3{}_n}{b_n^M})\varepsilon_n{}^2 , \tag{29}$$

From quadratic form (29) there is determined the condition of positive definiteness, i.e. the condition of aperiodic robust stability of system (29)

$$\begin{cases} -(k_1^3 - \dfrac{a_n}{b_n^M} + \dfrac{d_1^3}{b_n^M}) \geq 0 \\ -(k_2^3 - \dfrac{a_{n-1}}{b_n^M} + \dfrac{d^3{}_2}{b_n^M}) \geq 0 \\ \ldots, \\ -(k_n^3 - \dfrac{a_1}{b_n^M} + \dfrac{d^3{}_n}{b_n^M}) \geq 0 \end{cases} \tag{30}$$

ARPN Journal of Engineering and Applied Sciences

From the conditions of aperiodic robust stability with known values of the system parameters with the desired dynamics b_n^m and d_i^3, $i=1,...,n$, and measurements of the control object parameters of the $a_1,a_2,...,a_n$, there can be obtained an algorithm for calculating the tunable parameters in the form:

$$\begin{cases} k_1^3 = \dfrac{d_1^3}{b_n^м} - \dfrac{a_n}{b_n^м} \\ k_2^3 = \dfrac{d^3{}_2}{b_n^м} - \dfrac{a_{n-1}}{b_n^м} \\ ,..., \\ k_n^3 = \dfrac{d^3{}_n}{b_n^м} - \dfrac{a_1}{b_n^м} \end{cases} \quad (31)$$

Aperiodic robust stability of another stationary state (26) of system (24) is studied by the gradient-speed method of the Lyapunov vector-functions [11, 12]. System (24) is represented in deviations from stationary state (26) of the Lyapunov vector functions in the a quadratic form [11, 12, 13]:

$$V(\varepsilon) = \frac{1}{2}(k_1^3 - \frac{a_n}{b_n^м} + \frac{d_1^3}{b_n^м})\varepsilon^2{}_1 + \frac{1}{2}(k_2^3 - \frac{a_{n-1}}{b_n^м} + \frac{d^3{}_2}{b_n^м} - 1)\varepsilon_2{}^2 +,...,+ \frac{1}{2}(k_n^3 - \frac{a_1}{b_n^м} + \frac{d^3{}_n}{b_n^м} - 1)\varepsilon_n{}^2 \quad (32)$$

From the boundary conditions of aperiodic robust stability for known values of the system parameters with the desired dynamics b_n^M and d_i^3, $i=1,...,n$ when measuring the control object parameters of the $a_1,a_2,...,a_n$,, there can be obtained an algorithm for calculating the tunable parameters in the form:

$$\begin{cases} k_1^3 = \dfrac{a_n}{b_n^м} - \dfrac{d_1^3}{b_n^м} \\ k_2^3 = \dfrac{a_{n-1}}{b_n^м} - \dfrac{d^3{}_2}{b_n^м} \\ ,..., \\ k_n^3 = \dfrac{a_1}{b_n^м} - \dfrac{d^3{}_n}{b_n^м} \end{cases} \quad (33)$$

Thus, during the functioning of the adaptive control system (24) in the area (30) of stationary state (25), the tuning coefficients of the adaptive system are calculated by the formula (31), and in stationary states (26) the tunable coefficients are calculated by the formula (33). The adaptive control system is self-organizing aperiodic robustly stable and provides a given control quality. Stationary states (25) and (26) do not exist simultaneously and are not aperiodic robustly stable. With the loss of stability of one, another stable stationary state appears and self-organization occurs in the system.

CONCLUSIONS

Synthesizing adaptive control systems for deterministic chaotic processes is inextricably linked with ensuring stability, robustness, and quality of control of a closed object with an adaptation loop. Known methods of synthesizing adaptation algorithms include the methods based on the theory of hyperstability, on the organization of sliding modes, on the introduction of an infinitely large gain factor that is mainly used to develop the simplest low-order systems, and on the use of the Lyapunov function including gradient-speed methods. These are the main methods of studying the stability and quality of motion of linear and nonlinear systems. But due to the absence of universal approaches to the construction of the Lyapunov function, these methods are limitedly applied to studying adaptive control systems. Currently, the method of Lyapunov functions is mainly a tool for theoretical research and cannot provide answers to many questions regarding the stability and quality of operation of adaptive control systems in real conditions.

Presentation of the reference model with the desired dynamics, the main circuit, and the "generalized tunable object" of the adaptive control system as gradient systems, and the Lyapunov functions as potential functions from catastrophe theories, allows proposing a universal approach to the construction of Lyapunov functions and applying the gradient-speed method of the Lyapunov vector function to study stability, robustness, and quality of the main circuit and the "generalized tunable object" of the adaptive control system parameters of the control object. The corresponding coefficients of the tunable controller of the main control circuit and the adapter are calculated.

REFERENCES

[1] Brock W. 2011. Chaos theory. M.: Nauka. p. 424.

[2] Andrievsky B. R., Fradkov A. L. 1999. Selected chapters of the theory of automatic control. Ch. 13 Control of nonlinear oscillatory and chaotic systems. St. Petersburg: Nauka. p. 475.

[3] Loskutov A. Yu. 2001. Chaos and control of dynamic systems. Nonlinear dynamics and control / Ed. S. V. Emelyanov, S. K. Emelyanov, S. K. Korovin. M.: Fizmat-lit. 1: 163-216.

[4] Meehan P. A., Asohanthan S. F. 2002. Control of chaotic motion in a dual spin spacecraft with nutational damping. J. Guid. Control Dyn. 25. 2. pp. 209-214.

[5] Polyak B. T., Shcherbakov P. S. 2002. Robust stability and control. M.: Nauka. p. 303.

[6] Dorato P., Vedevalli. 1990. Recent Advances in Robust Control. New York: IEE press.

[7] Methods of classical and modern theory of automatic control: Textbook in 5 vols. 2004. V.5: Methods of modern theory of automatic control / Ed. K. F. Pupkov, N. D. Egunov. M.: publishing house of the Moscow State Technical University named after N.E. Bauman. p. 784.

[8] Petrov B. N., Rutkovsky V. Yu., Krutova I. N., Zemlyakov S. D. 1972. Principles of construction and design of self-adjusting control systems. M.: Mechanical engineering. p. 260.

[9] Fradkov A. L. 1990. Adaptive control in complex systems. M.: Nauka. p. 292.

[10] Gilmore R. 1984. Applied theory of catastrophes. In 2 volumes. V. 1. M.: Mir. p. 287.

[11] Beisenbi M. A. 2015. Studying robust stability of automatic control systems by the function method of A.M. Lyapunov. Astana. p. 204.

[12] Beisenbi M. A. 2019. Controlled chaos in the development of the economic system. Nur- Sultan: Master Po LLP. p. 168.

[13] Beisenbi M. A., Basheyeva Zh. O. 2019. Solving output control problems using Lyapunov gradient-speed vector function. International Journal of Electrical and Computer Engineering. 9(4): 2874-2879.

[14] Malkin I. G. 1966. The theory of stability of motion. M.: Nauka. p. 540.

[15] Della Rossa M., Goebel R., Tanwani A., *et al.* 2021. Piecewise structure of Lyapunov functions and densely checked decrease conditions for hybrid systems. Math. Control Signal Syst. 33, 133-149. https://doi.org/10.1007/s00498-020-00273-9.

[16] Bailey F. N. Application of Lyapunov Second Method to Interconnected system. Journal of the society for Industrial and Applied Mathematics series A Control, 3(3): 443-162. (p. 20). https://doi.org/10.1137/0303030.

[17] Boukas E. K., Liu Z. K. 2002. Robust Stability and Robust Stabilizability. Deterministic and Stochastic Time-Delay Systems. Control Engineering. Birkhauser Boston. https://doi.org/10.1007/978-1-4612-0077-2_4.

[18] Beisenbi M. A. 2011. Methods of increasing the robust stability potential of the control system. Astana, P. 292.

[19] Beisenbi M. A., Erzhanov B. A. 2002. Control systems with increased potential for robust stability. Astana. p. 164.