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EFFECT OF MAGNETO HYDRODYNAMIC USING MICRO-POLAR FLUID BETWEEN CIRCULAR STEPPED PLATE

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ABSTRACT

The impact of MHD on a circular stepped plate lubricated with micro-polar fluid is being investigated. The modified Reynolds equation is derived considering MHD and squeezing film technique. The expressions for pressure, workload, and time height relationship are obtained. The comparison of the effect of MHD in its presence and absence is carried out to enumerate its importance. The variation between the parameters like coupling number, micro-polar parameter, step size, Hartman number, and height are analyzed.

Keywords: circular stepped plate, MHD, micro-polar fluid, squeeze film technique.

INTRODUCTION

Magneto-hydrodynamics defines the physical phenomenon of a motion of a fluid that is capable of conducting electricity like ionized gases, plasma, certain liquid metals like mercury, etc, in a magnetic field. this induces electromagnetic effects in the fluid resulting in electric currents. Which have limited lubricating properties, as well as a poor capacity for transporting work-loads because of their low viscosity. However, because they're thermally stable and have high thermal conductivity, they are good lubricants for bearings that work at extremely high temperatures. This deficiency of the lubricant caused due to its low viscosity can be overcome through certain means. the most practical means can be subjecting them to a magnetic field which will cause an induced electromagnetic force to support the workload in a better way. According to recent research, applying a magnetic field to a rough surface can cause an increase in pressure. N. B. Naduvinamani *et al* [12] work proved that the applied magnetic field increases the load-carrying capacity and squeeze film time. Jaw-Ren Lin *et al*. [8] have solved the squeezing film characteristics between circular stepped plates with ferrofluids in the presence of applied magnetic fields analytically, by taking into consideration the effect of the inertial forces. S Sangeetha and A Govindarajan [14] concluded that the dimensionless pressure decreases as the viscosity variation parameter(pressure dependency) increases and the workload reduces as the couple stress parameter rises with the effect of MHD. It was way back in 1966 when Eringen [4] put forth the concept of the micropolar theory of fluid a particular kind of fluid. The liquid crystals of the micro-polar fluid are dumb-bell shaped, a special characteristic that helps to identify the micro-polar fluid. Some polymeric fluids and fluids with specific additives are well-denoted by the mathematical model that underpins the micropolar fluids due to their physical properties. The lubrication theory for micro-polar fluid was studied by Prakesh and Sinha [13]. Albert E Yousif and Thamir [1] studied friction forces and found their value to be higher when micropolar is used as a lubricant. Squeeze-film techniques have many applications in the field of applied science and industrial engineering, including industrial equipment, gearboxes, bushings, automotive engines, synovial joints, rolling elements, etc. Naduvinamani *et al.* [9] investigated the circular stepped plate using a squeeze-film technique where the plate is lubricated with couple-stress fluid. In comparison to the corresponding Newtonian cases, the gain in load was 62 percent greater. From the study of Hanumagowda *et al* [6] it is concluded that the design engineers can make up the difference for the drawbacks of porosity by selecting appropriate parameters which can prolong the bearing's life and effectiveness under extreme operating conditions. B N Hanumagowda *et al* [5] figured out that as the porosity of the surfaces decreases the time of approach of the bearing surfaces increases and the work-load increases as the permeability parameter decreases. Amit Kumar Rahul, and Pentyala Srinivasa Rao [2] concluded that the circular stepped plates with porous walls have a lower load capacity and squeeze time than non-porous plates. Based on the studies of Hanumagowda *et al* [7] and Cowling [3] this article aims at studying the behaviour of a circular stepped plate that is lubricated with micro-polar fluid and subjected to an external magnetic field. The variation is analyzed with MHD and without MHD using the parameters like Hartmann number, coupling number, micro-polar parameter, and step size.

MATHEMATICAL FORMULATION

The physical arrangement of circular stepped plates moving toward one another at $V=\frac{d(2h)}{dt}$ is given in Figure-1. The lubricant used between the plates is considered a micro-polar fluid. The two plates are subjected to a constant magnetic field B_0 which acts in a direction perpendicular direction fluid flow. assuming the basic assumption which holds true for a hydrodynamic film study that has taken up, the basic equation pertaining to incompressible non-Newtonian micro-polar fluid is given up.

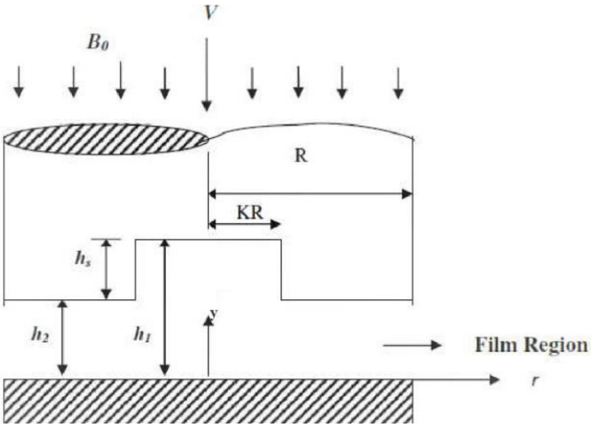

Figure-1. Circular stepped plate.

Equation of conservation of
Linear momentum:

$$\left(\frac{2\mu+\chi}{2}\right)\frac{\partial^2 u}{\partial y^2}+\chi\frac{\partial v_3}{\partial y}=\frac{\partial p}{\partial r} \tag{1}$$

Angular momentum:

$$\gamma\frac{\partial^2 v_1}{\partial y^2}-2\chi v_1-\chi\frac{\partial u}{\partial y}=0 \tag{2}$$

Mass:

$$\frac{1}{r}\frac{\partial(ru)}{\partial r}+\frac{\partial v}{\partial y}=0 \tag{3}$$

In the above equation u and v represent the components of velocity in the direction of r and y respectively, v_1 is the micro-rotational velocity and s represents the electrical conductivity of the fluid. μ represent the viscosity of the micro-polar fluid, χ its spin viscosity and γ its viscosity coefficient.

(i) At the lower plate $(y=-h)$

$$u=0, v=0, v_1=0 \tag{4}$$

(ii) At the upper plate $(y=h)$

$$u=0, v=-V, v_1=0 \tag{5}$$

The solving of eqn. (1) to (3) by applying the boundary conditions (4) and (5) the expression for velocity in the direction of r is obtained as.

$$u=-\frac{a_{11}-b_{11}}{\sigma B_0^2[a_{22}-b_{22}]}\frac{\partial p}{\partial x} \tag{6}$$

where

$a_{11}=\phi_2\sinh(k_2h)[\cosh(k_1h)-\cosh(k_1y)]$
$b_{11}=\phi_1 sinh(k_1h)[cosh(k_2h)-cosh(k_2y)]$
$a_{22}=\phi_2 sinh(k_2h)cosh(k_1h)$
$b_{22}=\phi_1 sinh(k_1h)cosh(k_2h)$

In the above expression the following representation hold true.

$$k_1=\frac{\sqrt{\xi_1+\sqrt{\xi_1^2-4\xi_2}}}{2}, k_2=\frac{\sqrt{\xi_1-\sqrt{\xi_1^2-4\xi_2}}}{2} \tag{7}$$

$$\phi_1=\frac{2\sigma B_0^2-(2\mu+\chi)k_1^2}{2\chi k_1}, \phi_2=\frac{2\sigma B_0^2-(2\mu+\chi)k_2^2}{2\chi k_2} \tag{8}$$

The modified Reynolds equation is determined by integrating the conservation of mass equation(3) through the film thickness and applying the boundary conditions (4) and (5)

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial p}{\partial r}f(h,N,L,M)\right]=-12\frac{dh}{dt} \tag{9}$$

On integrating equation (9) using the boundary condition

$$\frac{dp}{dr}=0 \ \ at \ \ r=0$$

The rate of the pressure with respect to r is obtained as

$$\frac{dp_i}{dr}=\frac{-6\frac{dh}{dt}r}{f(h_i,N,L,M)} \qquad i=1,2 \tag{10}$$

In the above expression

$h_i=h_1 \ \ for \ \ 0\le r\le KR;$
$h_i=h_2 \ \ for \ \ KR\le r\le R;$

$$f(h_i,N,L,M)=\frac{(a_1-b_1)}{\sigma B_0^2k_1k_2(a_2-b_2)}$$

$$a_1=k_2\phi_2 sinh(k_2h_i)[cosh(k_1h_i)hk_1-sinh(k_1h_i)]$$

$$b_1=k_1\phi_1 sinh(k_1h_i)[cosh(k_2h_i)hk_2-sinh(k_2h_i)]$$
$$a_2=\phi_2 sinh(k_2h_i)cosh(k_1h_i)$$
$$b_2=\phi_1 sinh(k_1h_i)cosh(k_2h_i)$$
$$N=\left(\frac{\chi}{2\mu+\chi}\right)^{\frac{1}{2}}, L=\left(\frac{\gamma}{4\mu}\right)^{\frac{1}{2}}$$
$$M=B_0\left(\frac{\sigma}{\mu}\right)^{\frac{1}{2}}$$

The relevant boundary condition for the pressure are

$$p_1=p_2 \ \ at \ \ r=KR, \tag{11}$$

$$p_2=0 \ \ \ at \ \ r=R \tag{12}$$

Integrating equation (10) and applying the above the boundary condition (11) and (12), the expression for pressure taken the form.

$$p_1=-3\frac{dh}{dt}\left[\frac{(K^2R^2-r^2)}{f_1(h_1,N,L,M)}+\frac{R^2(1-K^2)}{f_2(h_2,N,L,M)}\right] \tag{13}$$

$$p_2=-3\frac{dh}{dt}\frac{(R^2-r^2)}{f_2(h_2,N,L,M)} \tag{14}$$

The following non-dimensional variable and parameters are introduced as

$$p_1^* = -3\left[\frac{(K^2-r^2)}{f_1(h_1^*,N,L,M)} + \frac{(1-K^2)}{f_2(h_2^*,N,L,M)}\right] \quad (15)$$

$$p_2^* = -3\frac{(1-r^2)}{f_2(h_2^*,N,L,M)} (16)$$

Integrating the film pressure equation (15) and (16), the expression for work-load is obtain as

$$W = 2\pi\int_0^{KR} rp_1dr + 2\pi\int_{KR}^{R} rp_2dr \quad (17)$$

In a dimensionless form, the work-load is expressed as

$$W^* = \left[\frac{k^4}{f_1(h_1^*,N,L,M)} + \frac{1-k^4}{f_2(1,N,L,M)}\right] \quad (18)$$

The Time-height relation is calculated by using the equation of the work-load. The squeezing time required to reduce the film thickness from an initial h_0 of h_2 to a final h_f value is given by

$$T = \frac{-3\pi R^4}{2W}\int_{h_0}^{h_f}\left[\frac{k^4}{s_1(h_2,h_s,N,L,M)} + \frac{1-k^4}{s_2(h_2,N,L,M)}\right]dh_2 \quad (19)$$

where $s_1(h_2,h_s,N,L,M) = \frac{(a-b)}{\sigma B_0^2 k_1 k_2(c-d)}$

$$a = \left(k_2\phi_2 sinh(k_2h_2(1+\frac{h_s}{h_2}))\right)\left[cosh(k_1h_2(+\frac{h_s}{h_2}))h_2(1+\frac{h_s}{h_2})k_1 - sinh(k_1h_2(1+\frac{h_s}{h_2}))\right]$$

$$b = \left(k_1\phi_1 sinh(k_1h_2(1+\frac{h_s}{h_2}))\right)\left[cosh(k_2h_2(1+\frac{h_s}{h_2}))h_2(1+\frac{h_s}{h_2})k_2 - sinh(k_2h_2(1+\frac{h_s}{h_2}))\right]$$

$$c = \left(\phi_2 sinh(k_2h_2(1+\frac{h_s}{h_2}))cosh(k_1h_2(1+\frac{h_s}{h_2}))\right)$$

$$d = \left(\phi_1 sinh(k_1h_2(1+\frac{h_s}{h_2}))cosh(k_2h_2(1+\frac{h_s}{h_2}))\right)$$

$$s_2(h_2,N,L,M) = f_2(h_2,N,L,M)$$

The non-dimensional time-height relation is

$$T^* = \int_{h_f}^{1}\left[\frac{k^4}{s_1(h_2^*,h_s^*,N,L,M)} + \frac{1-k^4}{s_2(h_2^*,N,L,M)}\right]dh_2 \quad (20)$$

RESULTS AND DISCUSSIONS

The effects of MHD is analysed on the circular stepped plate lubricated with micro-polar fluid. The Hartmann number $M = B_0\left(\frac{\sigma}{\mu}\right)^{\frac{1}{2}}$ helps to estimate the effect of MHD on the plates. The effect of micro-polar fluid is analyzed using the coupling number $N = \left(\frac{\chi}{2\mu+\chi}\right)^{\frac{1}{2}}$ and micro-polar parameter $l = \left(\frac{\gamma}{4\mu}\right)^{\frac{1}{2}}$

Dimensionless Pressure

Figure-2 depicts a change of pressure(p^*) with distance(r^*) for distinct values of N and M . It has been noticed that when the value of the coupling number $N(0.1,0.2,0.3)$ rises, likewise does the pressure. Figure-3 depicts a change of pressure(P^*) and distance(r^*) for varying values of l and M. Pressure rises when the value of the micro-polar parameter $l(0.1,0.25,0.3)$ increases. The rise in pressure can be attributed to the fluid that gets built up in the fluid film region due to the resistance offered to the flow by the viscosity of the particles and the spin of the particles of the micro-polar fluid. In consideration with respect to the Hartmann number M, it is observed that when $M = 0$ (absence of MHD) the pressure built is much lesser than when $M = 4$. The same case holds true for figure 3 and Figure 4 too. A variation of P^* with k^* for varying values of H and M shown in Figure-4. Pressure drops as the value of $H(1.1,1.2,1.4)$ rises, and pressure increases as the step size k^* rises.

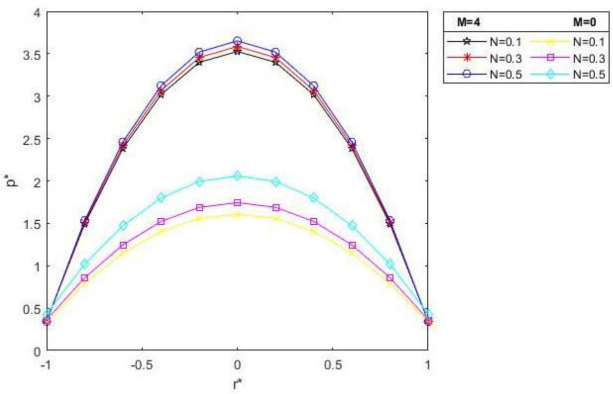

Figure-2. Plot of p^* with r^* for distinct values of N and M.

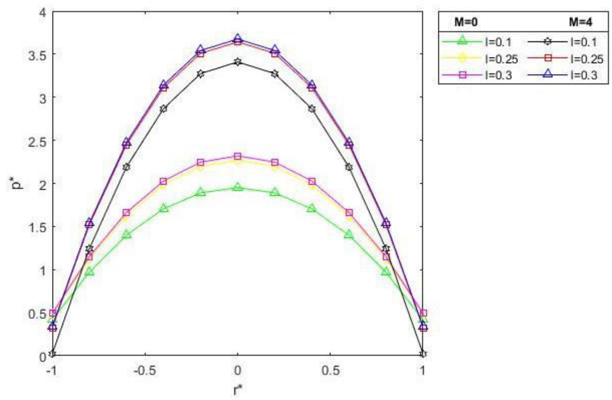

Figure-3. Plot of p^* with r^* for distinct values of M and l.

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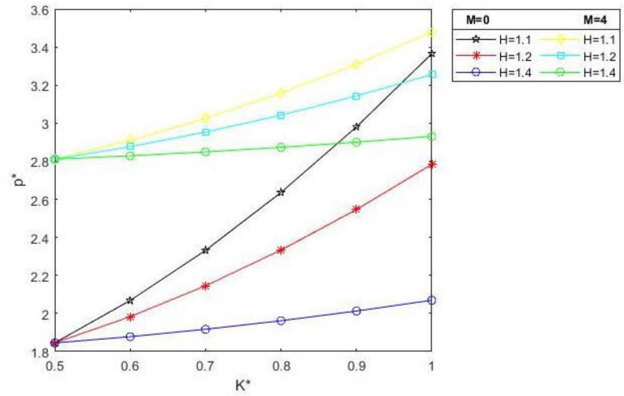

Figure-4. Plot of p* with k^* for distinct values of H and M.

Dimensionless Work Load

Figure=5 depicts the change in H^* and w^* for diverse M and k values. It is noticed that increasing the step size $k(0.4,0.6,0.8)$ decreases the work-load w^* while raising the height H^* reduces the work-load $w*$. The change in work-load(w^*) and H^* for varying values of l and M shown in Figure-6. It is noticed that raising the micro-polar parameter $l(0.1,0.15,0.2)$ it enhances the work-load w^*, and raising the height H reduces the work-load w^*. Figure-7 depicts the change in w^* and k for distinct values of N and M. It is found that raising the coupling number $N(0.2,0.4,0.6)$ enhances the work-load $w*$ and raising the height H^* reduces the work-load w^*.

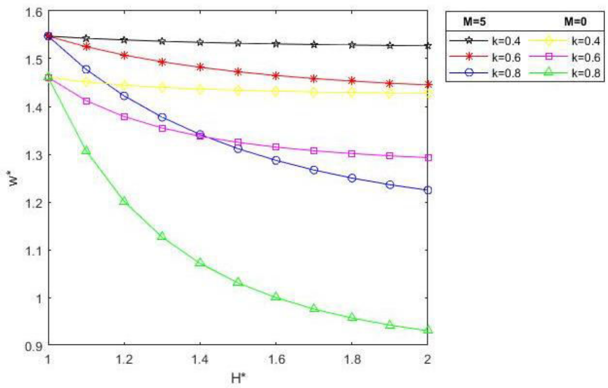

Figure-5. Plot of w^* with H^* for distinct values of k and M.

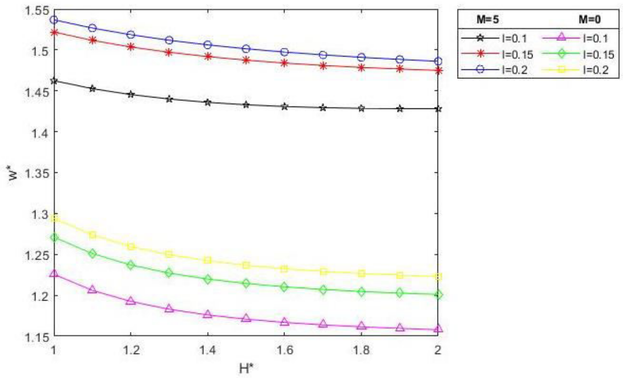

Figure-6. Change in w^* with H^* for various values of l and M.

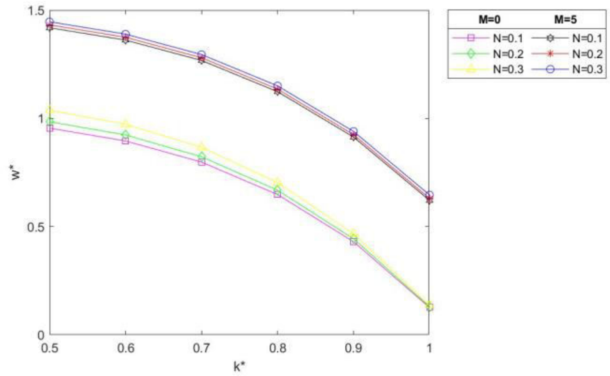

Figure-7. Change in w^* with k^* for various values of N and M.

Dimensionless Time and Height Relationship

Figure-8 shows the relationship between T^* and h_f for distinct values of M and N. It is noted that raising the coupling number $N(0.1,0.3,0.5)$ raises the squeezing time T^*, and increasing the final height h_f decreases the squeeze time $T*$.The variation of T^* with k for various values of N and M. It is noticed that raising the coupling number $N(0.1,0.3,0.5)$ increases the squeeze time T^*, and raising the final height k reduces the squeezing time T^* shown in Figure-9.

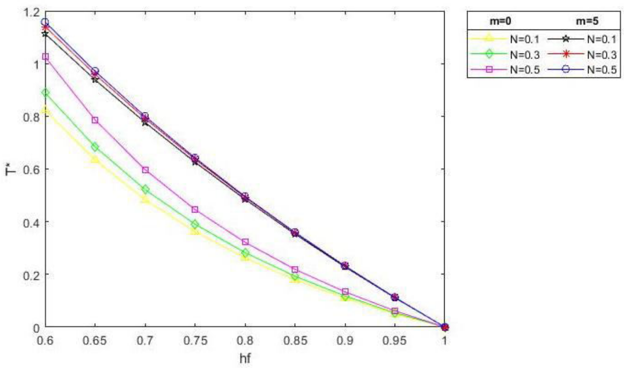

Figure-8. Change in of T^* with h_f for various values of N and M.

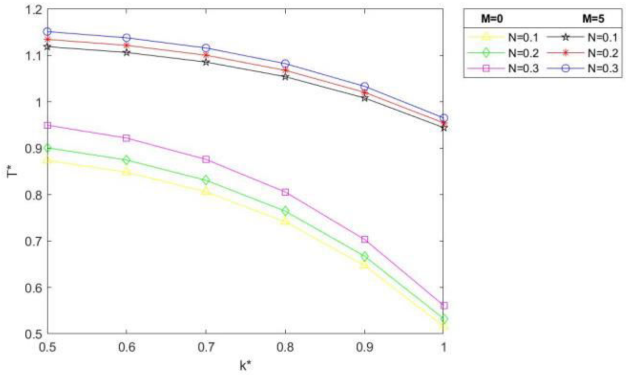

Figure-9. Change in T^* with k for various values of N and M.

The relative percentage increase in dimensionless the squeeze film time T, R_{T^*} and load bearing capacity W,R_{W^*} are determined by $R_{W^*} = \left(\frac{W^*_{magnetic}}{W^*_{non-magnetic}} - 1\right) \times 100$ and $R_{T^*} = \left(\frac{T^*_{magnetic}}{T^*_{non-magnetic}} - 1\right) \times 100$ the values of R_{W^*} and R_{T^*} for the values of M and k are take and listed in the table 1. It is observed from the table for the values of $k = 0.5, N = 0.2$ and $M = 5$ it decreases 39 percent and 21 percent respectively R_{W^*} and R_{T^*}.

Table-1. Change in R_{W^*} and R_{T^*} for distinct values of step height $k$and $N = 0.2,0.3$ with $M = 5$, $l = 0.2$

k	N	R_W	R_T
0.5	**0.1**	**48.5623**	**28.0602**
	0.2	45.4533	25.9499
	0.3	39.4572	21.2448
0.6	0.1	52.0256	30.4534
	0.2	48.8725	28.2780
	0.3	42.8096	23.4617
0.7	0.1	58.8510	34.7259
	0.2	55.6112	32.4344
	0.3	49.4172	27.4197

CONCLUSIONS

The following results are obtained based on the observations on a circular stepped plate lubricated with micro-polar fluid in the presence of MHD.

- An increase in the values of coupling number N and micro-polar parameter l^* increases the pressure, work-load and squeeze time.
- As the value taken by the step size k rises with height H^* there is a dip in the workload, pressure, and squeeze time.
- The comparison work is made between the with MHD and without MHD ($M = 0$). It is observed that the MHD case is greater than the non-MHD cases.
- The relative workload and the relative time drops when rising the range of coupling number N and increasing the rang of step size k, the values of R_W and R_T are also increased.
- The relative work load is greater than the relative time.

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