ACCURACY DOMINANT FREQUENCIES OF LOMB - SCARGLE PERIODOGRAM IN MODELING MONTHLY RAINFALL

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ABSTRACTS

In the field of hydrology, many researchers have developed time series models. Usually, the modelling uses time series data, such as daily rainfall, monthly rainfall, etc. In order to produce a good model, the frequencies used in the model must represent the data being modelled. Otherwise, the inaccurate frequencies used to cause the resulting model to be inaccurate. Several methods or techniques have been used to estimate the frequencies contained in time series data. These methods are the Fast Fourier Transform and Lomb-Scargle periodogram methods. This study used the Lomb-Scargle periodogram method to predict monthly rainfall time series data. In this research, the Lomb periodogram results in rainfall frequencies. The frequencies are created by using monthly rainfall time series data. Using the frequencies, the dominant frequencies can be selected. Periodic modelling of monthly rainfall time series can be adequately simulated using the dominant frequencies. The correlation coefficient between the monthly rainfall data with the monthly rainfall model can be used to measure the accuracy of the monthly rainfall model.

Keywords: monthly rainfall, lomb periodogram, dominant frequency.

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INTRODUCTION

In recent years, the modelling of time series data has been carried out in hydrology. This modelling is intended to simulate changes in hydrological variables in a time series. Using this time series data model, it is expected that future variations of hydrological variables can be predicted well.

It is known that the time series hydrological model is assumed to consist of several components, such as a trend component, a periodic component, and a stochastic component or a random time series component [1][2][3][4][5][6]. These components are assumed to compose the time series data. For rainfall modelling, a trend component is neglected [1] [2] [5].

The Fourier equation is always used to model the periodic components of the time series. However, an accurate time series periodic component frequency is needed to produce a periodic model close to the existing reality. Therefore, in recent years many researchers have conducted studies to obtain the correct method to produce precise frequencies [7][8][9][10]. Fast Fourier Transform (FFT) [11][4][12] and Lomb and Lomb - Scargle periodogram methods [13][14][15][16] are methods that usually used to get frequencies of periodicities. Then it is hoped that this frequency can be used to produce a periodic model that can predict hydrological time series in the future.

Many researchers have also studied stochastic components in this field [17][18][19]. Many variables in the field of hydrology, this component is not too dominant compared to the periodic components. Some studies consider the stochastic component a random value or noise.

In this study, the Lomb-Scargle periodogram is used to produce the dominant period or frequency. This dominant frequency is used to model the periodic monthly rainfall model. In addition, this periodic model is also used to predict the monthly rainfall in the future.

MATERIALS AND METHODS

Study Area

The study area comes under the humid region of the Province of Jakarta, Indonesia.

Collection of Rainfall Data

Daily rainfall data of the Sukarno Hatta region was collected from the Indonesian Meteorological, Climatological, and Geophysical Agency, Province of Jakarta. Rainfall data for 21 years (1998-2020) was used in the study.

Research Methodology

The mathematical procedure adopted for formulating a predictive model has been discussed as follows: The principal aim of the analysis was to obtain a reasonable model for estimating the generation process and its parameters by decomposing the original data series into its various components.

In general, time series data can be decomposed into an equation with a deterministic component, where this can be formulated into values in the form of components that are exact solutions and components which is stochastic, where this value is always represented as a function consisting of several time series data functions. Time series data x(t) is presented as a model consisting of the following functions [2][3][4][5][6]:

$$x(t) = T(t) + P(t) + S(t) \tag{1}$$

Where t is a time, x(t) represents the observed rainfall value, T(t) is a trend component, P(t) is a periodic component, and S(t) is a stochastic or random component or residual component. The trend component describes the change in the increase in the value of the data concerning the length of the lengthy recording of rainfall data during the recording of rainfall data and ignoring the fluctuation component with short duration.

In this study, it is assumed that there is no trend for the rainfall time series [1][2][5]. So this Equation can be represented as follows:

$$x(t) \approx P(t) + S(t) \tag{2}$$

Equation (2) is the approximate Equation for the rainfall data used in this research. This shows that the rainfall data consists of periodic (P) and stochastic (S) or random components.

Lomb Periodogram

Lomb-Scargle (L.S.) periodogram [20][21] is a method that can be used to find frequencies or recurrence of a data series such as rainfall time series. This method can be presented in the following Equation [22][23][24][25][26]:

$$P(f) = \frac{1}{2\sigma^2}\left\{\frac{\left[\sum_{i=1}^{n}(x_i-\bar{x})\cos(\omega[t_i-\tau])\right]^2}{\sum_{i=1}^{n}\cos^2(\omega[t_i-\tau])} + \frac{\left[\sum_{i=1}^{n}(x_i-\bar{x})\sin(\omega[t_i-\tau])\right]^2}{\sum_{i=1}^{n}\sin^2(\omega[t_i-\tau])}\right\} \tag{3}$$

Here, $\omega = 2\pi f$ is the angular frequency. Where τ is defined as follows:

$$\tan(2\omega\tau) = \frac{\sum_{i=1}^{n}\sin(2\omega t_i)}{\sum_{i=1}^{n}\cos(2\omega t_i)} \tag{4}$$

This method is generally used by experts in astronomy and medical scientists. However, this method can also be used in other fields, such as civil engineering, with serial data to be analysed. Like the spectral method, using the Lomb-Scargle Periodogram, we get a power spectrum or Intensity of rainfall series $P(f)$ in the frequency domain.

Periodic Component

The periodic component P(t) corresponds to an oscillating displacement for a given interval [8][5]. The existence of P(t) is identified by using the Lomb-Scargle periodogram method. The oscillating part indicates P(t) using period or angular frequency; some peak periods or angular frequencies can be estimated. The frequencies obtained from the spectral method clearly show periodic variations. Frequencies of the periodic component P(t) can also be written in terms of the angular frequency (ω_r). Furthermore, an equation can be expressed in Fourier form as follows [10][19][27][28][5]:

$$\hat{P}(t) = S_o + \sum_{r=1}^{k} A_r \sin(\omega_r.t) + \sum_{r=1}^{k} B_r \cos(\omega_r.t) \tag{5}$$

$P(t)$ is a periodic component, but $\hat{P}(t)$ is a model of the periodic component. S_o is an average value, ω_r is an angular frequency (radian), t is time (day), A_r and B_r are coefficients of Fourier components, *and* k is a few significant components.

Determination of Periodic Parameters

The least square method can be applied to determine periodic parameters or Fourier coefficients A_r and B_r. If $P(t)$ is a periodic component or observed time series data and $\hat{P}(t)$ is a model of time series data, then some squares of errors (E^2) can be presented as follows:

$$E^2 = \sum_{t=1}^{t=n}\left\{P(t) - \hat{P}(t)\right\}^2 \tag{6}$$

$$E^2 = \sum_{t=1}^{t=n}\left\{P(t) - S_o - \sum_{r=1}^{k} A_r \sin(\omega_r.t) - \sum_{r=1}^{k} B_r \cos(\omega_r.t)\right\}^2 \tag{7}$$

Where E^2 depends on A_r, B_r, and ω_r. And a required condition for E^2 is some squares of errors E^2 should be minimum if satisfying conditions as follows [16][5],

$$\frac{\partial E^2}{\partial A_r} = \frac{\partial E^2}{\partial B_r} = 0 \tag{8}$$

Where r = 1, 2, 3, ... , k. Furthermore, k is several frequencies. Based on Equation (8), we can findequations as follow:

$$\hat{P}(t) = S_o + \sum_{r=1}^{r=k} C_r\, Cos(\omega_r t - \varphi_r) \tag{9}$$

Equation (9) is a periodic model of monthly rainfall series. $C_r = \sqrt{(A_r)^2 + (B_r)^2}$ are rainfall amplitudes of monthly rainfall series, and $\varphi_r = atan\left(\frac{B_r}{A_r}\right)$ are phases of monthly rainfall series.

RESULT AND DISCUSSIONS

To test the statistical characteristics of the monthly rainfall time series, 21 years of data (1998-2020) of monthly rainfall from station Sukarno Hatta was taken. Variations of monthly rainfall time series for 21 years from Sukarno Hatta station can be presented as follows:

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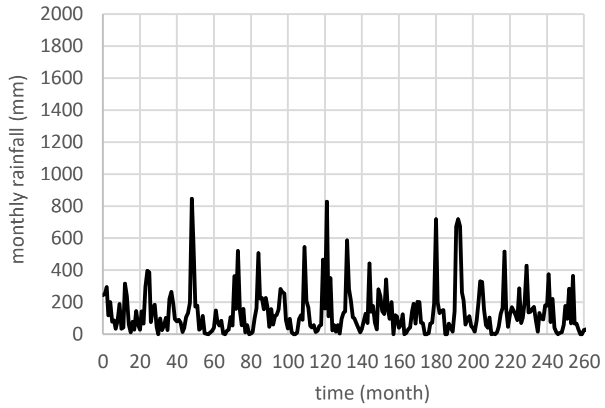

Figure-1. Monthly rainfall from Sukarno Hatta station.

13 Dominant Frequencies

Based on rainfall data series from the station of Sukarno Hatta (Figure-1), rainfall frequencies can be generated by transforming the data using a Lomb-Scargle (L.S.) periodogram method, the rainfall series in the time domain transformed to be rainfall series in the frequency domain. Using 21 years or 264 months of rainfall time series, 264 frequencies can be generated, as shown in the figures below:

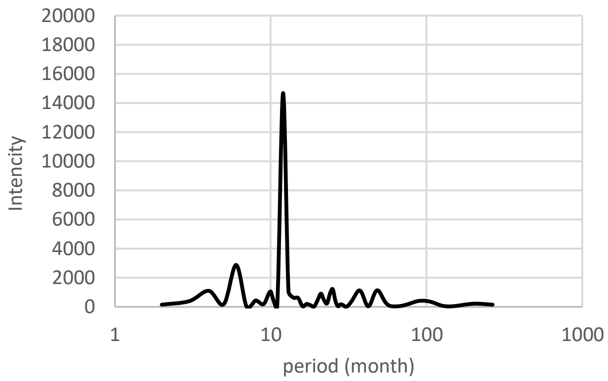

Figure-2. L.S. period of monthly rainfall from Sukarno Hatta station.

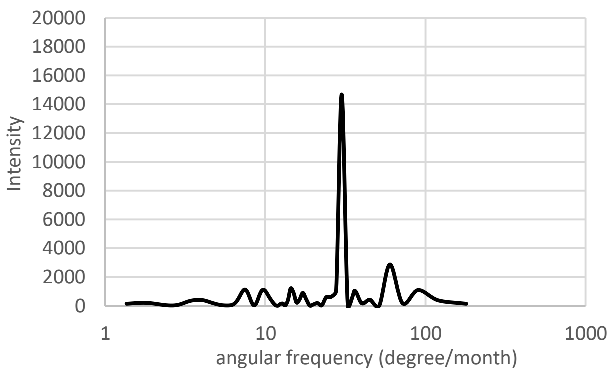

Figure-3. L.S. frequency of monthly rainfall from Sukarno Hatta station.

From Lomb periods of monthly cumulative rainfall (Figure-2), and by using the L.S. periodogram method, dominant periods of rainfall time series can be assumed and calculated as,

If Intensity (period (i)) > Intensity (period (i-1)) and
Intensity (period (i)) > Intensity (period (i+1))
then
period(i) selected as a dominant period

Dominant frequency or period is identified using the periodogram's peak intensity. Based on this assumption, 13 dominant periods were generated from the L.S. periods. In the next step, 13 dominant periods are ordered or ranked from maximum to minimum Intensity. Thus, dominant periods of rainfall time series are presented such as in the following Table-1.

Table-1. 13 Periods and angular frequencies rank based on the maximum Intensity.

Rank	Intensity	period (month)	angular frequency (degree/month)
1	14675	12	30.0
2	2871	6	60.0
3	1208	25	14.4
4	1124	48	7.5
5	1122	37	9.7
6	1094	4	90.0
7	1052	10	36.0
8	901	21	17.1
9	426	8	45.0
10	413	94	3.8
11	211	208	1.7
12	189	17	21.2
13	159	28	12.9

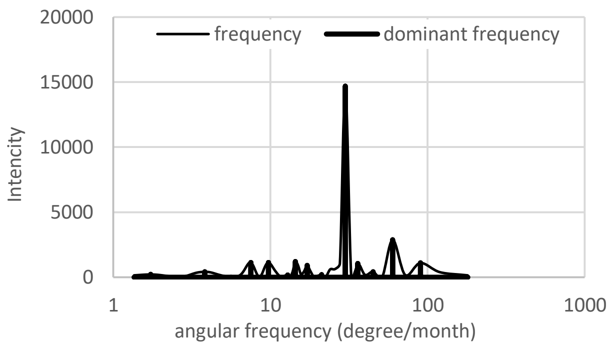

Figure-4. Intensity versus frequency and dominant frequency.

Periodic Model Using 13 Dominant Frequencies

By using 13 dominant frequencies, we can compute periodic modelling of monthly rainfall as presented as follows:

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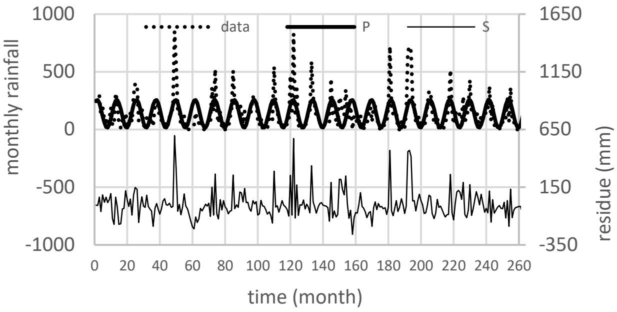

Figure-5. Periodic modelling of monthly rainfall using 1frequency (R1).

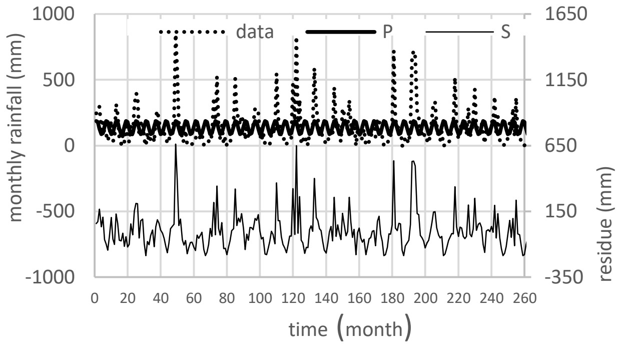

Figure-6. Periodic modelling of monthly rainfall using 1frequency (R2).

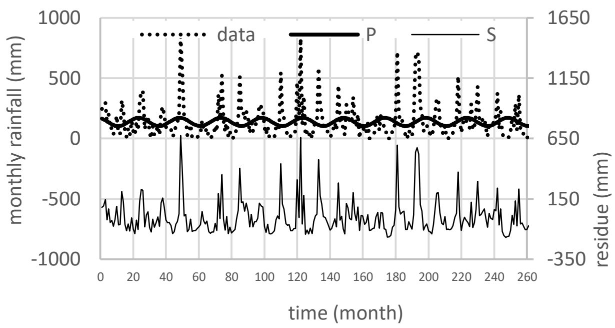

Figure-7. Periodic modelling of monthly rainfall using 1frequency (R3).

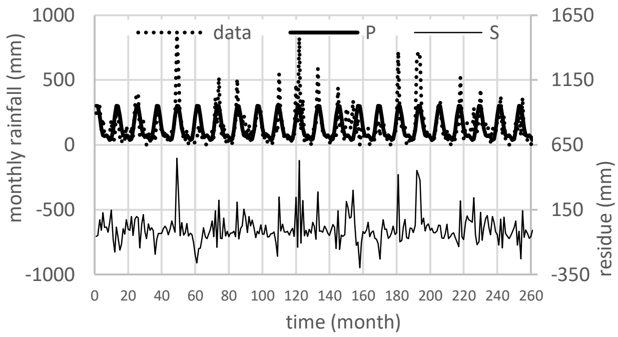

Figure-8. Periodic modelling of monthly rainfall using 2frequencies (R1-2).

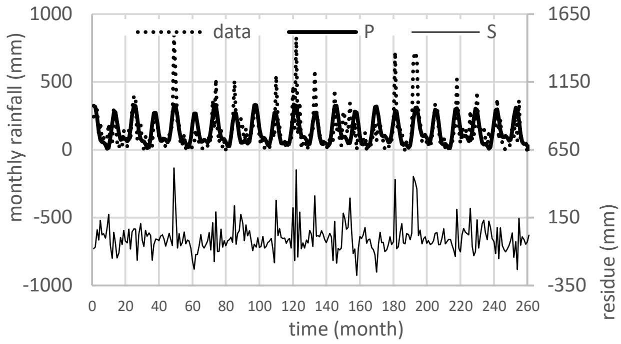

Figure-9. Periodic modelling of monthly rainfall using 3frequencies (R1-3).

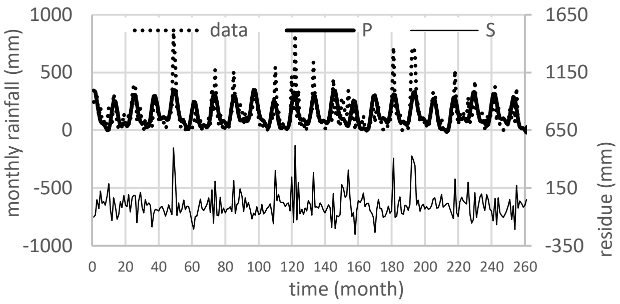

Figure-10. Periodic modelling of monthly rainfall using 4frequencies (R1-4).

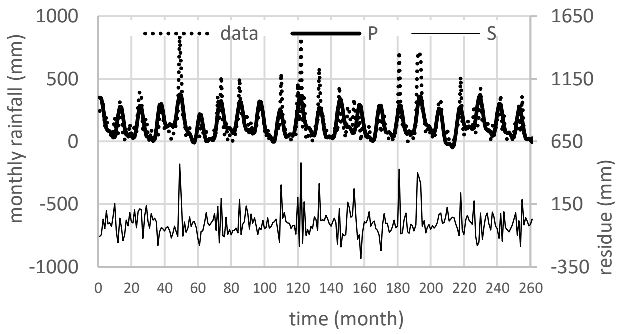

Figure-11. Periodic modelling of monthly rainfall using 5frequencies (R1-5).

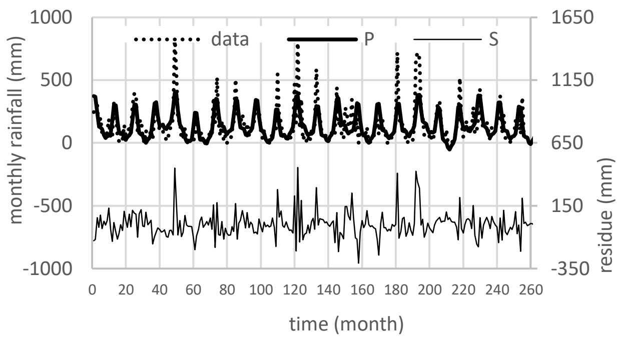

Figure-12. Periodic modelling of monthly rainfall using 6frequencies (R1-6).

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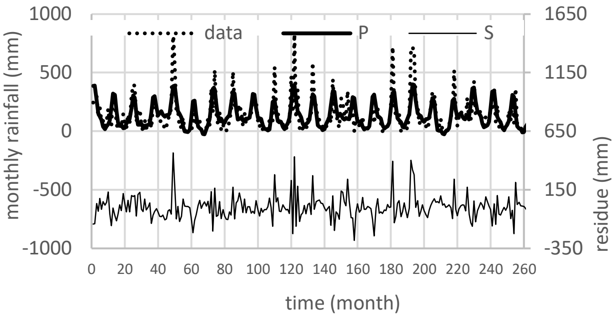

Figure-13. Periodic modelling of monthly rainfall using 7frequencies (R1-7).

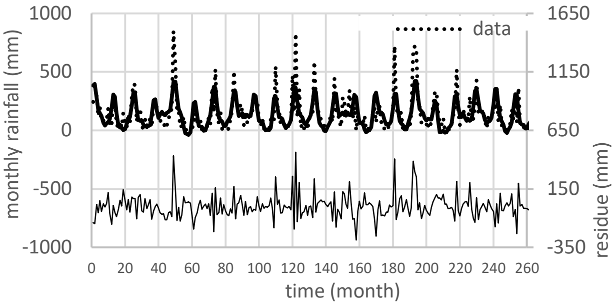

Figure-14. Periodic modelling of monthly rainfall using 8frequencies (R1-8).

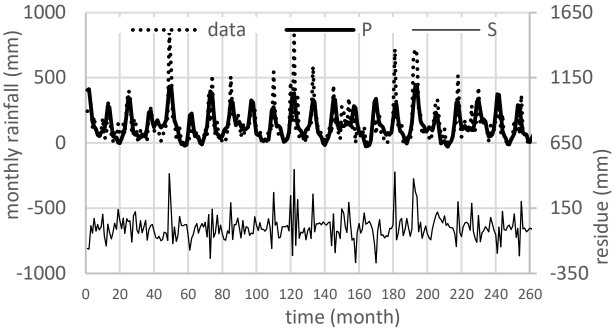

Figure-15. Periodic modelling of monthly rainfall using 9frequencies (R1-9).

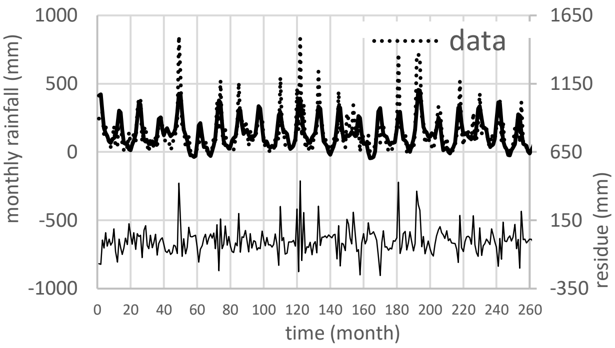

Figure-16. Periodic modelling of monthly rainfall using 10 frequencies (R1-10).

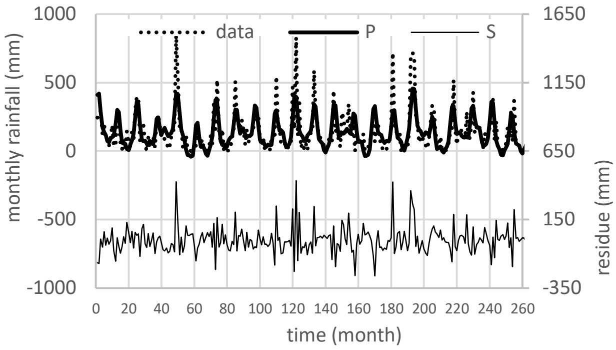

Figure-17. Periodic modelling of monthly rainfall using 11frequencies (R1-11).

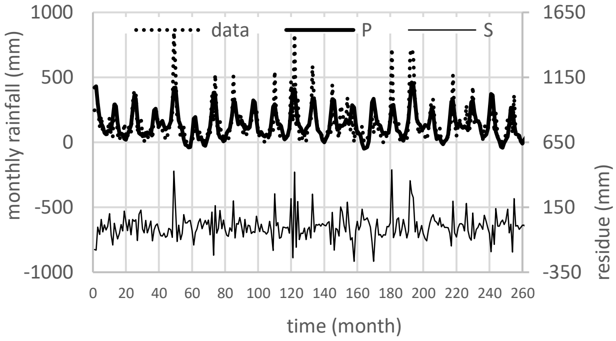

Figure-18. Periodic modelling of monthly rainfall using 12frequencies (R1-12).

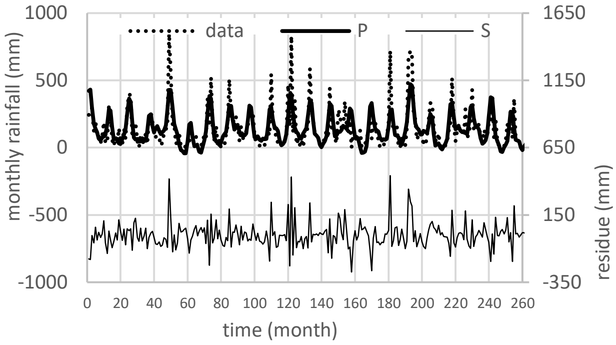

Figure-19. Periodic modelling of monthly rainfall using 13frequencies (R1-13).

Correlation coefficients of the periodic modelling of monthly rainfall can be arranged based on some dominant frequencies, as presented in Table-2,

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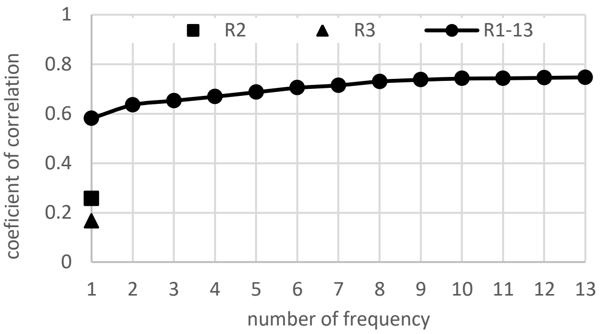

Figure-20. Correlation coefficient versus the number of frequencies.

Table-2. The correlation coefficient of periodic modelling.

Number Of Frequencies	Coefficient Of Correlation	Rank
1	0.5810	R1
1	0.2570	R2
1	0.1667	R3
2	0.6353	R1-2
3	0.6526	R1-3
4	0.6690	R1-4
5	0.6870	R1-5
6	0.7047	R1-6
7	0.7148	R1-7
8	0.7302	R1-8
9	0.7372	R1-9
10	0.7422	R1-10
11	0.7432	R1-11
12	0.7452	R1-12
13	0.7469	R1-13

Dominant Frequencies of Rainfall Residue

The residue of the monthly rainfall model using the 13 dominant frequencies is presented as shown in Figure-21 below:

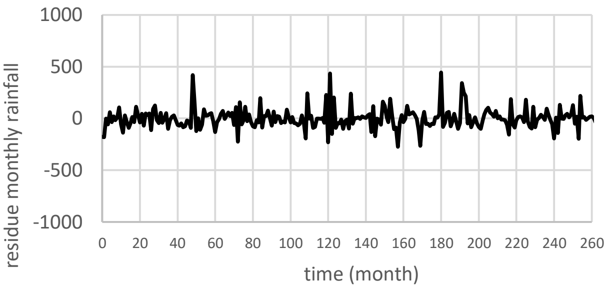

Figure-21. The residue of the periodic modelling of monthly rainfall for 13 frequencies.

To get a better monthly rainfall model. We should get more dominant frequencies. The dominant frequencies can be decomposed from the residue of the periodic rainfall model that has been generated. Using the Lomb periodogram method resulted in frequencies and periods of monthly rainfall residues such as in Figure-22 and Figure-23.

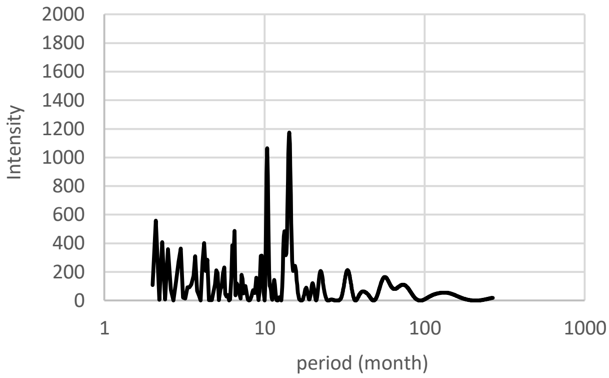

Figure-22. Lomb periods of monthly rainfall residue.

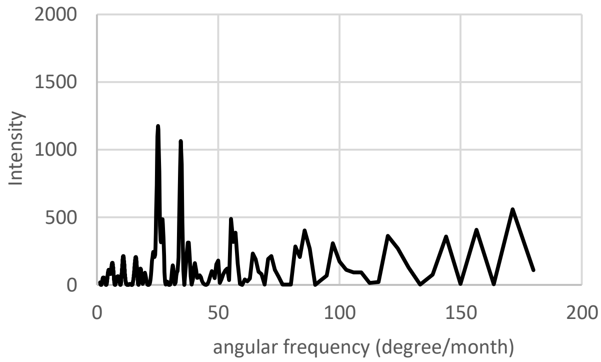

Figure-23. Lomb frequencies of monthly rainfall residue.

From Figure-22, Figure-23, and by using Equation (38), 29 dominant frequencies of monthly rainfall residue are calculated and presented in Table-3 as follows:

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Table-3. 29 Dominant frequencies of rainfall residue.

Rank	Intensity	period (month)	angular frequency (degree/month)
1	1175	14.3	25.17
2	1065	10.4	34.62
3	558	2.1	171.43
4	488	6.5	55.38
5	486	13.3	27.07
6	408	2.3	156.52
7	402	4.2	85.71
8	386	6.3	57.14
9	364	3	120.00
10	358	2.5	144.00
11	314	9.6	37.50
12	309	3.7	97.30
13	284	4.4	81.82
14	243	15.5	23.23
15	233	5.6	64.29
16	212	5	72.00
17	206	22.4	16.07
18	180	7.2	50.00
19	160	8.9	40.45
20	144	11.5	31.30
21	121	20	18.00
22	120	6.7	53.73
23	103	7.6	47.37
24	93	3.3	109.09
25	89	18.2	19.78
26	72	8.5	42.35
27	40	5.9	61.02
28	24	12.5	28.80
29	7	26.2	13.74

They use 42 frequencies (13 frequencies + 29 frequencies from rainfall residue) generated periodic modelling of monthly rainfall time series as presented in Figure-24.

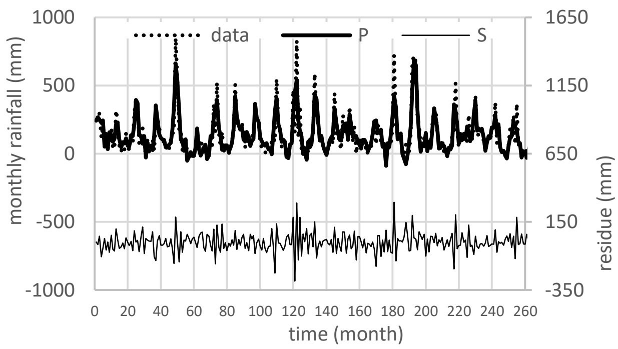

Figure-24. Periodic modelling of monthly rainfall using 42 dominant frequencies.

Using 13 frequencies, the correlation coefficient is about 0.7469 (Figure-19), but by using 42 frequencies, the correlation coefficient increases to 0.8666 (Figure-24). It indicates that periodic modelling using 42 dominant frequencies is better than 13 dominant frequencies. This is not only indicated by the correlation coefficient but also can be seen by the Root Mean Square of error (RMS error) of the model. The model using 13 frequencies gives an RMS error of about 67.0171 mm, but the model using 42 frequencies makes an RMS error smaller, which is about 53.6957 mm.

Selection of dominant frequencies based on maximum Intensity or power spectrum is the most appropriate way to get the suitable frequencies. It is indicated in Figure-5, Figure-6, Figure-7, Figure-20, and Table-2. Figure-5 is the periodic modelling using 1 frequency with the 1st maximum Intensity. Figure-6 uses 1 frequency with the 2nd maximum Intensity, and Figure-7 uses 1 frequency with 3rd maximum Intensity. Even though the models all use only 1 frequency, the model in Figure-5 indicates a best fit than the others, where the correlation coefficient is about 0.5820 > 0.2570 > 0.1667.

Prediction Modelling of Monthly Rainfall

To illustrate that the periodic model using the Lomb-Scargle periodogram can be used to predict monthly rainfall time series, 132 months of rainfall time series are used to predict the next 132 months. Using 132 months of the rainfall series and Lomb-Scargle periodogram method are decomposed frequencies and periods as presented below?

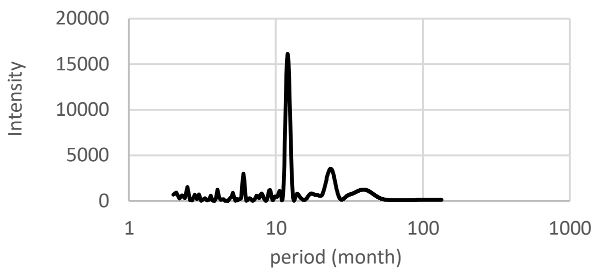

Figure-25. Lomb periods of rainfall time series using 132 data lengths.

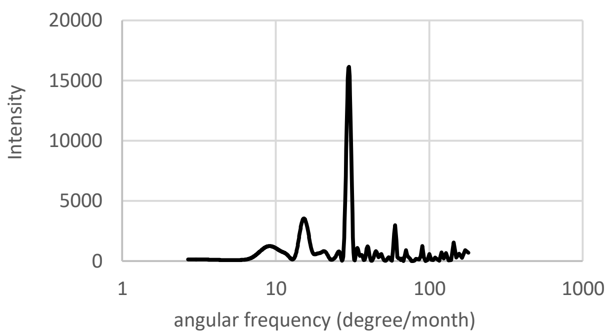

Figure-26. Lomb-Scargle frequencies of rainfall time series using 132 data length.

Using the same way as before, 22 dominant frequencies have been extracted, as presented below:

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Table-4. Periods and angular frequencies rank monthly rainfall using 132 data lengths.

Rank	Intensity	Period (month)	angular frequency (degree/month)
1	16144	12	30.00
2	3540	23.6	15.25
3	2988	6	60.00
4	1542	2.5	144.00
5	1243	4	90.00
6	1220	9.1	39.56
7	1100	10.6	33.96
8	916	2.1	171.43
9	899	5.1	70.59
10	832	8	45.00
11	819	17.5	20.57
12	804	14	25.71
13	714	3	120.00
14	672	2.8	128.57
15	602	2.3	156.52
16	586	7.4	48.65
17	583	3.6	100.00
18	505	9.9	36.36
19	316	6.6	54.55
20	304	3.3	109.09
21	217	5.4	66.67
22	182	4.4	81.82

Using 22 dominant frequencies, periodic modelling of monthly rainfall time series and predicted periodic modelling of monthly rainfall time series have been generating such as illustrated in Figure-27 as follows:

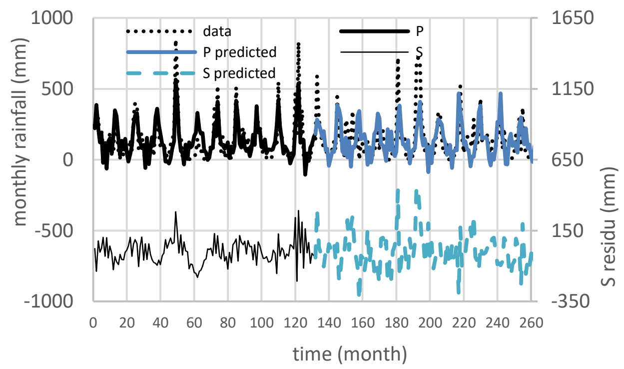

Figure-27. Predicted periodic modelling of monthly rainfall.

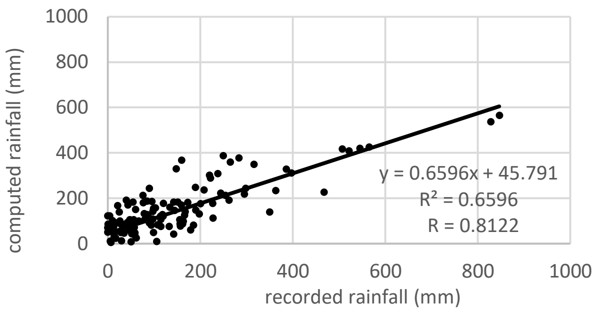

Figure-28. Correlation between recorded rainfall (data) versus computed rainfall.

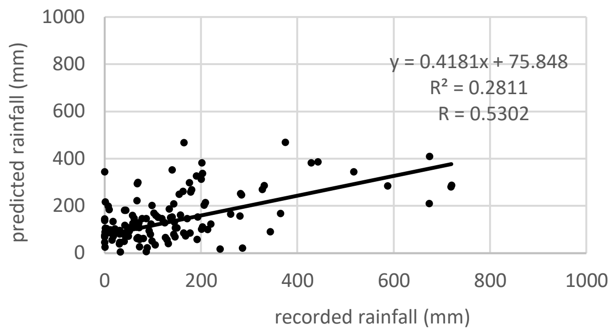

Figure-29. Correlation between recorded rainfall (data) versus predicted rainfall.

The correlation coefficient (r) of the periodic modelling of monthly rainfall time series (computed rainfall) using data length of 132 months with recorded rainfall data (r) is about 0.8122. However, the correlation coefficient of the predicted periodic modelling of monthly rainfall time series (predicted rainfall) with recorded rainfall data (r) is about 0.5302.

The validation model indicated that computed rainfall correlates well with the recorded rainfall data. The verification model indicated that predicted rainfall or forecasting monthly rainfall has a good enough correlation.

CONCLUSIONS

The periodic modelling of monthly rainfall time series has been modelled using 42 dominant frequencies from the Lomb-Scargle periodogram; in order to do that needed, two steps. The first step, 13 frequencies are extracted from the monthly rainfall time series. In the second step, 29 frequencies are extracted from the residue of the periodic modelling of monthly rainfall time series. Periodic modelling using 42 dominant frequencies gives better results than only 13 dominant frequencies. In modelling monthly rainfall time series, the correlation between the calculated periodic modelling of monthly rainfall with recorded monthly rainfall is about 0.8122. In predicting monthly rainfall time series, the correlation between the predicted periodic modelling of monthly rainfall and with recorded monthly rainfall time series is about 0.5302.

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