VOL. 18, NO. 10, MAY 2023 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

PERFORMANCE EVALUATION OF NHPP SOFTWARE RELIABILITY MODELAPPLYING GAMMA FAMILY LIFETIME DISTRIBUTION

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ABSTRACT

In this paper, after applying the Gamma family distribution (Erlang, Log-Logistic, Rayleigh) to the finite failure-type NHPP reliability model in which failures do not occur during flaw repair, the reliability performance of the software was evaluated by comparing with the Goel-Okumoto basic model. In this process, software failure time data collected randomly during the operation of the computer system was used, and the estimator solution of the applied NHPP model was performed by applying the numerical analysis (bisection method).Conclusively, first, as a result of calculating the criteria for efficient model selection, it was evaluated that the efficiency of the Rayleigh and Goel-Okumoto basic models was excellent. Second, as a result of analyzing the reliability attributes functions, the Rayleigh model showed the best performance. Third, as a result of future reliability testing, the Rayleigh model with the highest reliability was efficient. Thus, the Rayleigh model showed the best reliability performance in this work. Through this study, the software reliability performance of the Gamma family lifetime distribution was newly identified, and this analysis data can help software developers to utilize it as basic data for reliability improvement in the testing process.

Keywords: erlang, gamma family, goel-okumoto, log-logistic, rayleigh, software reliability.

Manuscript Received 10 February 2023; Revised 17 July 2023; Published 25 July 2023

1. INTRODUCTION

With the rapid advent of the artificial intelligence era, software convergence technology is rapidly spreading to various related industries. This software convergence technology is a technology that designs, develops, and operates software that can reliably process large amounts of data required in various industries such as engineering, finance, and medical, as well as the software used in computers and mobile devices. Therefore, to improve the quality of applied products by improving the reliability of software convergence technology, developers are currently investing a lot of research. For this reason, Non-homogeneous Poisson Process (NHPP) model widely known to be suitable for software reliability research is attracting attention. In particular, research on NHPP software reliability models for predicting reliability attributes such as failure rates using strength functions and mean value functions is becoming a focus [1]. Regarding research on the NHPP model, Goel and Okumoto [2] predicted error behavior that could occur when software was running, Huang [3] analyzed software reliability using a reliability attribute function, and Rao [4] presented a new method of predicting software defects after testing with various performance metrics by applying a characterization code related to software quality and comparing it with existing learning-based methods. Also, Kim [5] compared and analyzed the predictive power of software failure time using the finite failure NHPP reliability model, Pham [6] proposed a new reliability distribution function applying a failure rate function in the form of Vtub, and this function was used for software reliability modeling. Tokuno, Fukuda, and Yamada [7] investigated and explained the correlation between the characteristics of software reliability and system performance along with the probabilistic performance evaluation of software systems considering real-time properties using the NHPP model. Yang [8] analyzed reliability attributes using the NHPP exponential-type distribution model, which represents a continuous probability distribution. Also, Yang [9] applied the NHPP reliability model to the Weibull distribution, and then analyzed reliability performance.

Thus, in this work, the Gamma family lifetime distribution, which is well known to be suitable for software reliability quality testing, is applied to the NHPP model. Based on this applied model, the reliability performance is newly analyzed according to the proposed algorithm sequence, and the optimal model is also presented.

2. RELATED RESEARCH

2.1 NHPP Software Reliability Model

2.1.1 NHPP model

A software reliability model in which software failures depend on the NHPP is classified as a model having a time domain. In this stochastic process, the parameter λ(t) represents the intensity function related to the software execution time point t. Therefore, if N(t) represents the accumulated number of failures in time t, and m(t) represents the mean value function.

That is, N(t) is known as a Poisson probability density function with a mean value function m(t) as a parameter, as shown in Equation (1).

$$P\{N(t)=n\}=\frac{[m(t)]^n \cdot e^{-m(t)}}{n!} \quad (1)$$

Note that $n = 0,1,2, \cdots \infty$.

As such, time-related models can be explained as stochastic failure processes by NHPP. Thus, m(t)and λ(t) satisfy the relationship as shown in Equations (2) and (3).

$$m(t) = \int_0^t \lambda(s)ds \quad (2)$$

$$\frac{dm(t)}{d(t)} = \lambda(t) \quad (3)$$

These NHPP models are classified into finite failures in which failures do not occur during repairs and infinite failures in which failures continue to occur even during repairs.

In this paper, we will develop this work based on the finite failure NHPP model by applying the actual software development situation.

2.1.2 NHPP Software reliability model

The finite-failure NHPP models assume that the expected value of a defect has a finite value given sufficient test time.

When given sufficient testing time in the NHPP model, if the detectable residual failure rate is θ, the cumulative distribution function is F(t), and the probability density function is f(t), then m(t) and λ(t) can be expressed as the following functional expressions, respectively.

$$m(t|\theta, b) = \theta F(t) \quad (4)$$

$$\lambda(t|\theta, b) = \theta F(t)' = \theta f(t) \quad (5)$$

Note that $b > 0, \theta > 0$.

Applying Equations (4) and (5), the likelihood function of the NHPP model is as follows.

$$L_{NHPP}(\Theta|\underline{x}) = \left(\prod_{i=1}^{n} \lambda(x_i)\right) exp[-m(x_n)] \quad (6)$$

Note that $\underline{x} = (x_1, x_2, x_3 \cdots x_n)$.

2.2 NHPP Goel-Okumoto Basic Model

In the field of software reliability, the Goel-Okumoto model is well known as the basic model. In particular, in the Goel-Okumoto basic model, the lifetime distribution following the distribution of failure occurrence time per software defect assumes an exponential distribution.

Therefore, the attributes functions of the reliability performance are as follows [10].

$$m(t|\theta, b) = \theta F(t) = \theta\left(1 - e^{-bt}\right) \quad (7)$$

$$\lambda(t|\theta, b) = \theta f(t) = \theta b e^{-bt} \quad (8)$$

That is, if applying the values of m(t) and λ(t) to Equation (6) and rearranging it, the following equation can be written.

$$\ln L_{NHPP}(\Theta|\underline{x}) = n\ln\theta + n\ln b - b\sum_{k=1}^{n} x_k$$
$$-\theta(1 - e^{-bx_n}) \quad (9)$$

Accordingly, using Equation (9), the estimators $\hat{\theta}_{MLE}$ and $\hat{\tau}_{MLE}$for the parameters must satisfy the following conditional expression.

$$\frac{\partial \ln L_{NHPP}(\Theta|\underline{x})}{\partial\theta} = \frac{n}{\hat{\theta}} - 1 + e^{-\hat{b}x_n} = 0 \quad (10)$$

$$\frac{\partial \ln L_{NHPP}(\Theta|\underline{x})}{\partial b} = \frac{n}{\hat{b}} - \sum_{i=1}^{n} x_n - \hat{\theta}x_n e^{-\hat{b}x_n} = 0 \quad (11)$$

2.3 NHPP Erlang Distribution Model

Among the software reliability distributions, the Gamma distribution is most widely used in reliability data analysis because it can express various distributions according to the values of shape and scale parameters.

Therefore, the attributes functions of the reliability performance are as follows [11].

$$m(t|\theta, b) = \theta\left[1 - e^{-bt}\sum_{i=0}^{a-1}\frac{(bt)^i}{i!}\right] \quad (12)$$

$$\lambda(t|\theta, b) = \theta\left[\frac{b^a}{\Gamma(a)} t^{a-1} e^{-bt}\right] \quad (13)$$

Note thata, $b > 0$, $a = 1,2,3 \ldots$, $t \in [0, \infty]$

The Erlang distribution belonging to the Gamma-family distribution to be studied in this paper considers the case where the value of the shape parameter (a) is 2. Here, the shape parameter (a) refers to a value that makes the shape of the failure distribution.

$$\ln L_{NHPP}(\Theta|\underline{x}) = n\ln\theta - n\ln\Gamma(a) + na\ln b - b\sum_{i=1}^{n} x_i$$
$$+(a-1)\sum_{i=1}^{n}\ln x_i - \theta + \theta e^{-bx_n}\left(\sum_{i=0}^{a-1}\frac{(bx_n)^i}{i!}\right) \quad (14)$$

Therefore, in Equation (14), the estimators $\hat{\theta}_{MLE}$ and $\hat{b}_{MLE}$for the parameters must satisfy the following Equations (15) and (16).

$$\frac{\partial \ln L_{NHPP}(\Theta|\underline{x})}{\partial\theta} = \frac{n}{\theta} - 1 + e^{-bx_n}(1 + bx_n) = 0 \quad (15)$$

$$\frac{\partial \ln L_{NHPP}(\Theta|\underline{x})}{\partial b} = \frac{2n}{b} - \sum_{i=1}^{n} x_i - \theta b {x_n}^2 e^{-bx_n} = 0 \quad (16)$$

2.4 NHPP Log-Logistic Distribution Model

In general, the Log-Logistic distribution is a measurable continuous distribution defined using scale and shape parameters and has been used to model binary responses in fields such as biostatistics and economics of growth models.

But, compared to a general distribution model in which the failure rate per flaw monotonically increases and decreases, the log-logistic distribution model, which has nonlinear characteristics in which the failure occurring rate per flaw increases and then decreases, is known as an appropriate model for reliability testing [12].

$$m(t|\theta,\tau,k) = \theta \frac{(\tau t)^k}{[1+(\tau t)^k]} \quad (17)$$

$$\lambda(t|\theta,\tau,k) = \theta f(t) = \theta \frac{\tau k(\tau t)^{k-1}}{[1+(\tau t)^k]^2} \quad (18)$$

Note that $\tau > 0,\ k > 0$

As shown in Equations (17) and (18), the Log-Logistic distribution belonging to the Gamma-family lifetime distribution to be studied in this work considers the case where the value of the shape parameter (k) is 2.

$$\ln L_{NHPP}(\Theta|\underline{x}) = n\ln 2 + n\ln\theta + 2n\ln\tau + \sum_{i=1}^{n} x_i$$

$$-2\sum_{i=1}^{n} \ln[1+(\tau x_i)^2] - \theta\frac{(\tau x_n)^2}{[1+(\tau x_n)^2]} = 0 \quad (19)$$

That is, if using Equation (19), the estimators $\hat{\theta}_{MLE}$ and $\hat{\tau}_{MLE}$for the parameters must satisfy the following conditional expression.

$$\frac{\partial \ln L_{NHPP}(\Theta|\underline{x})}{\partial\theta} = \frac{n}{\hat{\theta}} - \frac{(\hat{\tau}x_n)^2}{[1+(\hat{\tau}x_n)^2]} = 0 \quad (20)$$

$$\frac{\partial \ln L_{NHPP}(\Theta|\underline{x})}{\partial\tau} = \frac{2n}{\hat{\tau}} - 2\hat{\tau}\sum_{i=1}^{n} {x_i}^2 \frac{1}{\ln[1+(\hat{\tau}x_i)^2]}$$

$$-\hat{\theta}\left(\frac{2\hat{\tau}{x_n}^2(1+\hat{\tau}^2\, x_n - \hat{\tau}^2\, {x_n}^2)}{[1+(\hat{\tau}x_n)^2]^2}\right) = 0 \quad (21)$$

2.5 NHPP Rayleigh Distribution Model

The Rayleigh distribution was originally known as a distribution widely used as a distance distribution in spatial Poisson process and specific functional modelling of electromagnetic waves, but it is also recognized as an appropriate model for reliability lifetime test and analysis. Also, the Rayleigh distribution is a special case with a shape parameter of 2 in the Weibull distribution. Therefore, the property functions (m(t), λ(t)) of the NHPP Rayleigh model considering the shape parameter (α) can be written as follows.

$$m(t|\theta,b) = \theta\left(1-e^{-bt^{\alpha}}\right) = \theta\left(1-e^{-bt^2}\right) \quad (22)$$

$$\lambda(t|\theta,b) = \theta\left(2bt^{a-1}e^{-bt^{\alpha}}\right) = \theta\left(2bt^{a-1}e^{-bt^2}\right) \quad (23)$$

Note that$\theta > 0, b = \frac{1}{2\beta^2} > 0,\ t \in [0,\ \infty]$

Therefore, if applying the values of m(t) and λ(t) to Equation (6) and rearranging it, the following equation can be written.

Table-1. Collected software failure time data.

Failure umber	Failure time (hours)	Failure time Interva1 (hours)	Failure time (hours)× 10^{-2}
1	30.02	30.02	0.30
2	31.46	1.44	0.31
3	53.93	22.47	0.53
4	55.29	1.36	0.55
5	58.72	3.43	0.58
6	71.92	13.20	0.71
7	77.07	5.15	0.77
8	80.90	3.83	0.80
9	101.90	21.00	1.01
10	114.87	12.97	1.14
11	115.34	0.47	1.15
12	121.57	6.23	1.21
13	124.97	3.40	1.24
14	134.07	9.10	1.34
15	136.25	2.18	1.36
16	151.78	15.53	1.51
17	177.50	25.72	1.77
18	180.29	2.79	1.80
19	182.21	1.92	1.82
20	186.34	4.13	1.86
21	256.81	70.47	2.56
22	273.88	17.07	2.73
23	277.87	3.99	2.77
24	453.93	176.06	4.53
25	535.00	81.07	5.35
26	537.27	2.27	5.37
27	552.90	15.63	5.52
28	673.68	120.78	6.73
29	704.49	30.81	7.04
30	738.68	34.19	7.38

$$\ln L_{NHPP}(\Theta|\underline{x}) = n\ln 2 + n\ln\theta + n\ln b + \sum_{i=1}^{n} \ln x_i$$

$$-b\sum_{i=1}^{n} {x_i}^2 - \theta\left(1-e^{-bx_n^2}\right) \quad (24)$$

That is, if using Equation (24), the estimators $\hat{\theta}_{MLE}$ and $\hat{b}_{MLE}$ for the parameters must satisfy the following conditional expression.

$$\frac{\partial \ln L_{NHPP}(\Theta|\underline{x})}{\partial\theta} = \frac{n}{\hat{\theta}} - 1 + exp\left(-\hat{b}x_n^2\right) = 0 \quad (25)$$

$$\frac{\partial \ln L_{NHPP}(\Theta|\underline{x})}{\partial b} = \frac{n}{\hat{b}} - \sum_{i=1}^{n} x_i^2 - \hat{\theta}x_n^2\, exp\left(-\hat{b}x_n^2\right) = 0 \quad (26)$$

 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

3. RELIABILITY PERFORMANCE ANALYSIS USING SOFTWARE FAILURE TIME DATA

In this paper, the reliability performance is analyzed by the following algorithm method and the solution sequence (Step 1 to Step 5).

Step 1: After collecting the failure time generated randomly during the operation of the software system, verify the suitability of the collected data.
Step 2: Estimate the parameter (maximum likelihood estimator) of the proposed NHPP reliability model.
Step 3: Check the efficiency of the proposed NHPP reliability model to select an efficient model.
Step 4: Analyzing the property functions $(m(t), \lambda(t), \widehat{R}(\tau))$ of the reliability performance.
Step 5: Presenting evaluation results of reliability performance based on performance property data.

Table-1 is the software failure time data cited in this work, which means failures that occur randomly during the normal operation of the software system [13].

Additionally, these failures were caused by analysis errors and insufficient testing during the software development process, indicating that 30 failures occurred in a total of 738.68 hours.

3.1 Step 1: After Collecting the Failure Time Generated Randomly during the Operation of the Software System, Verify the Suitability of the Collected Data

The cited software failure time data was verified using Laplace trend analysis to determine whether it was applicable to this study. If the result of the Laplace trend analysis is distributed between '-2 and 2', it is said to be reliable because the distribution of the cited data is stable. That is, as shown in Figure-1, the estimated result value was distributed between 0 and 2 [14].

Therefore, the software failure time data cited in this work is applicable.

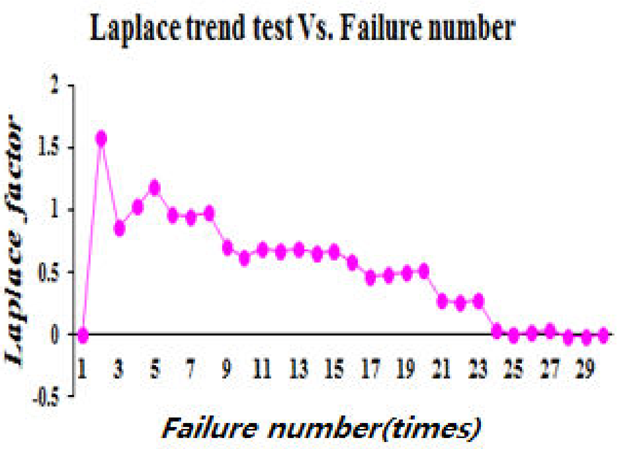

Figure-1. Analysis results of Laplace Trend Test.

3.2 Step 2: Estimate the Parameter (Maximum Likelihood Estimator) of the Proposed NHPP Reliability Model

Table-2 shows the estimated results of the parameters $(\hat{\theta}, \hat{b}(\hat{\tau}))$ applying the cited failure time data.

These calculation results were processed by the maximum likelihood estimation (MLE) and the estimator parameters were performed by applying the numerical analysis method.

Table-2. Parameter estimation using MLE.

Type	NHPP model	MLE	
		$\hat{\theta}$	$\hat{b}\ (\hat{\tau})$
Basic model	Goel-Okumoto	33.4092	0.3090
Gamma family distribution	Erlang	30.5978	0.7922
	Log-Logistic	32.2412	0.4953
	Rayleigh	24.0116	0.3707

3.3 Step 3: Check the Efficiency of the Proposed NHPP Reliability Model to Select an Efficient Model

3.3.1 Coefficient of Determination (R^2)

When a model is least squares estimated in data analysis, the R^2 is a numerical value indicating how well the estimated model can explain the subject [15].

$$R^2 = 1 - \frac{\sum_{i=1}^{n}\left(m(x_i) - \hat{m}(x_i)\right)^2}{\sum_{i=1}^{n}\left(m(x_i) - \sum_{j=1}^{n} m(x_j)/n\right)^2} \tag{27}$$

Note that $m(x_i)$means the cumulative number of failures up to the observation point $(0, x_i)$.

When comparing the efficiency of the model, the larger the value of the coefficient of determination, the smaller the error, and it is considered an efficient model.

3.3.2 Mean Square Error (MSE)

The smaller the MSE value, which is the criterion for selecting an efficient model, the more efficient it is.

$$MSE = \frac{\sum_{i=1}^{n}\left(m(x_i) - \hat{m}(x_i)\right)^2}{n-k} \tag{28}$$

Figure-2 shows the results of confirming the goodness of fit using the efficiency of the models applied in this work through the pattern trend analysis of MSE according to the total number of failures.

ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

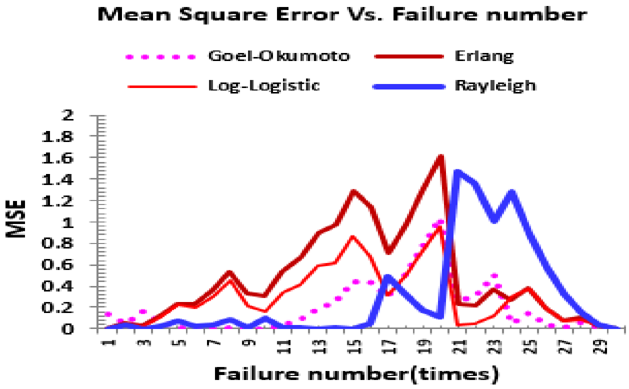

Figure-2. Analysis of MSE.

As shown in Table-3, the coefficients of determination of the proposed models are all estimated to be over 80%, and the models cited in this study are judged to be useful in the field of software reliability [16].

Table-3. Model efficiency.

Type	NHPP model	MSE	R^2
Basic model	Goel-Okumoto	5.8424	0.9814
Gamma family distribution	Erlang	14.1910	0.9550
	Log-Logistic	8.9730	0.9715
	Rayleigh	8.6587	0.9725

Table-4 shows the detailed analysis data of the MSE function that determines the efficiency of the model.

Table-4. Detailed analysis data of MSE.

Failure number (times)	MSE			
	Goel-Okumoto	Erlang	Log-Logistic	Rayleigh
1	0.137	0.002	0.003	0.000
2	0.042	0.050	0.054	0.030
3	0.161	0.028	0.025	0.000
4	0.055	0.115	0.109	0.021
5	0.010	0.233	0.220	0.069
6	0.015	0.236	0.200	0.020
7	0.000	0.357	0.300	0.040
8	0.013	0.530	0.447	0.084
9	0.000	0.336	0.215	0.012
10	0.000	0.305	0.159	0.092
11	0.034	0.538	0.335	0.016
12	0.084	0.669	0.417	0.015
13	0.188	0.902	0.591	0.001
14	0.254	0.971	0.610	0.012
15	0.442	1.295	0.861	0.000
16	0.435	1.140	0.678	0.054
17	0.299	0.705	0.309	0.483
18	0.496	0.970	0.487	0.325
19	0.761	1.309	0.732	0.180
20	1.031	1.610	0.954	0.106
21	0.260	0.230	0.041	1.467
22	0.305	0.219	0.044	1.352
23	0.501	0.375	0.128	1.003
24	0.050	0.268	0.303	1.285
25	0.144	0.385	0.370	0.897
26	0.039	0.191	0.181	0.575
27	0.004	0.084	0.074	0.324
28	0.055	0.099	0.089	0.144
29	0.013	0.025	0.022	0.036
30	0.000	0.000	0.000	0.000

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3.4 Step 4: Analyzing the Property Functions $(m(t),\ \lambda(t),\ \hat{R}(\tau))$ of the Reliability Performance

3.4.1 Mean Value Function $(m(t))$

Figure-3 shows the trend of the expected value of failure occurrence for the entire failure time range cited in this study. In this simulation, all the models showed error-estimated trends concerning the real value, but the Rayleigh and Goel-Okumoto basic models showed the smallest error over the entire failure time range and were relatively efficient.

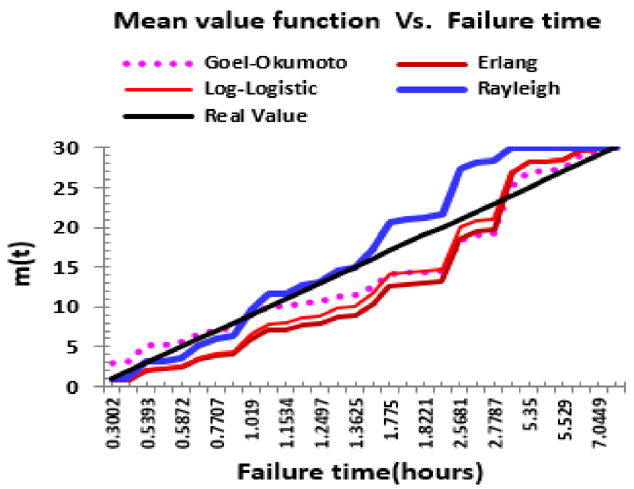

Figure-3. Analysis of Mean Value Function

3.4.2 Intensity Function $(\lambda(t))$

In general, the failure rate initially increases before the defect is repaired, but eventually decreases as the defect is repaired with the passage of time. Therefore, the intensity function also reflects this physical phenomenon.

Figure-4 shows the results of analyzing the trend of the intensity function, in this simulation; it showed a similar pattern as the general failure phenomenon. That is, although the intensity function of the proposed model initially increased, the failure rate was eliminated over time, resulting in an efficient pattern trend in which the intensity function greatly decreased. However, only the Goel-Okumoto basic model showed inefficiency in which the failure rate continuously decreased.

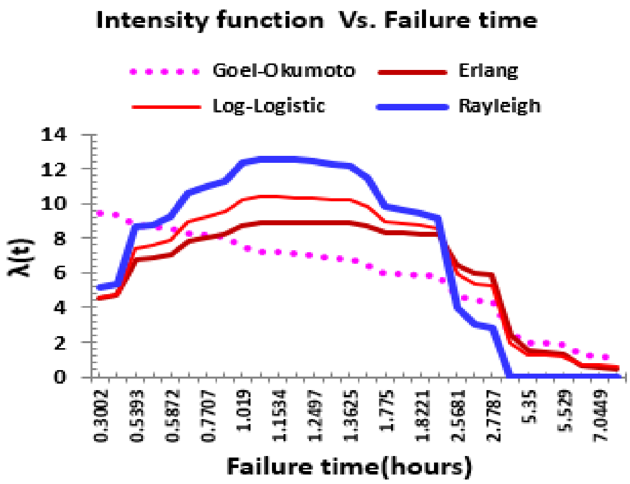

Figure-4. Analysis of Intensity Function.

Table-5 shows the performance trend analyzed in detail with failure time data that occurred during a total of 738.68 failure times for the property functions representing the reliability performance of the NHPP model applied in this work.

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Table-5. Detailed analysis data of reliability performance attributes.

Failure Number	Failure Time (hours)	Failure Time (hours) $\times 10^{-2}$	ReliabilityPerformance Attributes							
			Mean ValueFunction$(\mathbf{m(t)})$				Intensity Function$(\boldsymbol{\lambda}(\mathbf{t}))$			
			Goel-Okumoto	Erlang	Log-Logistic	Rayleigh	Goel-Okumoto	Erlang	Log-Logistic	Rayleigh
1	30.02	0.3002	2.959	0.739	0.697	0.986	9.408	4.544	4.545	5.168
2	31.46	0.3146	3.094	0.806	0.764	1.081	9.367	4.708	4.743	5.398
3	53.93	0.5393	5.128	2.111	2.147	3.067	8.738	6.755	7.432	8.619
4	55.29	0.5529	5.246	2.203	2.249	3.215	8.702	6.851	7.568	8.788
5	58.72	0.5872	5.543	2.442	2.514	3.601	8.610	7.081	7.896	9.199
6	71.92	0.7192	6.657	3.428	3.630	5.236	8.266	7.812	8.959	10.569
7	77.07	0.7707	7.079	3.836	4.100	5.931	8.135	8.036	9.287	11.008
8	80.9	0.809	7.389	4.147	4.460	6.465	8.040	8.184	9.501	11.299
9	101.9	1.019	9.024	5.930	6.545	9.589	7.534	8.728	10.238	12.344
10	114.87	1.1487	9.982	7.073	7.884	11.611	7.238	8.878	10.370	12.538
11	115.34	1.1534	10.016	7.115	7.933	11.684	7.228	8.882	10.371	12.5391
12	121.57	1.2157	10.462	7.669	8.579	12.660	7.090	8.911	10.358	12.5138
13	124.97	1.2497	10.702	7.972	8.930	13.191	7.0164	8.916	10.333	12.469
14	134.07	1.3407	11.331	8.783	9.866	14.5994	6.821	8.900	10.214	12.258
15	136.25	1.3625	11.480	8.977	10.088	14.932	6.776	8.890	10.175	12.188
16	151.78	1.5178	12.507	10.349	11.641	17.236	6.458	8.757	9.801	11.502
17	177.5	1.775	14.104	12.553	14.055	20.679	5.965	8.353	8.933	9.827
18	180.29	1.8029	14.270	12.786	14.303	21.018	5.913	8.299	8.827	9.619
19	182.21	1.8221	14.383	12.945	14.472	21.2471	5.879	8.261	8.754	9.474
20	186.34	1.8634	14.624	13.284	14.830	21.728	5.804	8.176	8.595	9.157
21	256.81	2.5681	18.300	18.457	19.925	27.410	4.668	6.447	5.927	3.965
22	273.88	2.7388	19.0761	19.520	20.889	28.153	4.428	6.006	5.370	3.022
23	277.87	2.7787	19.252	19.758	21.101	28.299	4.374	5.904	5.247	2.826
24	453.93	4.5393	25.192	26.740	26.916	30.000	2.539	2.391	1.958	0.038
25	535	5.35	27.013	28.284	28.221	30.013	1.976	1.482	1.315	0.002
26	537.27	5.3727	27.057	28.317	28.251	30.013	1.9625	1.462	1.301	0.002
27	552.9	5.529	27.357	28.536	28.447	30.014	1.870	1.329	1.210	0.001
28	673.68	6.7368	29.242	29.665	29.584	30.014	1.287	0.622	0.723	0.000
29	704.49	7.0449	29.620	29.838	29.794	30.014	1.170	0.509	0.641	0.000
30	738.68	7.3868	30.000	29.995	30.000	30.014	1.053	0.407	0.564	0.000

3.4.3 Reliability Function $(\hat{\mathbf{R}}(\boldsymbol{\tau}))$

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Table-6. Analysis data of reliability.

Mission Time (hours)		Reliability Function $\hat{R}(\tau)$			
		Goel-Okumoto	Erlang	Log-Logistic	Rayleigh
1	0.1	0.901	1.047	1.031	0.999
5	0.5	0.613	0.916	0.841	0.999
10	1	0.404	0.808	0.677	0.999
15	1.5	0.282	0.740	0.564	0.999
20	2	0.207	0.694	0.482	0.999
25	2.5	0.159	0.664	0.421	0.999
30	3	0.127	0.643	0.374	0.999
35	3.5	0.105	0.629	0.338	0.999
40	4	0.089	0.619	0.309	0.999
45	4.5	0.077	0.612	0.286	0.999
50	5	0.068	0.607	0.266	0.999
55	5.5	0.061	0.604	0.250	0.999
60	6	0.056	0.601	0.237	0.999
65	6.5	0.052	0.600	0.225	0.999
70	7	0.048	0.599	0.216	0.999
75	7.5	0.046	0.598	0.207	0.999
80	8	0.044	0.597	0.200	0.999
85	8.5	0.042	0.597	0.193	0.999
90	9	0.040	0.597	0.187	0.999
95	9.5	0.039	0.597	0.182	0.999
100	10	0.038	0.596	0.177	0.999
105	10.5	0.037	0.596	0.173	0.999
110	11	0.037	0.596	0.170	0.999
115	11.5	0.036	0.596	0.166	0.999
120	12	0.035	0.596	0.163	0.999
125	12.5	0.035	0.596	0.161	0.999
130	13	0.035	0.596	0.158	0.999
135	13.5	0.034	0.596	0.156	0.999
140	14	0.034	0.596	0.154	0.999
145	14.5	0.034	0.596	0.152	0.999

After the final failure time ($x_n = 7.3868$) cited in this study, a random mission time (τ) was input and the reliability performance was analyzed as follows.

The reliability $\hat{R}(\tau|x_n)$ is expressed by the following Equation (29). That is, the attribute functions ($\hat{R}(\tau|x_n)$) of the reliability performance are as follows [17].

$$\hat{R}(\tau|x_n) = exp[-\{m(x_n+\tau) - m(x_n)\}]$$
$$= exp[-\{m(7.3868+\tau) - m(7.3868)\}] \quad (29)$$

Figure 5 shows the analysis results of future reliability for the mission time of the proposed models. The Rayleigh model showed high reliability and stable efficiency as the mission time passed, but other models (Goel-Okumoto, Erlang, Log-Logistic) showed inefficient properties with continuously decreasing reliability.

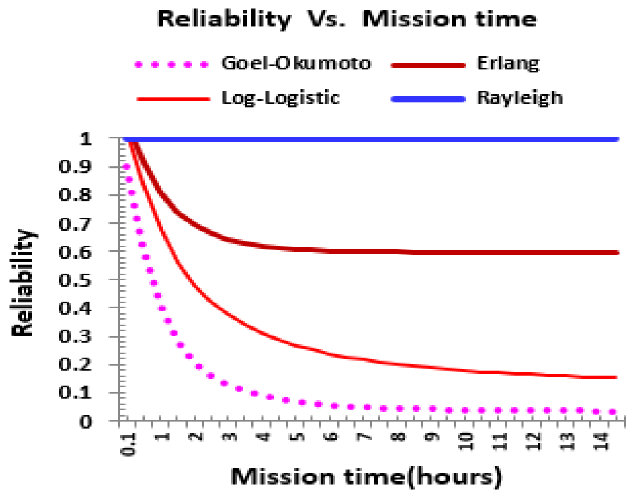

Figure-5. Analysis of Reliability Function.

Table-6 shows the detailed estimation result values of the attribute functions ($\widehat{R}(\tau)$) that determine the future reliability performance.

For reference, the data presented in Table-5 is simulated by numerical conversion ($\times 10^{-1}$) of mission time to facilitate reliability calculation.

3.5 Step 5: Presenting Evaluation Results of Reliability Performance based on Performance Property Data

In conclusion, Table-7 shows the evaluation results of the reliability performance based on the performance property functions.

As a result of comprehensively evaluating reliability performance as shown in Table-7, the Rayleigh model showed the best reliability [18].

Therefore, if software developers use this research data, these can be used as basic development data needed in each software convergence field as well as reliability performance data required for software quality improvement.

Table-7. Reliability performance evaluation.

NHPP model	Model Efficiency		Performance Evaluation		
	MSE	R^2	m(t)	λ(t)	$\widehat{R}(\tau)$
Goel-Okumoto	Best	Good	Good	Worst	Worst
Erlang	Worst	Good	Bad	Best	Good
Log-Logistic	Good	Good	Bad	Best	Bad
Rayleigh	Good	Good	Best	Best	Best

4. CONCLUSIONS

If a software developer can predictively model failure time data collected during normal operation from the design, analysis stage, and testing process, then the reliability of software can be efficiently improved by predicting failure occurrence time in advance. Therefore, in this paper, the reliability performance of the Gamma family distribution, which has been studied as appropriate for software reliability research, was newly evaluated and identified.

The results of this work are as follows:

First, as a result of calculating the criteria for efficient model selection (MSE andR^2), it was evaluated that the efficiency of the Rayleigh and Goel-Okumoto basic models was excellent.

Second, as a result of analyzing the attributes functions (m(t), λ(t)), the Rayleigh model with excellent estimation ability for true values showed the best performance.

Third, as a result of testing future reliability, the Rayleigh model was the most efficient as it showed the highest and stable reliability regardless of time. But, it was found that other models are inefficient as their reliability decreases as time goes by.

In conclusion, it was found that the reliability performance of the Rayleigh model was the best. Therefore, the data of this study can be used as basic design information along with new testing data on the Gamma family distribution. In the future, a study will be required to research a suitable model through performance evaluation using the attribute function of the searched model after exploring applicable statistical models for each software application field.

ACKNOWLEDGMENTS

Funding for this paper was provided by Namseoul University.

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