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A NEW APPROACH FOR MODELLING THE VIBRATION OF BEAMS UNDER MOVING LOAD EFFECT

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ABSTRACT

In this paper, a new equivalent lumped parameter model is proposed for describing the vibration of beams under the moving load effect. Also, an analytical formula for calculating such vibration for low-speed loads is presented. Furthermore, a MATLAB/Simulink model is introduced to give a simple and accurate solution that can be used to design beams subjected to any moving loads, i.e., loads of any magnitude and speed. In general, the proposed Simulink model can be used much easier than the alternative FEM software, which is usually used in designing such beams. The obtained results from the analytical formula and the proposed Simulink model were compared with those obtained from Ansys R19.0, and very good agreement has been shown. It was found that the maximum vibration occurs when the load speed is about 0.58 of the critical speed of the beam. Furthermore, the vibration amplitude resulting from a moving load can amplify to 1.65 times the deflection produced by an equivalent static load.

Keywords: equivalent beam model, analytical formula, critical speed, passing time, transient structural analysis.

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1. INTRODUCTION

The need to study the moving load problems did not appear until the collapse of Stephenson's Bridge in England in 1847. where it was believed that the effects of moving loads were the cause of such failure, and this gave the motivation to study such problems [1]. Today, the moving load problem has become the subject of numerous studies related to the design of many real-life applications. For example, bridges, cranes, rails, tunnels, pipelines, etc. [2 and 3]. A review of many of the published works related to structural problems caused by moving loads can be found in [3]. Furthermore, this work presents several important concepts related to moving load problems. Generally, the governing equation of the vibration caused by a moving load can be derived based on either Timoshenko beam theory [4–8] or Euler-Bernoulli beam theory [9–16]. Unfortunately, the obtained equations for different applications have no exact solutions. Therefore, approximated analytical solutions or numerical methods have been extensively used in the literature. Usually, these solutions are complicated to achieve. This is why the need to find a simple and accurate solution has become necessary. This paper introduces such a solution, which is characterized by simplicity in handling and confidence in results.

2. LITERATURE REVIEW

The following review presents several published works related to the moving load problem. The review shows the main research lines, the mathematical approximations, and the experimental procedures available in the literature.

For example, Katz *et al.* [17] investigated the dynamic stability and transverse vibration of a beam under a moving load of constant velocity. They determined that a continuous sequence of passing loads could be the cause of the instability. And a single moving load passing over and leaving the beam cannot cause any instability. They used Galerkin's method to obtain the governing differential equations of periodic coefficients. The obtained differential equations were solved using numerical methods.

Furthermore, Yavari *et al.* [18] presented a numerical approach to analysing the dynamic response of a Timoshenko beam. This approach was called the discrete element technique, where the continuous flexible beam was replaced by several rigid bars and joints. The results of this approach were compared with those obtained from the numerical solutions of Euler-Bernoulli beams and Timoshenko beams. The effect of beam thickness and moving load velocity on the dynamic response of the beams under moving loads was studied.

Alternatively, Bilello *et al.* [19] studied the dynamic response of a small-scale bridge model subjected to the moving load effect. The analysis was developed based on Euler–Bernoulli beam theory. The proposed solution was validated by doing several experiments using a small-scale model that is designed to satisfy both static and dynamic excitations. It can be noted that the investigated load speeds ranged from 0.85 m/s to 2.1 m/s, which are considered very low, and the dynamic effect of the load is almost neglected.

However, Aied and Gonzalez [20] used a simply supported beam made of viscoelastic material to study the variation in its strain rate and the modulus of elasticity under the effect of a moving load. They discussed the effect of the load speed and magnitude on the deformation and strain of the beam. They concluded that the variations in strain and deformation are smaller than five percent for loads at low speeds, which are much lower than the critical speed of the beam.

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Similarly, Hassan and Sadiq [21] introduced an experimental and numerical study of the vibration of a beam under a constant-speed moving load. Two load speeds were investigated: 0.2 m/s and 0.25 m/s. A finite element model (FEM) of a beam under a moving load is established using ANSYS software. Despite employing an exceptionally slow load, the authors arrived at the conclusion that the vibration generated in the beam due to the moving load could exhibit a notably greater magnitude compared to what was observed with stationary loads.

Additionally, Froio *et al*. [22] introduced a numerical model for describing the vibration of a simply supported beam of elastic foundation subjected to a moving load. The Euler–Bernoulli theory of beams was used to obtain the equation of motion. They developed a finite element method approach coupled with a direct integration algorithm to solve the obtained equation. Similar to the aforementioned works, the solution obtained is very complicated.

Similarly, Songsuwan *et al*. [23] investigated the vibration of functionally graded sandwich beams of an elastic foundation subjected to a moving load. They used Timoshenko's beam theory and Lagrange's equations to derive the beam equation of motion. They applied Ritz and Newmark methods to solve the obtained equation of motion.

Furthermore, Sarparast and Ebrahimi-Mamaghani [24] studied the forced and free vibrations of a laminated curved beam subjected to moving loads. They investigated the effect of different stacking sequences and load speeds on the vibration of such beams. They used a numerical method for solving the governing equation.

However, Zhang *et al*. [25] introduced a comparison between the Timoshenko beam theory and the Euler-Bernoulli beam theory to find out which was best at describing the vibration of the beam under the moving load effect. FEM was introduced as the reference for these two theories. The authors claimed that the Timoshenko beam is much better than the Euler-Bernoulli beam in determining the dynamic response of beams at higher frequencies but that it makes no difference at low frequencies. Also, they found that the Timoshenko beam model is very accurate in describing beam deflection. They stated that a quasi-static model gives satisfactory results for moderate load speeds.

Additionally, Ebrahimi-Mamaghanin *et al.* [26] investigated the vibrations of axially functionally graded beams under the effect of moving loads. They introduced a mathematical model that addresses the effect of several important factors that considerably affect beam vibration: axial material gradation and rotary inertia factors on the critical speed, vibration magnification factors, mechanisms of cancellation, and maximum free vibration of the system. The mathematical model was solved using the fourth-order Runge-Kutta method. The authors verified their results by comparing them with the available results in the literature, and a good agreement was observed.

Also, Jahangiri *et al*. [27] introduced a new approach for addressing the nonlinear behaviour of a beam subjected to a moving load. The energy method was used to obtain the beam equations, which describe the vibration of the beam under a moving load in large oscillations. Galerkin and perturbation methods were used to solve the proposed equation.

Additionally, Zheng *et al.* [28] suggested that the bridge influence line can reflect the structural behaviour of the bridge under moving loads. They proposed the dynamic fluctuation part elimination method based on empirical mode decomposition to identify the bridge static influence line. They used a simply supported beam subjected to a moving load to validate their model, and they used three loads of different speeds. Also, the proposed solution is a very long-term solution and difficult to achieve.

Alternatively, Camara [29] presented an algorithm called MS5 embedded in the Python library MDyn. M55 uses an efficient indexing strategy to deactivate specific structural nodes and movements. This is done by a novel modal truncation based on a new dynamic participation factor and the vectorization of the Modal Superposition (MS) algorithm. For a range of load speeds, the results of MS5 were compared with the results of the conventional MS obtained from ABAQUS software. These results of MS5 were almost identical to those obtained from ABAQUS, but it is on average nine times faster. They stated that time and effort are very important to achieve a good beam design.

However, Assie *et al*. [30] investigated the dynamic response of a thick, perforated beam. They used different beams with different numbers and dimensions of square holes. They derived their equation of motion based on Timoshenko theory and the Lagrange procedure. Then, they solved their equation using the Ritz method with the Newmark average acceleration method. They presented the effect of moving load speed and magnitude on the dynamic response. Also, they addressed the influence of the holes' number and dimensions on the beam vibration.

Similarly, Akbaş *et al*. [31] used the Timoshenko beam theory to study the dynamic response of a composite beam subjected to a moving load. They obtained the beam equation of motion based on the Lagrange method. The equation was solved using the Ritz method in addition to the Newmark average acceleration method. They investigated the effects of fibre orientation, volume fraction, and the speed of the load. They concluded that the fibre orientation and the volume fraction have a significant effect on the beam response, but the major effect is coming from the speed of the load, which not only affects the beam response but also governs the required fibre orientation and the volume fraction.

On the other hand, Zhang *et al.* [32] proposed a structure containing multiple beams that were connected via elastic elements. The dynamic response of the structure under the effect of the moving load was investigated. To solve the derived equation of motion, the Fourier series in addition to three special transformations were proposed. They used their solution to study the influences of the beam material properties and load speed on the response of the structure.

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It can be concluded from all the above-mentioned works that there is no simple equation that can be used to calculate the vibration of a beam under the effect of a moving load. In addition, the available software for modelling such a case requires sufficient experience and consumes time. Therefore, it was my goal in this paper to develop an approach that can easily be used to describe the vibration of the beam under the effect of a moving load. This approach is based on deriving an equivalent lumped parameter model for the beam and the moving load. This developed model is characterized by simplicity in handling and confidence in results.

3. METHOD

This section contains the proposed mathematical model and the equivalent lumped parameter system of the beam under the effect of a moving load.

3.1 Mathematical Modelling

Figure-1 shows a simply supported beam subjected to the load P, which has the position x from the left-hand side of the beam

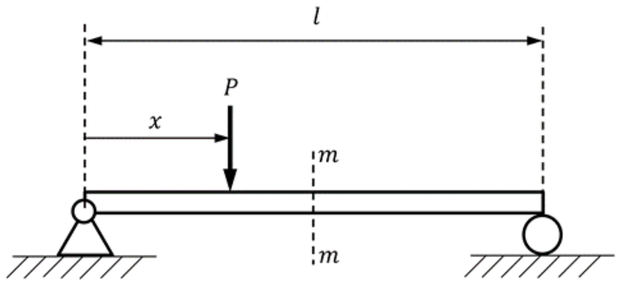

Figure-1. Simply supported beam subjected to moving load.

From writing down the moment equation about the right-hand side of the beam shown in Figure-1, the following expression can be obtained:

$$R = \left(\frac{l-x}{l}\right) P \tag{1}$$

Where l is the length of the beam and R is the vertical reaction of the left-hand support of the beam. Therefore, the moment in the section m-m can be expressed as

$$M|_{m-m} = \begin{cases} \frac{P}{2}x & 0 \le x \le l/2 \\ \frac{P}{2}(l-x) & l/2 \le x \le l \end{cases} \tag{2}$$

Now, the engineering equation of beams states that

$$M = EI\frac{dy^2}{dx^2} \tag{3}$$

Substituting (2) into (3) gives

$$EI\frac{dy_l^2}{dx^2} = \frac{P}{2}x \qquad 0 \le x \le l/2 \tag{4}$$

and

$$EI\frac{dy_r^2}{dx^2} = \frac{P}{2}(l-x) \qquad l/2 \le x \le l \tag{5}$$

where y_l and y_r are the deflection at the left- and right-hand side of the mid-span, respectively. Integrating (4) twice gives

$$\frac{dy_l}{dx} = \frac{1}{EI}\left(\frac{P}{4}x^2 + C_1\right) \tag{6}$$

and

$$y_l = \frac{1}{EI}\left(\frac{P}{12}x^3 + C_1 x + C_2\right) \tag{7}$$

Also, integrating (5) twice gives

$$\frac{dy_r}{dx} = \frac{1}{EI}\left[\frac{P}{2}\left(lx - \frac{x^2}{2}\right) + C_3\right] \tag{8}$$

and

$$y_r = \frac{1}{EI}\left[\frac{P}{2}\left(l\frac{x^2}{2} - \frac{x^3}{6}\right) + C_3 x + C_4\right] \tag{9}$$

Where C_1 , C_2, $C_{3,}$ and C_4 are constants that can be calculated using the proper boundary conditions. For completing the solution and finding the closed-form solution, the following boundary conditions are applied: at $x = 0$ then $y = 0$, at $x = \frac{l}{2}$ then $\frac{dy_l}{dx} = \frac{dy_r}{dx}$ and $y_l = y_r$ and finally at $x = l$ then $y = 0$. These give that the constants are: $C_1 = -Pl^2/16$, $C_2 = 0$, $C_3 = -9Pl^2/48$ and $C_4 = Pl^3/48$. Thus, the deflection of the beam can be written as

$$y = \frac{P}{48EI}\begin{cases} (4x^3 - 3l^2x) & 0 \le x \le l/2 \\ (12lx^2 - 4x^3 - 9l^2x + l^3) & l/2 \le x \le l \end{cases} \tag{10}$$

Investigating (10) for the entire length of the beam shows that the deflection of the mid-span fits exactly with the first mode shape of vibration of the simply supported beam, thus the deflection is found to be expressed as

$$y = y_s \, sin\left(\frac{\pi x}{l}\right) \tag{11}$$

where y_s is the maximum deflection of the beam at the mid-span, which can be expressed as

$$y_s = \frac{Pl^3}{48EI} \tag{12}$$

Substituting (12) into (11) gives

$$y = \left(\frac{Pl^3}{48EI}\right) sin\left(\frac{\pi x}{l}\right) \tag{13}$$

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Now, if it is considered that the load is moving at a constant speed, v, then the position x of the force P can be expressed as

$$x = v.t \tag{14}$$

By substituting (14) into (13), the vibration of the beam mid-span can be expressed as

$$y(t) = \left(\frac{Pl^3}{48EI}\right) sin\left(\frac{\pi v}{l}\right) t \tag{15}$$

As will be shown later in the results section (see Figure-8) that (15) gives accurate results for loads of speeds that are less than 20% of the critical speed of the beam.

3.2 Lumped-Parameter Model

To obtain a full understanding of the behaviour of the beam under the effects of moving loads, even those of high velocities, the lumped parameter model shown in Figure-2 is proposed.

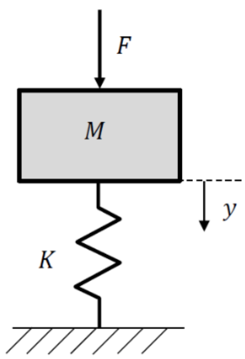

Figure-2. Equivalent lumped parameter system of the beam.

For the system shown in Figure-2, the relationship between the applied force and the mass displacement y can be expressed as

$$F(t) = K\, y(t) \tag{16}$$

where K is the equivalent stiffness of the beam. For a simply supported beam, the stiffness can be written as

$$K = \frac{48EI}{l^3} \tag{17}$$

As shown in Figure-3, the force $F(t)$ is only applied for a certain time depending on its speed and the beam length. Therefore, it is required to define a new variable to complete its definition. This variable is the passing time t_p, which is defined as the time required for the force to pass the beam and can be expressed as

$$t_p = \frac{l}{v} \tag{18}$$

Substituting (17) and (15) into (16) leads to the applied force, which can be formulated as

$$F(t) = P\, sin\, \omega t\,, \qquad t \leq t_p \tag{19}$$

where ω is effective frequency of the moving load, which is found to be expressed as

$$\omega = \frac{\pi v}{l} \tag{20}$$

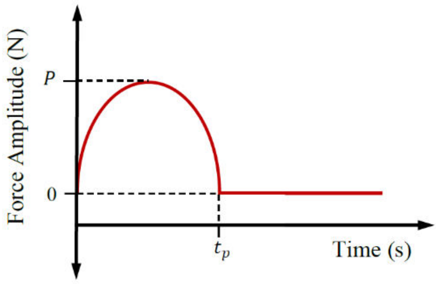

Figure-3. The force effect of the moving load on the mid-span of the beam.

The fundamental natural frequency of a simply supported beam, ω_n, can be written as

$$\omega_n = \left(\frac{\pi}{l}\right)^2 \sqrt{\frac{EI}{\rho A}} \tag{21}$$

where A and ρ are the cross-sectional areas and the density of the beam, respectively. Generally, the natural frequency can be expressed as

$$\omega_n = \sqrt{\frac{K}{M}} \tag{22}$$

Substituting (17) and (21) into (22) and then, after some mathematical simplifications, finding the equivalent mass of the beam to be

$$M = \left(\frac{48}{\pi^4}\right) \rho A l \tag{23}$$

The critical speed of the moving load, v_c, can be found by equalizing the equation of the excitation frequency (20) to that of the natural frequency (21). This gives

$$v_c = \frac{\pi}{l} \sqrt{\frac{EI}{\rho A}} \tag{24}$$

To give a general equivalent system for the vibrating beam, the damping is considered by the coefficient C. Thus, the general equation can be expressed as

$$M\ddot{y}(t) + C\dot{y}(t) + Ky(t) = F(t) \tag{25}$$

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The transfer function of the system can be written as

$$\frac{Y(s)}{F(s)} = \frac{1}{Ms^2+Cs+K} \tag{26}$$

Unfortunately, it is difficult to obtain one analytical solution for (25). This is because the moving load $F(t)$ changes its effect depending on its speed, v, as shown in Figure-4. Generally, the force $F(t)$ affects a sinusoidal wave at low speeds, but it acts more like a pulse at high speeds. Therefore, it can be said that the force effect is highly related to its speed, and this is the reason why (15) is valid only for load speeds that are less than 20% of the critical speed. Where force can be considered a sinusoidal effect. As will be shown next, a Simulink model is introduced to solve the proposed lumped-parameter model easily and with good accuracy.

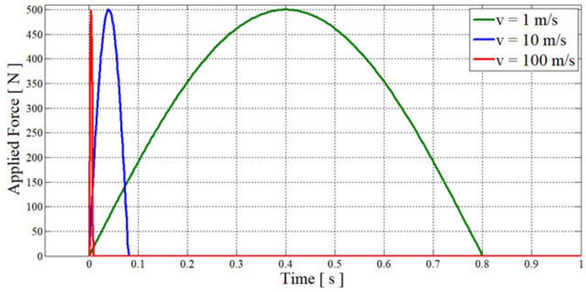

Figure-4. Effect of the moving load at the mid-span of the beam (load magnitude is 500 N).

4. RESULTS

A standard rectangular steel bar of 1 in x 1/2 in (25.4 mm x 12.7 mm) cross-sectional area with 800 mm length was chosen to be used for investigating the validity of the proposed model. The bar has the standard structural steel properties, with a modulus of elasticity equal to 200 GN/m^2 and a density of about 7800 kg/m^3. The bar was simulated as a simply supported beam using Ansys R19.0/Workbench, as shown in Figure-5a.

In Ansys R19.0/Workbench, a package called “Transient Structural Analysis” was used. In such an analysis, the moving force should be carefully defined as a function of time and location through tabular data. This requires defining different locations on the beam surface. The number of these locations is determined depending on the designer’s experience in getting the best results. It was found that ten locations are fair enough to define the moving force accurately, as shown in Figure-5(a). At each one of these locations, a force was applied based on the pattern shown in Figure-5(b), where it can be noted that the passing time t_p was used to set the load speed.

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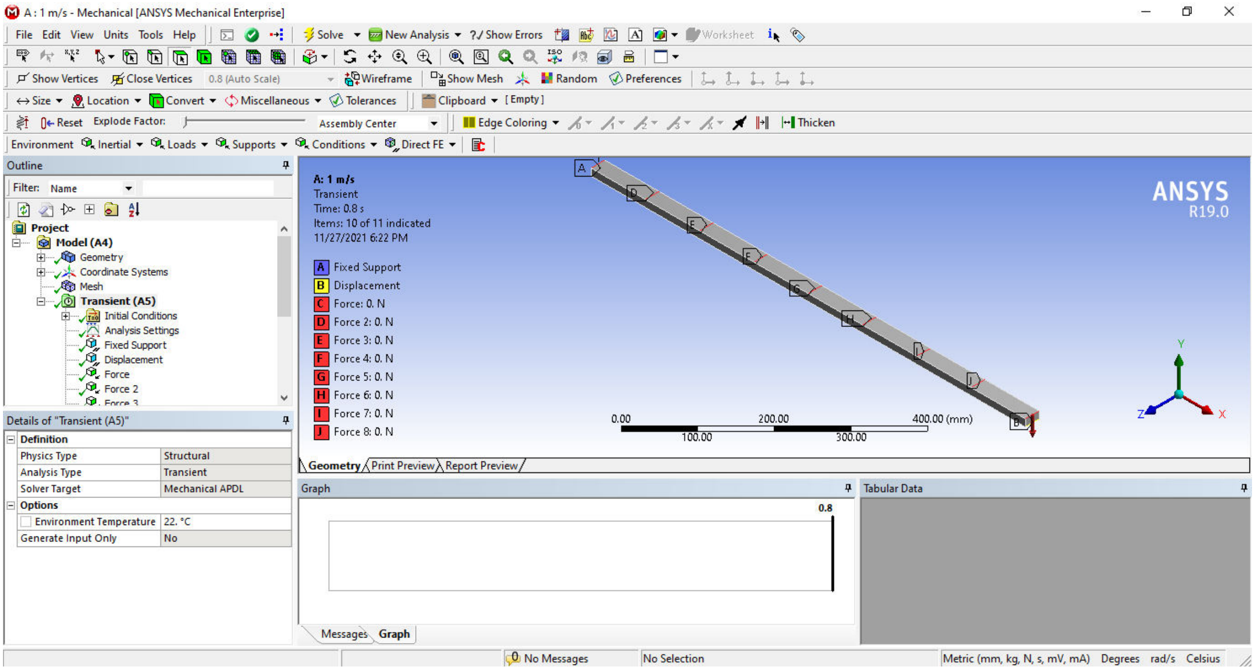
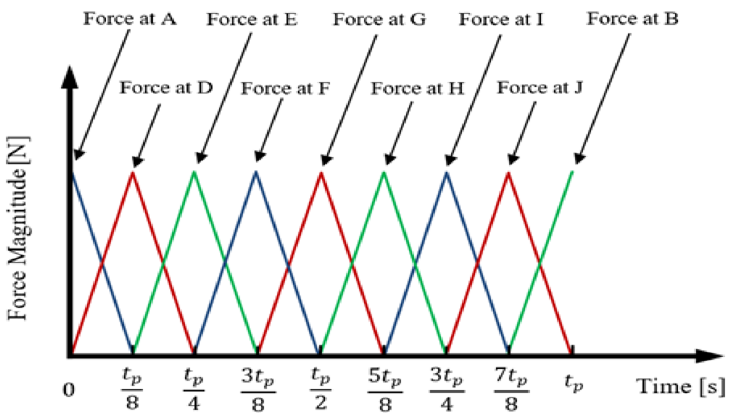


(a) Applied forces and their locations



(b) The concept of defining the moving load as a function of time

Figure-5. Moving load representation with Ansys R19.0.

As can be seen in Figure-6, a displacement probe was inserted at the mid-span of the beam to exactly measure the vibration at this location.

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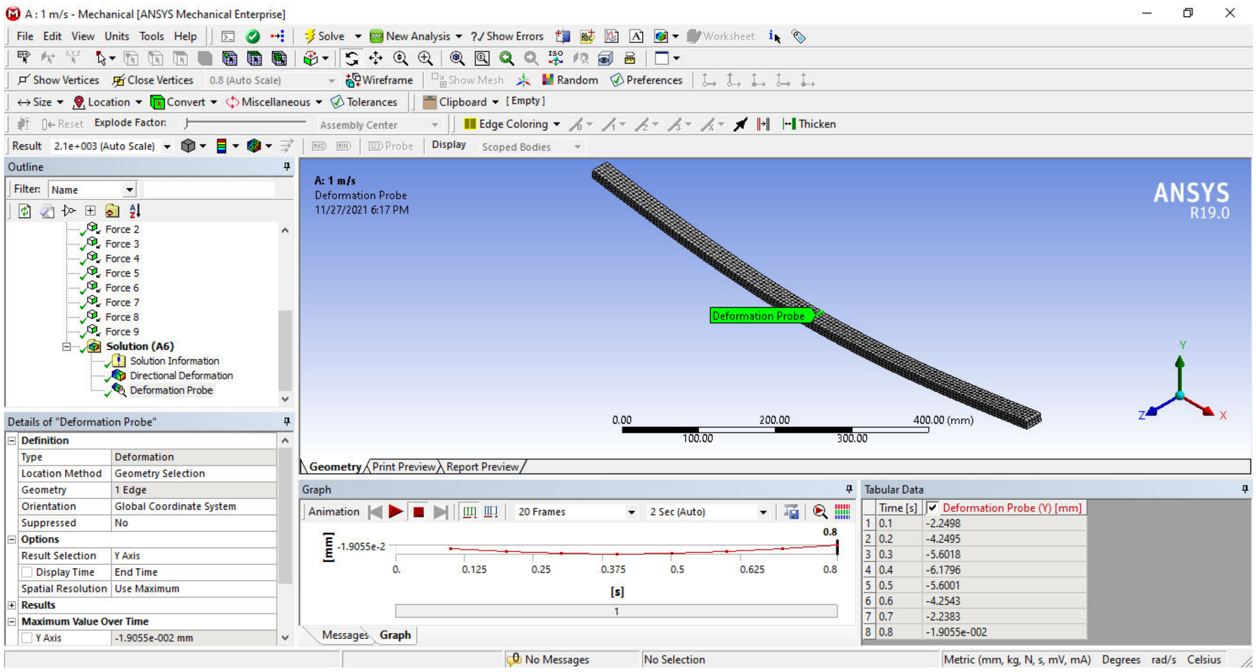

Figure-6. Displacement probe results of a beam subjected to moving load using workbench/transient structural analysis from Ansys R19.0.

Figure-7 shows the proposed Simulink model. It can be noted how easy it is to assemble such a model and investigate the vibration of the beam at different loading conditions.

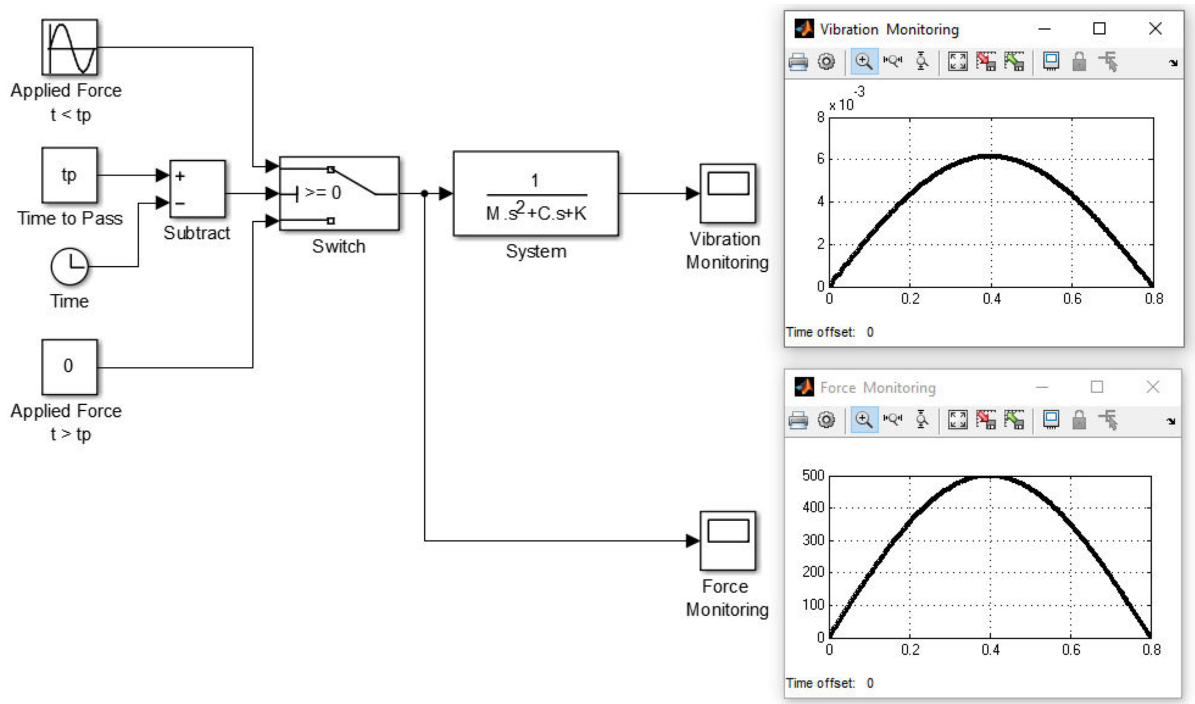

Figure-7. Simulink representation of a beam under moving load.

Figure-8 shows the results obtained from the derived analytical solution (equation 15) and the FEM model (Ansys R19.0) at different load speeds. To see the effect of the load movement clearly, the values of the vibration amplitude, y, were divided by the value of y_s, which is the maximum deflection caused by a static load of the same magnitude as the moving load. The value of y_s was calculated and found to be 6.17 mm. Also, the time, t, was divided by the corresponding passing time, t_p, of each speed.

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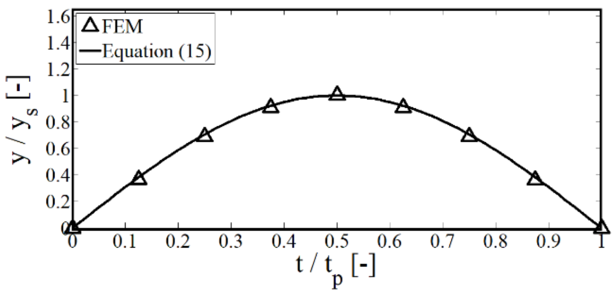
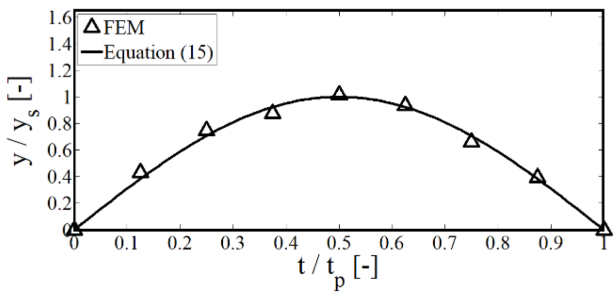
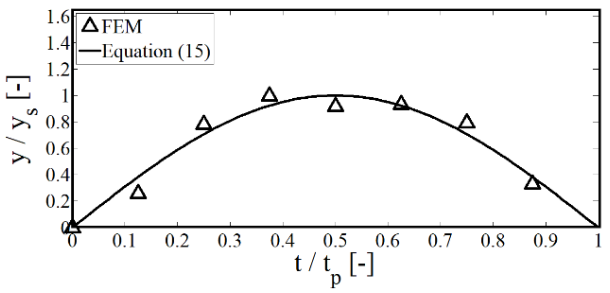


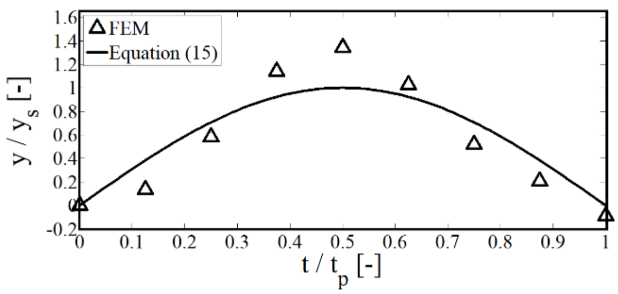
(a) $v = 1\ m/s$ & $t_p = 800\ ms$



(b) $v = 5\ m/s$ & $t_p = 160\ ms$



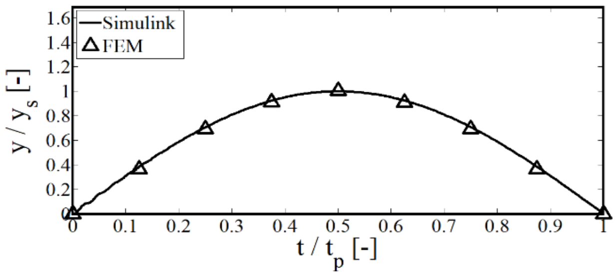
c) $v = 10\ m/s$ & $t_p = 80\ ms$



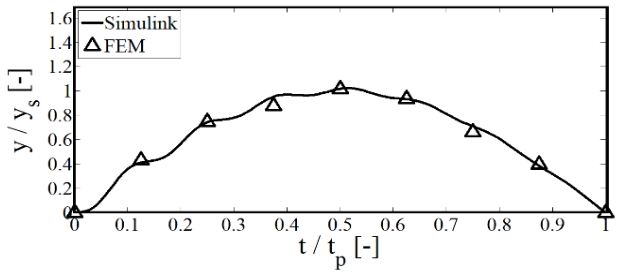
(d) $v = 20\ m/s$ & $t_p = 40\ ms$

Figure-8. FEM and equation (15) results for 500 N force moves with different speeds.

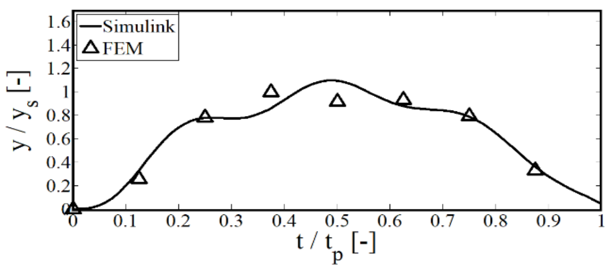
Figure-9 shows a comparison between the results obtained from the FEM model and those obtained from the proposed Simulink model.



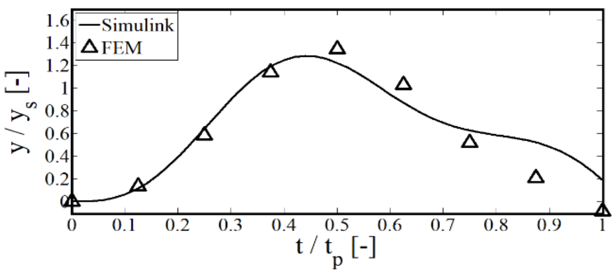
(a) $v = 1\ m/s$ & $t_p = 800\ ms$



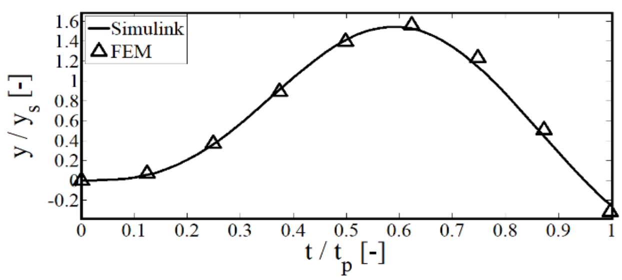
(b) $v = 5\ m/s$ & $t_p = 160\ ms$



(c) $v = 10\ m/s$ & $t_p = 80\ ms$



(d) $v = 20\ m/s$ & $t_p = 40\ ms$



(e) $v = 30\ m/s$ & $t_p = 26.6\ ms$

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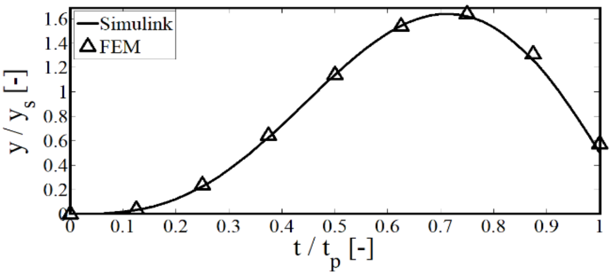
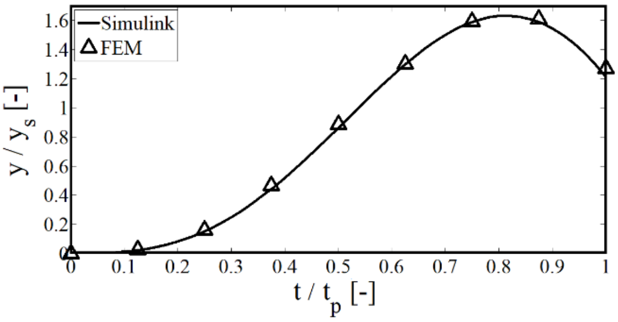
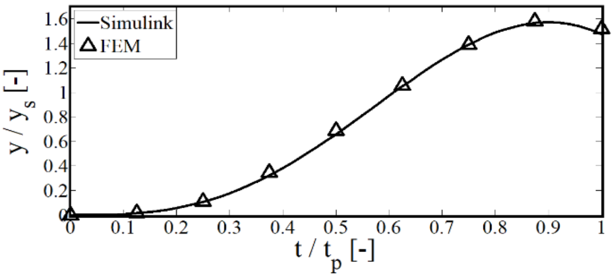


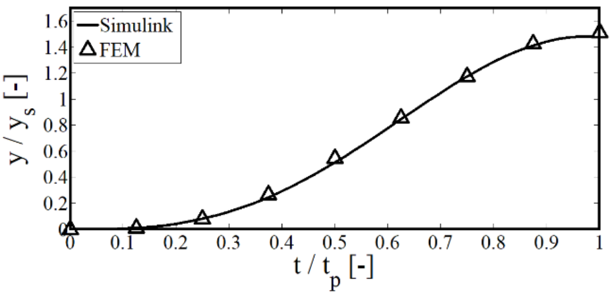
(f) $v = 40\ m/s$ & $t_p = 20\,ms$



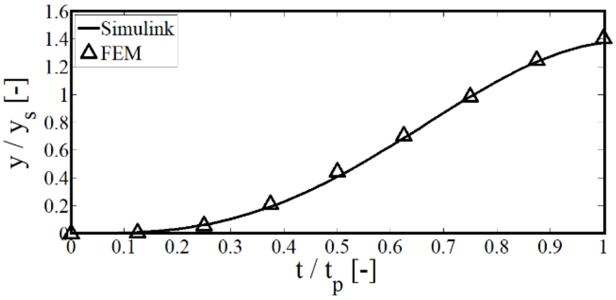
(g) $v = 50\ m/s$ & $t_p = 16\,ms$



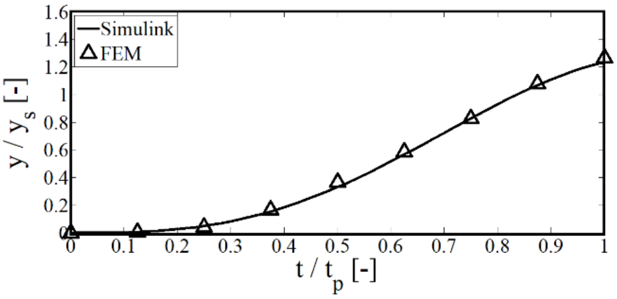
(h) $v = 60\ m/s$ & $t_p = 13.3\,ms$



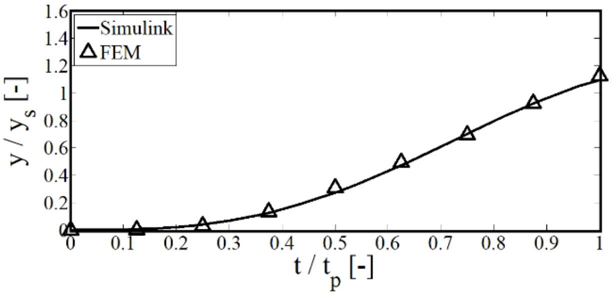
(i) $v = 70\ m/s$ & $t_p = 11.4\,ms$



(j) $v = 80\ m/s$ & $t_p = 10\,ms$



(k) $v = 90\ m/s$ & $t_p = 8.9\,ms$



(l) $v = 100\ m/s$ & $t_p = 8\,ms$

Figure-9. FEM and Simulink results for 500 N force moves with different speeds.

Figure-10 shows the maximum vibration amplitude of the beam with the corresponding speed of the load. In this figure, the load speed, v, was divided by the critical speed of the beam, v_c, which was calculated and found to be 72.67 m/s.

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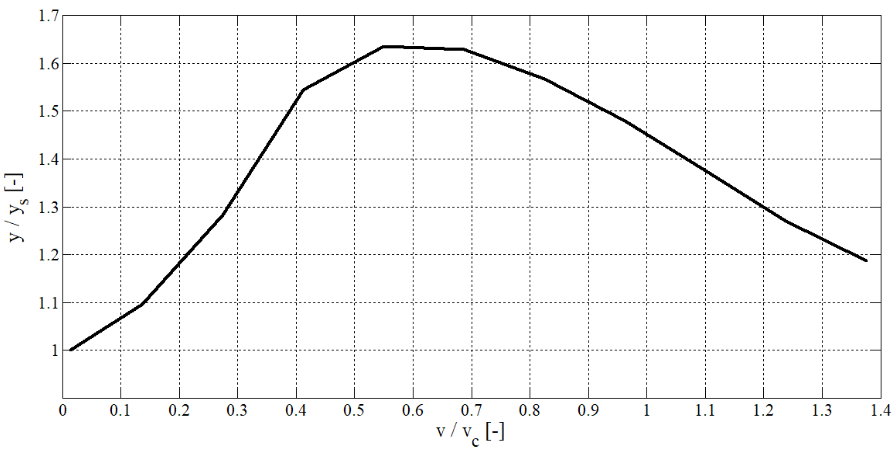

Figure-10. Maximum vibration amplitude of the beam at different load speeds.

5. DISCUSSIONS

In general, it can be concluded that using Transient Structural Analysis (Ansys) requires experience in defining the beam geometry and boundary conditions. Furthermore, the moving load needs to be set up carefully to achieve the required speed and magnitude. In addition, if it is required to investigate another beam of different geometry or different loading conditions, then a new model should be implemented. This can cause extra effort and time waste. Also, the Transient Structural Analysis does not offer any information about the steady-state response of the beam. Instead, the proposed Simulink model can be assembled and used without the need for any special knowledge of software handling, where the beam parameters and the load specifications can easily be adjusted.

It can be seen from Figure-8 that the results of the derived formula (equation 15) are very close to the results obtained from Ansys R19.0 for the load speeds of 1 m/s, 5 m/s, and 10 m/s. Therefore, it can be concluded that the derived formula can be used only for the low and intermediate load speeds, which are 20% of the critical speed.

Generally, there is a very good agreement between most of the results obtained from the FEM (Ansys R19.0) model and the proposed Simulink model, as shown in Figure 9. It was found that the differences do not exceed 1.5%. It is worth saying that the Simulink results were obtained much easier and faster than the results from Ansys R19.0.

Furthermore, Figure-10 shows that the maximum amplitude due to a moving load is about 1.64 times the deflection caused by a static load of the same magnitude, and this maximum vibration occurs when the load speed is about 0.58 times the critical speed of the beam.

6. CONCLUSIONS

This paper introduces an equivalent lumped parameter model to describe the beam vibration caused by a moving load. Furthermore, the paper proposes a simple analytical formula and Simulink model to calculate the vibration of such beams under any loading conditions. In general, the following points were withdrawn from the current work:

a) The derived formulae (equation 15) give accurate results for loads with speeds less than 20% of the critical speed of the beam.
b) Using the proposed Simulink model gives very good results in comparison with the results obtained from the FEM model (Ansys R19.0). However, the proposed Simulink model is easy to implement and can be used for designing any beam under any loading conditions.
c) The maximum vibration amplitude caused by a moving load in a simply supported beam can reach 1.64 times the deflection caused by a static load of the same magnitude.
d) Maximum vibration occurs when the load speed is about 0.58 of the critical speed of the beam.

REFERENCES

[1] Timoshenko S. P. 1953. History of Strength of Materials: With a Brief Account of the History of Theory of Elasticity and Theory of Structures, McGraw-Hill, New York.

[2] Fryba L. 1972. Vibration of Solids and Structures under Moving Loads. Noordhoff, Netherlands.

[3] Ouyang H. 2011. Moving-load Dynamic Problems: A Tutorial (with a brief overview)’, Mechanical Systems and Signal Processing. 25, pp. 2039-2060.

[4] Nielsen J. C. and Igeland A. 1995. Vertical dynamic interaction between train and track influence of wheel and track imperfections. Journal of sound and vibration. 187(5): 825-839.

[5] Andersson C. and Dahlberg T. 1998. Wheel/rail impacts at a railway turnout crossing. Proceedings of

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the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit. 212(2): 123-134.

[6] Sun Y. Q. and Dhanasekar M. 2002. A dynamic model for the vertical interaction of the rail track and wagon system. International journal of solids and structures. 39(5): 1337-1359.

[7] Markine V. L., Steenbergen M. J. M. M. and Shevtsov I. Y. 2011. Combatting RCF on switch points by tuning elastic track properties. Wear. 271(1-2): 158-167.

[8] Nielsen J. C., Lombaert G. and François S. 2015. A hybrid model for prediction of ground-borne vibration due to discrete wheel/rail irregularities. Journal of Sound and Vibration. 345, 103-120.

[9] Zhai W. and Cai Z. 1997. Dynamic interaction between a lumped mass vehicle and a discretely supported continuous rail track. Computers & structures. 63(5): 987-997.

[10] Andersen L. and Nielsen S. R. 2003. Vibrations of a track caused by variation of the foundation stiffness. Probabilistic Engineering Mechanics. 18(2): 171-184.

[11] Lou P., Zhong X. G., Tang J. F. and Zeng Q. Y. 2006. Finite-element analysis of discretely supported rail subjected to multiple moving concentrated forces. Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit. 220(3): 305-315.

[12] Huang H., Shen S. and Tutumluer E. 2010. Moving load on track with Asphalt trackbed. Vehicle System Dynamics. 48(6): 737-749.

[13] Triepaischajonsak N. and Thompson D. J. 2015. A hybrid modelling approach for predicting ground vibration from trains. Journal of Sound and Vibration. 335, 147-173.

[14] Koroma S. G., Thompson D. J., Hussein M. F. M. & Ntotsios E. 2017. A mixed space-time and wavenumber-frequency domain procedure for modelling ground vibration from surface railway tracks. Journal of Sound and Vibration. 400, 508-532.

[15] Germonpré M., Nielsen J. C. O., Degrande G. and Lombaert G. 2018. Contributions of longitudinal track unevenness and track stiffness variation to railway induced vibration. Journal of Sound and Vibration. 437, 292-307.

[16] Dai J., Ang K. K., Tran M. T., Luong V. H. and Jiang D. 2018. Moving element analysis of discretely supported high-speed rail systems. Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit. 232(3): 783-797.

[17] Katz R., Lee C. W., Ulsoy A. G. and Scott R. A. 1987. Dynamic Stability and Response of a Beam Subjected to a Deflection Dependent Moving Load. Journal of Vibration, Acoustics, Stress, and Reliability in Design. 109/361.

[18] Yavari A., Nouri M. and Mofid M. 2002. Discrete Element Analysis of Dynamic Response of Timoshenko Beams under Moving Mass. Advances in Engineering Software. 33, pp. 143-153.

[19] Bilello C., Bergman L. A. and Kuchma D. 2004. Experimental Investigation of a Small-Scale Bridge Model under a Moving Mass. Journal of Structural Engineering. 130, pp. 799-804.

[20] Aied H., Gonzalez A. 2014. Theoretical response of a simply supported beam with a strain rate dependent modulus to a moving load'. Engineering Structures. 77 pp. 95-108.

[21] Hassan A. R. and Sadiq S. E. 2016. Transient Stress Analysis of Simply Supported Beam Excited under Moving Load. Al-Qadisiyah Journal for Engineering Sciences. 9(3): 331-348.

[22] Froio D., Rizzi E., Simões F. M. and Pinto Da Costa A. 2018. Dynamics of a beam on a bilinear elastic foundation under harmonic moving load. Acta Mechanica. 229(10): 4141-4165.

[23] Songsuwan W., Pimsarn M. and Wattanasakulpong N. 2018. Dynamic responses of functionally graded sandwich beams resting on elastic foundation under harmonic moving loads. International Journal of Structural Stability and Dynamics. 18(09): 1850112.

[24] Sarparast H. and Ebrahimi-Mamaghani A. 2019. Vibrations of laminated deep curved beams under moving loads. Composite Structures. 226, 111262.

[25] Zhang X., Thompson D. and Sheng X. 2020. Differences between Euler-Bernoulli and Timoshenko beam formulations for calculating the effects of moving loads on a periodically supported beam. Journal of Sound and Vibration. 481, 115432.

ISSN 1819-6608
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[26] Ebrahimi-Mamaghani A., Sarparast H. and Rezaei M. 2020. On the vibrations of axially graded Rayleigh beams under a moving load. Applied Mathematical Modelling. 84 pp. 554-570.

[27] Jahangiri A., Attari N. K., Nikkhoo A. and Waezi Z. 2020. Nonlinear dynamic response of an Euler–Bernoulli beam under a moving mass–spring with large oscillations. Archive of Applied Mechanics. 90(5): 1135-1156.

[28] Zheng X., Yang D. H., Yi T. H. and Li H. N. 2020. Bridge influence line identification from structural dynamic responses induced by a high-speed vehicle. Structural Control and Health Monitoring. 27(7): e2544.

[29] Camara A. 2021. A fast mode superposition algorithm and its application to the analysis of bridges under moving loads. Advances in Engineering Software. 151, 102934.

[30] Assie A., Akbaş Ş. D., Bashiri A. H., Abdelrahman A. A. and Eltaher M. A. 2021. Vibration response of perforated thick beam under moving load. The European Physical Journal Plus. 136(3): 1-15.

[31] Akbaş Ş. D., Ersoy H., Akgöz B. and Civalek Ö. 2021. Dynamic analysis of a fiber-reinforced composite beam under a moving load by the Ritz method. Mathematics. 9(9): 1048.

[32] Zhang Y., Jiang L., Zhou W., Liu S., Feng Y., Liu X. and Lai Z. 2022. Dynamic response analysis of a multiple-beam structure subjected to a moving load. Earthquake Engineering and Engineering Vibration. 21(3): 769-784.