



# SYNTHESIZE THE OPTIMAL CONTROL LAW FOR THE ANTENNA TRANSMISSION SYSTEM OF THE MISSILE SEEKER

Dang Tien Trung<sup>1</sup> and Nguyen Van Bang<sup>2</sup>

<sup>1</sup>Faculty of Electrical Engineering, Electric Power University, Ha Noi, Viet Nam

<sup>2</sup>Faculty of Missile, Air Defense - Air Force Academy, Ha Noi, Viet Nam

E-Mail: [dangtientrung@gmail.com](mailto:dangtientrung@gmail.com)

## ABSTRACT

The appearance in the armed forces of some countries super-maneuverable aircraft has posed the problem of synthesizing the angle measuring equipment of the missile seeker with higher requirements for accuracy, fast acting, and stability when tracking the target. However, the operating principle of current angle measuring equipment does not allow simultaneous improvement of all the necessary criteria, even though such principles are based on the generation of the evaluation signals of angular coordinates and angular speed of line of sight in a loop circuit. Based on the application of optimal control theory, it is possible to synthesize the system's many loop circuits, which simultaneously achieve the best criteria in terms of accuracy, impact speed, as well as tracking stability, and consumption energy for the control process.

**Keywords:** control law, optimal, missile, seeker, antenna.

Manuscript Received 18 February 2023; Revised 28 July 2023; Published 13 August 2023

## INTRODUCTION

The system of determining the coordinates of the target angle of a tracking loop, simple structure [5], [6]. However, the transmission system not only tracks the change of the line of sight angle, but also the longitudinal axis angle change. Due to the direct use of the measuring signals to evaluate the angular coordinates, the noise of the measuring equipment cannot be excluded [4]. The accuracy of the line of sight angle coordinate and the speed of the line of sight angle depends on the accuracy of the antenna control. Because the antenna system has a large inertia, the accuracy of antenna control is not high, especially when the target is maneuvering. Simultaneously guaranteeing accuracy and high stability is difficult to achieve in maneuvering target conditions.

Multi-loop tracking target angle coordinates determination system, improved accuracy of tracking target angle coordinates. However, it is computationally complex and difficult to actualize the algorithm [1], [6]. Only take into account the maneuvering target situation with specific values of maneuvering intensity and frequency.

## SYNTHESIS OF CONTROL LAW

The target angular coordinate determination system is synthesized based on the following equations of state [5], [6]:

$$\dot{\varepsilon}_d = \omega_d; \varepsilon_d(0) = \varepsilon_{d0} \quad (1)$$

$$\dot{\omega}_d = -\frac{2\dot{D}}{D}\omega_d + \frac{1}{D}(j_{md} - j_d); \omega_d(0) = \omega_{d0} \quad (2)$$

$$\dot{j}_{md} = -\alpha_j j_{md} + \xi_{jmt}; j_{md}(0) = j_{md0} \quad (3)$$

$$\dot{j}_d = \xi_{jd}; j_d(0) = j_{d0} \quad (4)$$

$$\dot{\mathcal{G}} = \omega_g; \mathcal{G}(0) = \mathcal{G}_0 \quad (5)$$

$$\dot{\omega}_g = -\alpha_g \omega_g + \xi_{\omega g}; \omega_g(0) = \omega_{g0} \quad (6)$$

$$\dot{\varphi}_{ad} = \omega_{ad}; \varphi_{ad}(0) = \varphi_{ad0} \quad (7)$$

$$\dot{\omega}_{ad} = -\frac{1}{T}\omega_{ad} + \frac{b}{T}u_{\omega} + \xi_{\omega}; \omega_{ad}(0) = \omega_{ad0} \quad (8)$$

Where;

$\varepsilon_d$  and  $\omega_d$  - Line of sight angle and line of sight angle speed;

$D$  and  $\dot{D}$  - Missile-to-target distance and approach speed;

$j_{md}$  and  $j_d$  - Normal acceleration of the target and of the missile;

$\alpha_j$  - Coefficient to account for the maneuverability of the target;

$\mathcal{G}$  and  $\omega_g$  - Nod angle and nod angle speed;

$\alpha_g$  - Characteristic coefficient for the width of the missile's angular oscillation spectrum;

$\varphi_{ad}$  and  $\omega_{ad}$  - Antenna rotation angle and rotation speed;

$u_{\omega}$  - Antenna transmission system control signal;

$b$  and  $T$  - Gain and time constant of the transmission system;

$\xi_{jmt}$ ,  $\xi_{jd}$ ,  $\xi_{\omega g}$  and  $\xi_{\omega_a}$  - Central white noise with known one-sided spectral density  $G_{jmt}$ ,  $G_{jd}$ ,  $G_{\omega_g}$ ,  $G_{\omega_a}$ .

The model of the observed equations is described as follows [5]:



$$z_1 = K_{\Delta}(\varepsilon_d - \vartheta - \varphi_{ad}) \cos \psi_p + \xi_{\Delta} \quad (9)$$

$$z_2 = K_j \dot{j}_d + \xi_{jdd} \quad (10)$$

$$z_3 = K_g \vartheta + \xi_{gd} \quad (11)$$

$$z_4 = K_{\varphi} \varphi_{ad} + \xi_{\varphi d} \quad (12)$$

In there,

$z_1, z_2, z_3, z_4$  - The voltage at the output of the single-pulse directional equipment, accelerometer, position gyroscope, and antenna angular position sensor;

$K_{\Delta}, K_j, K_g, K_{\varphi}$  - Their transmission coefficients;

$\psi_p$  - Current phase of the medium frequency signal at the output of the minus channel receiver of the directional machine;

$\xi_{\Delta}, \xi_{jdd}, \xi_{gd}, \xi_{\varphi d}$  - Central white noise of measurements with known one-sided spectral density  $G_{\Delta}, G_{jdd}, G_{gd}, G_{\varphi d}$ .

Since the original models (1)-(8) are linear, the noise effects are Gauss, the quality function is quadratic, so the synthesis problems of the optimal filter circuit, the optimal controller can be performed appear separately [4]. The process of synthesizing the system of determining angular coordinates is divided into two problems: the optimal regulator synthesis problem (optimal control law for the transmission system) and the optimal filter synthesis problem. The optimal regulator of the antenna transmission system in the angle measuring equipments must ensure the following requirements:

- Generates a control signal to ensure constant tracking by antenna with a strong maneuvering target;
- Generates mixed feedback signal for optimal filter circuit;
- Ensures high precision antenna stability in space when missile have angular oscillations.

To solve this problem, it is necessary to generate for the transmission system(7), (8) the control signal  $u_{\omega}$ , optimized according to the minimum criterion of the quality function [3], [4]:

$$I = M \left\{ \left[ \mathbf{x}_T(t) - \mathbf{x}_y(t) \right]^T \mathbf{Q} \left[ \mathbf{x}_T(t) - \mathbf{x}_y(t) \right] + \int_0^{t_k} \mathbf{u}^T(t) \mathbf{K} \mathbf{u}(t) dt \right\} \quad (13)$$

This transmission system is used to track the process (1), (2) when there are observations (9) - (12).

$$I = M_{dk} \left\{ \begin{bmatrix} \varepsilon_d - \vartheta & -\varphi_{ad} \\ \omega_d - \omega_g & -\omega_{ad} \end{bmatrix}^T \begin{bmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{bmatrix} \begin{bmatrix} \varepsilon_d - \vartheta & -\varphi_{ad} \\ \omega_d - \omega_g & -\omega_{ad} \end{bmatrix} + \int_0^t u_{\omega}^2 k_u dt \right\} \quad (14)$$

In there,  $q_{11}, q_{22}$  and  $q_{12} = q_{21}$  - Punishment coefficient according to angular tracking accuracy and angular speed,  $k_u$  - Punishment coefficient by the control signal  $u_{\omega}$ .

Using (1), (2) and (7), (8) and comparing (14) with (13), we get:

$$\mathbf{x}_T = \begin{bmatrix} \varepsilon_d - \vartheta & \omega - \omega_g \end{bmatrix}^T; \mathbf{x}_y = \begin{bmatrix} \varphi_{ad} & \omega_{ad} \end{bmatrix}^T; \mathbf{u} = u_{\omega}$$

$$\mathbf{F}_T = \begin{bmatrix} 0 & 1 \\ 0 & -2\dot{D}/D \end{bmatrix}; \mathbf{F}_y = \begin{bmatrix} 0 & 1 \\ 0 & -1/T \end{bmatrix} \quad (15)$$

$$\mathbf{B}_y = \begin{bmatrix} 0 \\ b/T \end{bmatrix}; \mathbf{Q} = \begin{bmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{bmatrix}; \mathbf{K} = k_u$$

Replace (15) in the following expression [3], [4]:

$$u = -\mathbf{K}^{-1} \mathbf{B}^T \mathbf{Q} \hat{\mathbf{x}} = -\mathbf{K}^{-1} \begin{bmatrix} 0 & \mathbf{B}_y^T \\ -\mathbf{Q} & \mathbf{Q} \end{bmatrix} \begin{bmatrix} \hat{\mathbf{x}}_T \\ \hat{\mathbf{x}}_y \end{bmatrix} = \mathbf{K}^{-1} \mathbf{B}_y^T \mathbf{Q} (\hat{\mathbf{x}}_T - \hat{\mathbf{x}}_y) \quad (16)$$

Find the optimal control signal:

$$u_{\omega} = \frac{bq_{21}}{Tk_u} (\hat{\varepsilon}_d - \hat{\vartheta} - \hat{\varphi}_{ad}) + \frac{bq_{22}}{Tk_u} (\hat{\omega}_d - \hat{\omega}_g - \hat{\omega}_{ad}) = K^{\varphi} \Delta\varphi + K^{\omega} \Delta\omega \quad (17)$$

In there,  $\Delta\varphi = \hat{\varepsilon}_d - \hat{\vartheta} - \hat{\varphi}_{ad}$  - Error of tracking to the angular.

$\Delta\omega = \hat{\omega}_d - \hat{\omega}_g - \hat{\omega}_{ad}$  - Error of tracking to the angular speed.

The gain according to the tracking errors:

$$K^{\varphi} = bq_{21} / (Tk_u), K^{\omega} = bq_{22} / (Tk_u) \quad (18)$$

From (16) - (18) it follows that:

- The regulator is a system with negative feedback in all coordinates that is stabilized and controlled.
- The control signal depends on the angular tracking errors as well as the angular speed.
- The weight of the errors in the control signal is determined by the parameters of the transmission system ( $b/T$ ) and by the correlation of the punishment coefficients according to the accuracy and energy consumption ( $q_{21}/k_u$  and  $q_{22}/k_u$ ).
- A filter circuit is required to generate optimal evaluation signals  $\hat{\varepsilon}_d, \hat{\vartheta}, \hat{\varphi}_{ad}$  and  $\hat{\omega}_d, \hat{\omega}_g, \hat{\omega}_{ad}$  for regulator operation.

### CALCULATE THE GAIN OF THE OPTIMAL CONTROL SIGNAL

The selection of the punishment coefficients of the quality function (14) allows to ensure the maximum permissible high precision of the regulator in steady mode with given constraints on the control signal magnitude



$u_{\omega} \leq U_{\omega_{cp}}$  and of the time constant  $T_{\varphi} \leq T_{\varphi_{cp}}$  of the tracking error handling process:

$$\Delta\varphi_0 = \hat{\varepsilon}_{d0} - \hat{\mathcal{G}}_0 - \hat{\varphi}_{ad0}, \Delta\omega_0 = \hat{\omega}_{d0} - \hat{\omega}_{g0} - \hat{\omega}_{ad0} \quad (19)$$

When, assume that the following conditions are satisfied:

- At the time the target is captured, the maximum error appears, maybe  $\Delta\varphi_0 = \Delta\varphi_{\max}$ ,  $\Delta\omega_0 = \Delta\omega_{\max}$  have the same sign.
- The allowable values of control signal  $U_{\omega_{cp}}$  of time constant  $T_{\varphi_{cp}}$  are known.
- All types of noise effects in models (1)-(12) are absent, that is  $\hat{\varepsilon}_d = \varepsilon_d$ ,  $\hat{\mathcal{G}} = \mathcal{G}$ ,  $\hat{\varphi}_{ad} = \varphi_{ad}$  and  $\hat{\omega}_d = \omega_d$ ,  $\hat{\omega}_g = \omega_g$ ,  $\hat{\omega}_{ad} = \omega_{ad}$ .

The variation law of  $\varepsilon_d$  is determined by assumptions  $\dot{\omega}_d = 0$  and  $\dot{\omega}_g = 0$ ; to simplify the process of calculating the tracking error in steady mode.

When guiding the missile according to the proportional approach method [5]:

$$j_d = \frac{3D_0}{D_0 - D_k} V_{ic} \omega_d + j_{mid} = -N_{01} \dot{D} \omega_d + j_{mid} \quad (20)$$

With,  $N_{01} = 3D_0 / (D_0 - D_k)$ .

Put (20) into (2), we get the equation:

$$\dot{\omega}_d = \frac{\dot{D}}{D} (N_{01} - 2) \omega_d, \omega_d(0) = \omega_{d0} \quad (21)$$

Thus, on the basis of (1), (5), (7) and (21), (7), (8), (17) the target tracking error by angle and angular speed will be determined as follows:

$$\Delta\dot{\varphi} = \omega_d - \omega_g - \omega_{ad} = \Delta\omega \quad (22)$$

$$\begin{aligned} \Delta\dot{\omega} = \dot{\omega}_d - \dot{\omega}_g - \dot{\omega}_{ad} &= \frac{\dot{D}}{D} (N_{01} - 2) \omega_d + \alpha_g \omega_g + \frac{1}{T} \omega_{ad} - \frac{b}{T} K^{\varphi} \Delta\varphi - \frac{b}{T} K^{\omega} \Delta\omega \\ &= \left[ \frac{\dot{D}}{D} (N_{01} - 2) + \frac{1}{T} \right] \omega_d + \left( \alpha_g - \frac{1}{T} \right) \omega_g - \frac{b}{T} K^{\varphi} \Delta\varphi - \left( \frac{b}{T} K^{\omega} + \frac{1}{T} \right) \Delta\omega \end{aligned} \quad (23)$$

When receiving (23) it must be noted that  $\omega_{ad} = \omega_d - \Delta\omega - \omega_g$ , after differentiating (22) with respect to time taking into account (23), we find:

$$\Delta\ddot{\varphi} + \left( \frac{b}{T} K^{\omega} + \frac{1}{T} \right) \Delta\dot{\varphi} + \frac{b}{T} K^{\varphi} \Delta\varphi = \left[ \frac{\dot{D}}{D} (N_{01} - 2) + \frac{1}{T} \right] \omega_d + \left( \alpha_g - \frac{1}{T} \right) \omega_g \quad (24)$$

From this equation, it can be inferred that the tracking stability is determined only by the parameters of the predetermined part  $(b, T)$  and of the regulator

$(K^{\omega}, K^{\varphi})$ . Meanwhile, the accuracy of tracking the target by the signal equalization direction depends on the parameters of the predetermined part, the parameter of the regulator  $(b, T, K^{\omega}, K^{\varphi})$  and the usage conditions  $(D, \dot{D}, \omega_d, \omega_g)$  and  $N_{01} = 3D_0 / (D_0 - D_k)$ .

Investigate the transition process in the regulator through the analysis of the homogeneous solution of equation (24):

$$\Delta\varphi = C_1 \exp(\lambda_1 t) + C_2 \exp(\lambda_2 t) \quad (25)$$

$C_1$  and  $C_2$  are determined by the initial errors  $\Delta\varphi_0$  and  $\Delta\omega_0$ .

$\lambda_1$  and  $\lambda_2$  find out from expression  $\det(E\lambda - F_y + B_y K^{-1} B_y^T Q) = 0$ .

$$\lambda_1 = -0,5 \left( \frac{bK^{\omega} + 1}{T} \right) + 0,5 \sqrt{\left( \frac{bK^{\omega} + 1}{T} \right)^2 - 4 \frac{bK^{\varphi}}{T}} \quad (26)$$

$$\lambda_2 = -0,5 \left( \frac{bK^{\omega} + 1}{T} \right) - 0,5 \sqrt{\left( \frac{bK^{\omega} + 1}{T} \right)^2 - 4 \frac{bK^{\varphi}}{T}} \quad (27)$$

From (25) it can be seen that, to eliminate the initial tracking errors, only conditions  $\lambda_1 < 0$  and  $\lambda_2 < 0$  need to be satisfied.

To avoid overcorrection in the processing of initial tracking error handling, it is necessary to ensure that the solutions  $\lambda_1$  and  $\lambda_2$  are real solutions, that is  $(bK^{\omega} + 1)^2 > 4bTK^{\varphi}$ .

So  $-\lambda_1 < -\lambda_2$  is true. Under that condition, the time constant  $T_{\varphi}$  of the regulator is determined by the modulus minimum solution with sufficient accuracy:

$$-\lambda_1 = 1/T_{\varphi} \quad (28)$$

Its value depends on the parameters of the predicate and of the regulator.

The magnitude of  $K^{\varphi}$  and  $K^{\omega}$  must be ensured such that (17) satisfies condition  $u_{\omega} \leq U_{\omega_{cp}}$ . By using the worst case  $u_{\omega} = U_{\omega_{cp}}$ ,  $T_{\varphi} = T_{\varphi_{cp}}$ ,  $\Delta\varphi = \Delta\varphi_0$  and  $\Delta\omega = \Delta\omega_0$ , we get:

$$K^{\omega} = (U_{\omega_{cp}} - K^{\varphi} \Delta\varphi_0) / \Delta\omega_0 \quad (29)$$

Instead, we find:

$$K^{\varphi} = \frac{T_{\varphi_{cp}} b U_{\omega_{cp}} + (T_{\varphi_{cp}} - T) \Delta\omega_0}{T_{\varphi_{cp}} b (T_{\varphi_{cp}} \Delta\omega_0 + \Delta\varphi_0)} \quad (30)$$



$$K^{\omega} = \frac{T_{\varphi_p}^2 b U_{\omega_p} - (T_{\varphi_p} - T) \Delta \varphi_0}{T_{\varphi_p} b (T_{\varphi_p} \Delta \omega_0 + \Delta \varphi_0)} \quad (31)$$

Analysis (30) and (31) can draw the following conclusions:

In the angle measuring equipment, this is only possible perform when the time constant  $T_{\varphi_p} > T \Delta \omega_0 / (b U_{\omega_p} + \Delta \omega_0)$ . Therefore, in order to increase the impact of the fast handling of the tracking error, it is necessary to increase  $b$  and  $U_{\omega_p}$ . In addition, it is necessary to reduce the initial tracking error according to the angular speed  $\Delta \omega_0$ .

The magnitude of the angular speed and angular target tracking error gain coefficients  $K^{\varphi}$  and  $K^{\omega}$  depend not only on the parameters of the angle measuring equipment, but also depend on the accuracy of the tracking equipment the target in the direction ( $\Delta \varphi_0$  and  $\Delta \omega_0$ ).

The values of the angular and angular speed target tracking errors in steady mode are determined only by the eigen roots of the heterogeneous equation (24). Find eigen roots of heterogeneous equation (24) in the class of functions:

$$\begin{aligned} \Delta \varphi_{xl} &= A \varepsilon_d + B \omega_d + C \vartheta + D_1 \omega_g \\ \Delta \dot{\varphi}_{xl} &= \Delta \omega_{xl} = A \omega_d + B \omega_g \\ \Delta \ddot{\varphi}_{xl} &= \Delta \dot{\omega}_{xl} = 0 \end{aligned} \quad (32)$$

On the basis of (24), we get the equation:

$$\begin{aligned} 0 + \left( \frac{b K^{\omega}}{T} \right) (A \omega_d + C \omega_g) + \frac{b}{T} K^{\omega} (A \varepsilon_d + B \omega_d + C \vartheta + D_1 \omega_g) \\ = \left[ \frac{\dot{D}}{D} (N_{01} - 2) + \frac{1}{T} \right] \omega_d + \left( \alpha_g - \frac{1}{T} \right) \omega_g \end{aligned}$$

From this equation, we get:

$$\begin{aligned} A = 0, \quad C = 0, \quad B = \frac{[D + (N_{01} - 2) \dot{D} T]}{D b K^{\omega}}, \quad D_1 = \frac{(1 - \alpha_g T)}{b K^{\omega}} \\ \Delta \varphi_{xl} = \frac{[D + (N_{01} - 2) \dot{D} T]}{D b K^{\omega}} \omega_d + \frac{(1 - \alpha_g T)}{b K^{\omega}} \omega_g \end{aligned} \quad (33)$$

The solution (33) is approximate. From (33) it follows that  $\Delta \varphi_{xl}$  depends on the parameters of the angle measuring equipment ( $b, K^{\omega}, T$ ), on the parameters of the self-guided system ( $N_{01}$ ) and on the conditions of use ( $D, \dot{D}, \omega_d, \omega_g$ ). If the establishment error  $\Delta \varphi_{xl}$  calculated by (33) is larger than the allowed value, then the new values  $K^{\varphi}, K^{\omega}$  are calculated according to (30) and (31) according to the new  $T_{\varphi}$  time of the transition process, determining the new value of  $\Delta \varphi_{xl}$  until an acceptable result is obtained.

## SURVEY RESULTS AND ANALYSIS

The target's equation of motion:

$$\begin{aligned} \dot{\vartheta}_m &= j_m / V_m \\ \dot{y}_m &= V_{my} = V_m \sin(\vartheta_m) \end{aligned} \quad (34)$$

$$\dot{x}_m = V_{mx} = V_m \cos(\vartheta_m)$$

Equation of the trajectory of the missile:

$$\begin{aligned} \dot{y} &= V_y = V \sin(\vartheta) \\ \dot{x} &= V_x = V \cos(\vartheta) \end{aligned} \quad (35)$$

$\vartheta$  is determined from the missile's response to the required acceleration.

Model of measuring sets:

+ Model of the directional equipment.

The angular deviation between the line of sight direction and the antenna axis.

$$\varphi_n = \arctan\left(\frac{y_m - y}{x_m - x}\right) \quad (36)$$

Using equation (9) with the coefficient  $K_A = I$ , get:

$$z_1 = \varphi_n - \vartheta - \varphi_a + \xi_{z_a} \quad (37)$$

+ Model of the missile's normal accelerometer.

According to expression(10), for  $K_{j_r} = I$ , the model of the normal accelerometer is determined by:

$$z_2 = j_r + \xi_{j_r} \quad (38)$$

+ Model of the missile's longitudinal axis angle measures.

According to expression(11), for  $K_g = I$ , the model of the longitudinal axis angler measure is determined by:

$$z_3 = \vartheta + \xi_{z_g} \quad (39)$$

+ Antenna angular position meter model.

According to expression(12), for  $K_{\varphi_a} = I$ , the antenna angular position meter model is determined by:

$$z_4 = \varphi_a + \xi_{z_{\varphi_a}} \quad (40)$$



**Table-1.** The parameters of the system define the target angular coordinates.

Parameters	Values	Parameters	Values
$\sigma_{z_{JT}}$	1 [m/s <sup>2</sup> ]	$b$	1 [1/s]
$\sigma_{z_{JT}}$	3 [m/s <sup>2</sup> ]	$T$	10 [s]
$\alpha_g$	0,2 [1/s]	$\Delta\phi_0$	5 [°]
$\sigma_{\omega_g}$	0,05 [°/s]	$\Delta\omega_0$	1 [°/s]
$\sigma_{z_g}$	0,01 [°]	$T_{cp}$	8 [s]
$\sigma_{\omega_a}$	0,01 [°/s]	$u_{cp}$	1
$\sigma_{z_{\phi_a}}$	0,1 [°]		

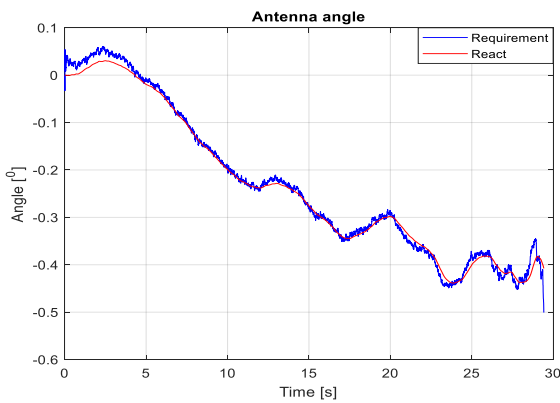
For coordinate determination system using optimization algorithm:

$$\sigma_{j_m} = 40(m/s^2); \alpha_{j_m} = 0,01(1/s); \sigma_{z_i} = 0,01(°/s).$$

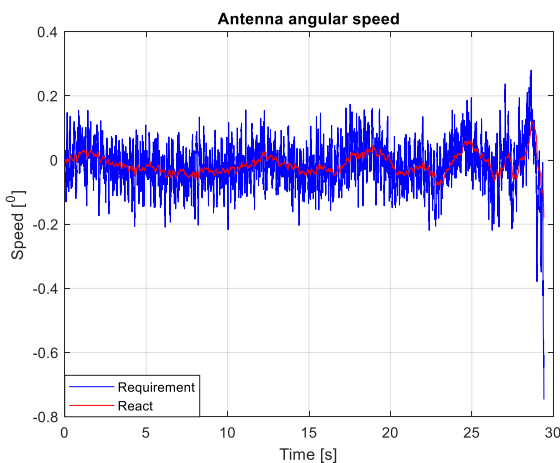
**The case of the target moving in a uniform straight**

Target parameter selected:

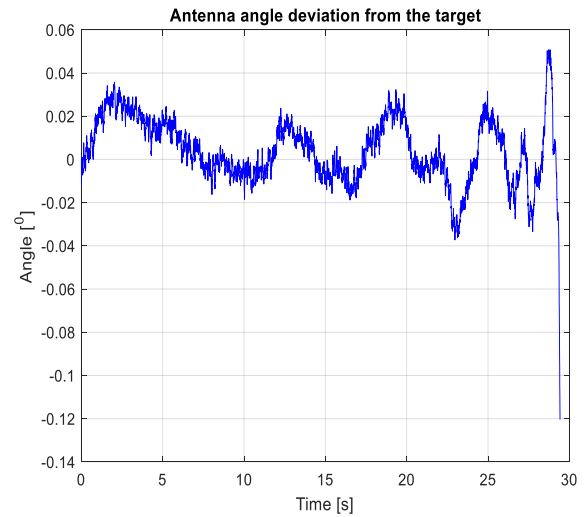
$$V_m = -350(m/s), \mathcal{G}_m = 1°, j_m = 0.$$



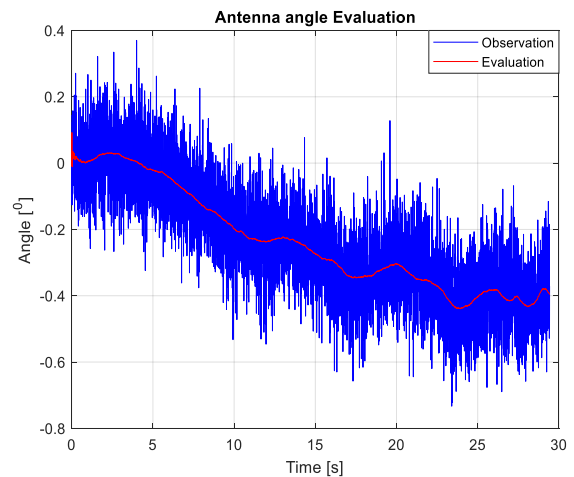
**Figure-1.** Antenna angle react  $\phi_a$ .



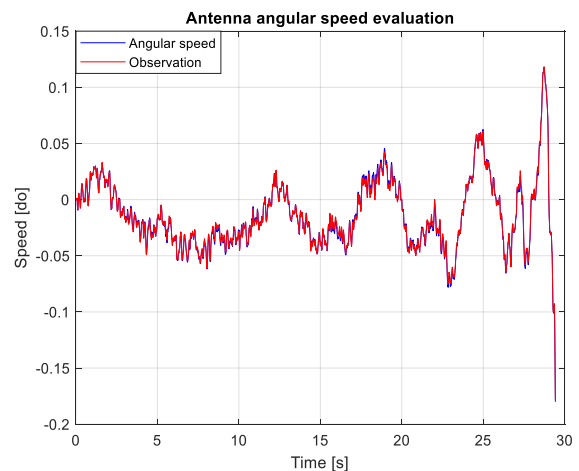
**Figure-2.** Antenna angular speed response react  $\omega_a$ .



**Figure-3.** Angle deviation of the antenna from the target.



**Figure-4.** Antenna angle  $\phi_a$  and antenna angle evaluation  $\hat{\phi}_a$ .



**Figure-5.** Antenna angular speed  $\omega_a$  and antenna angular speed evaluation  $\hat{\omega}_a$ .

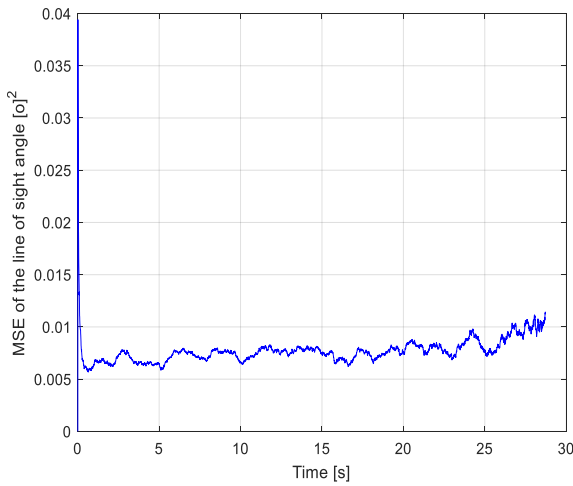


Figure-6. Mean squared error of the line of sight angle  $\varphi_m$ .

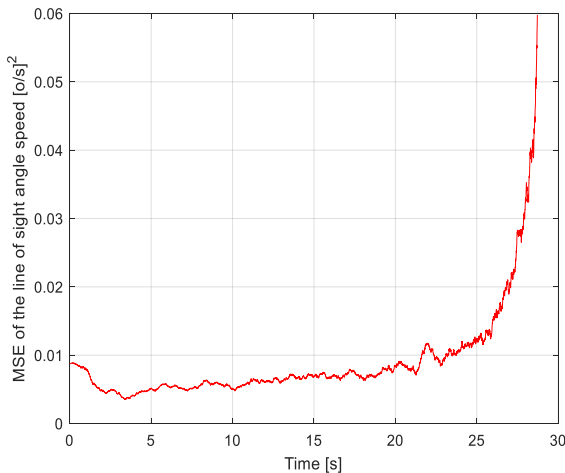


Figure-7. Mean squared error of the line of sight angle speed  $\omega_m$ .

- For the optimization algorithm, the error of evaluation of angle and angular speed in the first stage is very small. Near the encounter distance, the evaluation error fluctuates more strongly (Figures 3, 4, 5). The reason is that at this time, the small distance increases the influence of the determination error  $j_m$  and  $j_T$  on the evaluation of the state.
- The evaluation error of angular speed of the optimization algorithm is very small  $MSE(\varphi_m) \approx 0,4 \times 10^{-4} (^\circ)^2$   $MSE(\omega_m) \approx 0,6 \times 10^{-3} (^\circ/s)^2$  and fluctuates very little, but this error increases as the missile approaches the target.

**The Case of a Maneuvering Target**

The target parameter is selected:

$V_m = -350(m/s), \vartheta_m = 1^\circ$ .

The target's normal acceleration is generated from the following kinetic model:

$$j_m(k) = (1 - \tau\alpha_{j_m})j_m(k-1) + \tau\alpha_{j_m}u$$

Where,  $\alpha_{j_m} = 1(1/s)$ .

$$u = \begin{cases} 0 & \text{when } t < 5s \\ 40(m/s^2) & \text{when } 5s < t < 15s \\ 0 & \text{when } t \geq 15s \end{cases}$$

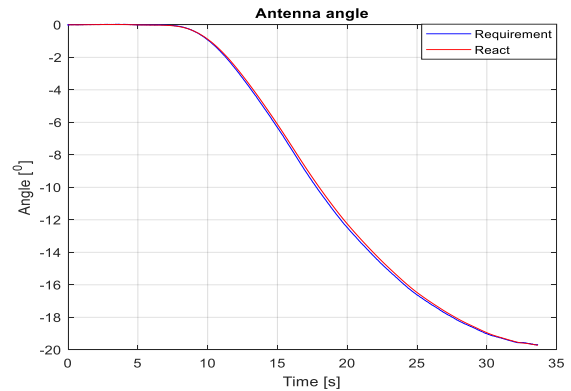


Figure-8. Antenna angle react  $\varphi_a$ .

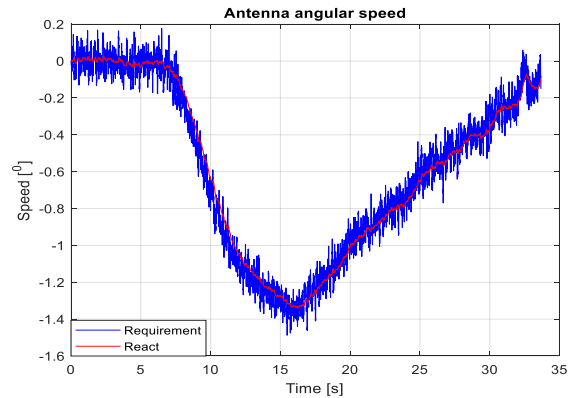


Figure-9. Antenna angular speed response react  $\omega_a$ .

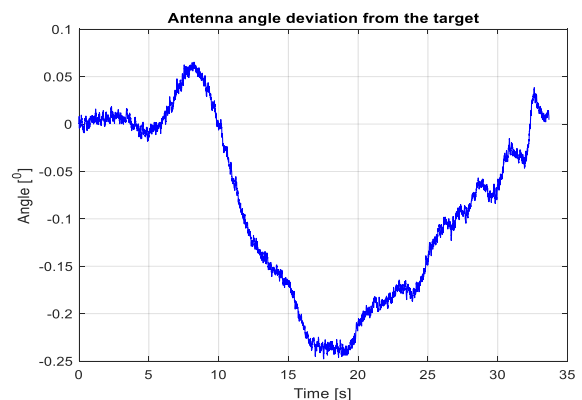
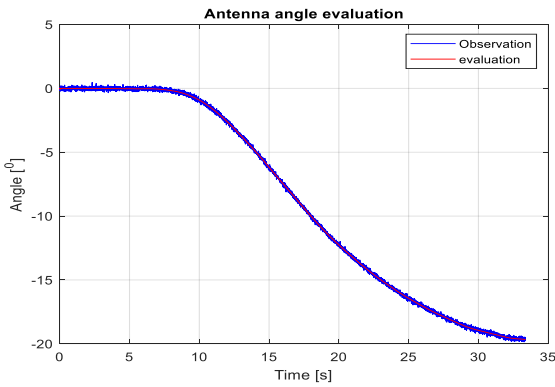
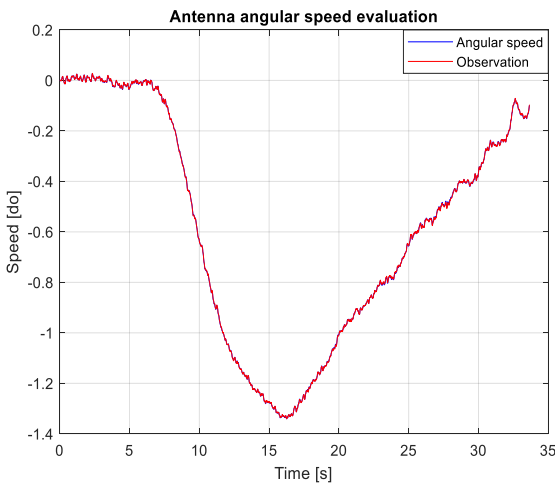


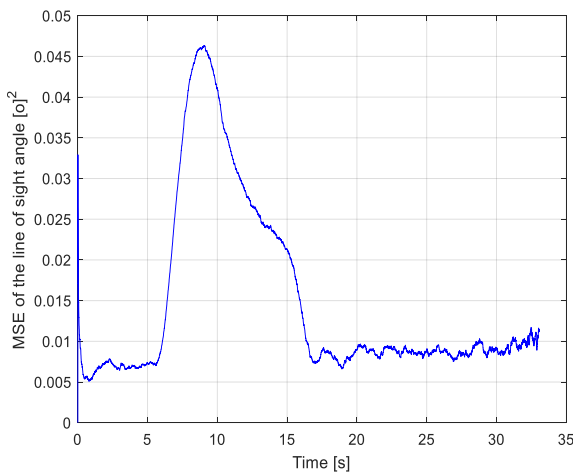
Figure-10. Angle deviation of the antenna from the target.



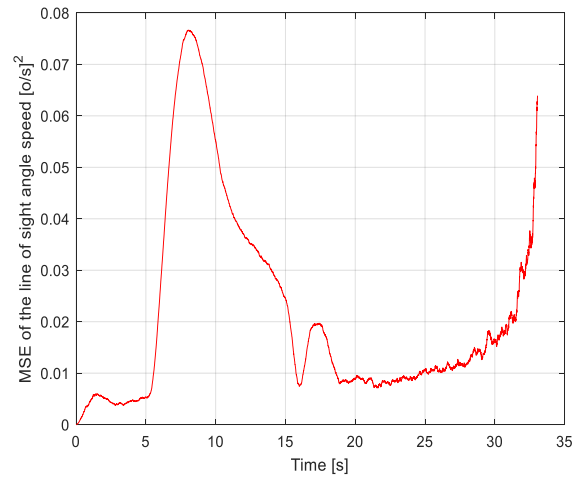
**Figure-11.** Antenna angle  $\varphi_a$  and antenna angle evaluation  $\hat{\varphi}_a$ .



**Figure-12.** Antenna angular speed  $\omega_a$  and antenna angular speed evaluation  $\hat{\omega}_a$ .



**Figure-13.** Mean squared error of the line of sight angle  $\varphi_m$ .



**Figure-14.** Mean squared error of the line of sight angle speed  $\omega_m$ .

- When the target is maneuvering with a large acceleration change (starting at 5s and 15s), the optimal target angular coordinate system appears to have errors in both angle and angular speed. Maximum error  $MSE(\varphi_m) \approx 5 \times 10^{-3} (o)^2$ ,  $MSE(\omega_m) \approx 0,08 (o /s)^2$ .

**5. CONCLUSIONS**

The optimal tracking system determines the maneuvering target angle coordinates is built from separate filters and combines with the antenna control system to form a multi-loop tracking coordinate determination system.

It is not possible to choose a state-space model suitable for all target movements, especially in the case of a maneuvering target. Because, each selection model only matches the movement of the target with a specific frequency and intensity.

The system of determining the target angular coordinates based on the application of the optimal control law of the antenna transmission system has high coordinate evaluation accuracy in the case of non-maneuver targets. In the case of a maneuvering target, the accuracy of target coordinates will decrease.

**REFERENCES**

[1] D. Trung, N. Tuan, N. Bang. 2021. Synthesis of line of sight angle coordinate filter on the basis of interactive multi-model evaluation algorithm, Informatics and Automation, Scopus Indexed - Q3, 20(6): 1333-1367.

[2] L. Bruyere, A. Tsourdos and B. A. White. 2015. Robust augmented lateral acceleration flight control design for a quasi-linear parameter-varying missile, Proceedings of the Institution of Mechanical



Engineers, Part G: Journal of Aerospace Engineering.  
Vol. 219.

- [3] Brian D.O. Anderson & John B Moore. 1995. Linear Optimal Control, Printice-Hall, Inc New Jersey.
- [4] Uhrmeister B. 2004. Kalman filter for a missile with radar and/or imaging sensor, J. Guid. Control Dyn.
- [5] Е. А. Федосов, В. Т. Бобронников, М. Н. Красильщиков, В. И. Кухтенко, А. А. Лебедев, В. В. Малышев, Е. В. Орлов, Б. В. Пучков, А. И. Силаев, В. А., Стефанов. Динамическое проектирование систем управления автоматических маневренных летательных аппаратов, Машиностроение, Москва, 2007, стр. 49-269.
- [6] Канащенко. А. И И Меркулова. В. И, Авиационные системы радиуправления (том 2), Радиотехника, Москва, 2003.