



DEVELOPMENT OF A NOVEL TOOL USED TO SUPPORT DESIGN AND FAILURE ANALYSIS OF MULTIAXIAL FATIGUE LOADING: A CASE OF STEERING KNUCKLE

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ABSTRACT

The steering knuckle requires a lot of attention when designing because once it is damaged it must be replaced with a new one. In automotive industry, since the structure of steering knuckles is very complex and different from each other; it is difficult to design it separately for each vehicle. In order to address this problem, an innovative idea to solve this problem was developed and named as analysis support tool for steering knuckle of the entire vehicle. In addition to that, the development of computer tool that can assist designers in the analysis phase can save the design time. For this purpose, analysis support tool for steering knuckles of entire vehicles has been developed. The part is first modeled by Free CAD 0.2 software considering the precision of the geometry. Based on scientific approach analysis support tool for steering knuckle of all vehicles is developed which can provide analysis of all forces acting on the component, maximum displacement, fatigue damage, maximum deflection, slope, factor of safety, life expectancy, stress and strain amplitude by stress and strain method and compare the results from Morrow, Smith Watson and Walker equations. In addition to that doing experiment of multiaxial is very expensive and time taking for the designer; therefore, developing simple, accurate and robustness tool used to determine life expectancy and cumulative fatigue damage of multiaxial which can be applied to entire materials is very crucial. The developed tool would be universal, effective, simple, and efficient method used to estimate the life expectancy and Cumulative fatigue damage of multiaxial. Finally, data's from open literatures are used to validate the accuracy and capabilities of the proposed tool for multiaxial fatigue loading. The end result is a computer tool that can be dedicated and run on any computer without using any internet connection.

Keywords: steering knuckle, multiaxial fatigue analysis, stress/ strain life, energy approach, factor of safety.

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INTRODUCTION

The steering knuckle is an essential component of the vehicle suspension that houses the wheel hub or axle and connected to the suspension components. The wheel and tire assembly is attached to the knuckle's hub or spindle, which rotates the tire/wheel while keeping it in a balanced plane of motion by the knuckle/suspension system [1], [2], [3]. The wheel assembly is attached at its center to the steering knuckle. Note the protruding arm of the steering knuckle that attaches the steering mechanism to rotate it and wheel assembly. It is the connection between the steering knuckles, tie rod and axle beam by means of kingpins, which are attached to the suspension system as well. The wheel hub is fixed with the steering knuckle using bearings. The main function of it is to convert the linear movement of the tie rod into the angular movement [9], [10]. When the steering is turned by the driver, half part of the component subjected to a tensile load and half part of the component subjected to a compression load, and due to this wheel rotation, it is subjected to a torsional load [8], [6], [7]. Components of this part: A. Suspension Mounting Upper Strut Mount, B. Steering Arm, C. Lower Ball Joint D, Ball Bearing Location / Stub Hole, E. Brake Caliper Mounting and shown in Figure-1 below.

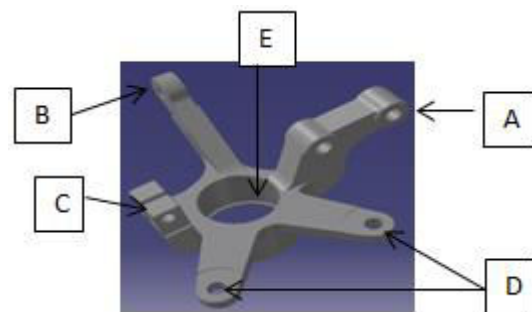


Figure-1. Components of Steering Knuckle by Free CAD 0.2 software.

Fatigue Analysis

Many structures that are used on daily basis exhibit multiaxial fatigue. It is necessary to assess the consequences of fatigue using modified models that take certain particular mechanisms into account. Engineers are frequently taken aback by the extensive list of criteria and are so many models to choose from. These models differ not just in the sorts of equations that they give, but also in the critical criteria that they use.

Recent technical demands for improving the performance of engineering components have highlighted the importance of proper component/system life. In today's



technological demands, proper estimation of component life is required to escape sudden or unexpected failure. Steering knuckles are subjected to cyclic or alternating stresses [11-15], [32-33]. In addition to that, a complex variable stress is frequently caused by material flaws, geometric discontinuities (notches), etc. Thus, fatigue loading is the primary cause of failure for different parts (such as aeronautical engines and automotive transmission components) [6-7]. Therefore, in engineering applications, fatigue analysis taking into account complex load time history is crucial to guaranteeing mechanical structures [12], [18], [36-37]. To survive many cycles experience during its lifetime, fatigue analysis is the key points included in the analysis [19-27], [35], [38]. The methods used to predict the fatigue life include stress and strain life method. Only when stress & strain are present in the elastic region is the stress life approach is appropriate but the strain approach is appropriate for issues brought by plastic strain brought on by a concentration of stress [28-34], [39]. In its original form, the Smith Watson Topper (SWT) was created to calculate the fatigue life expectancy of materials for different a load-condition [37]. Table-3 shows the stress and strain amplitude equation used in the developed tool.

Multi-axial fatigue stresses are frequently applied to numerous hazardous structural components in engineering techniques, including engine turbine discs and blades. Additionally, mechanical structures' local stresses and strains at their bears, connections, and joints will exhibit a multi-axial stress state [42], [43], [44], [45], [46]. It is one of considerable technical significance to conduct multi-axial fatigue load analysis on important parts of engineering structures that are subjected to complex loads in order to more fully use the load-bearing capabilities of materials. When the structure's stress level is primarily in the elastic region, the stress-life approach performs well in estimating fatigue life. The detailed procedure of stress life method is shown in Figure-2 below.

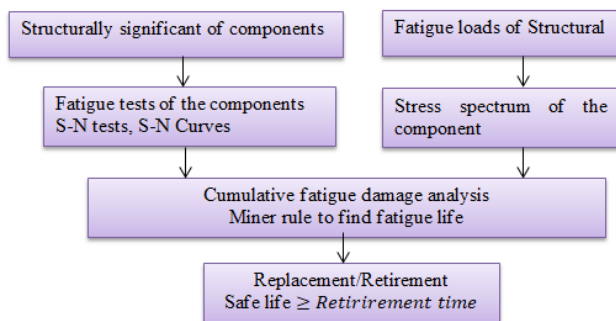


Figure-2. General Technique for calculating fatigue Safe-Life applying stress-life methods [47].

A multi-axial fatigue model based on strain was presented below for smooth and notched objects [48], [49].

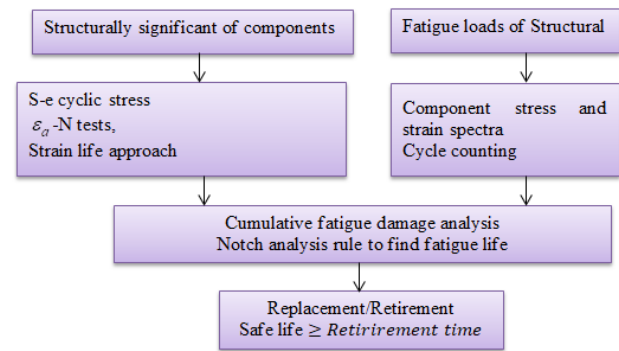


Figure-3. General Method for calculating the remaining and safe fatigue life using the strain-life approach [49].

The energy approaches not only have clear physical significance but also can escape the varied incremental plastic analysis, since energy comprises the interaction of the stress and strain [50], [51]. The energy criterion is suitable for the multi-axial fatigue loading failure mechanism; it cannot accurately depict the fatigue cumulative failure mechanism since the energy is a scalar [52]. Other researchers have also highlighted the drawbacks of the energy approach. First, the use of this criterion requires the use of an exact constitutive equation. Second, when the amount of plastic deformation is small, the plastic strain energy determination inaccuracy is undesired [53]. The strain energy density in the critical plane was believed to be more appropriate than stress or strain for fatigue life under uniaxial tension and compression [54], [55].

METHODOLOGY

A detail of the methodology is elaborated below. The corresponding flowchart is shown in Figure-4. The first stage includes preparation of revised design specification, detail design (i.e. force analysis acting on steering knuckle, maximum displacement, maximum deflection and slope), model it by using Free CAD 0.2 as per the design and procurement of standard part. Other key aspects of the design was multi-axial fatigue failure analysis by using Stress Life and Strain Life approach based on modified Morrow, Smith Watson Topper and Walker equation to find stress and strain amplitude. Finally, generate the results of factor of safety and life expectancy of the steering knuckle of all vehicles. In the design process, basic standards were followed and showed in Figure-4 below.



General Flow chart

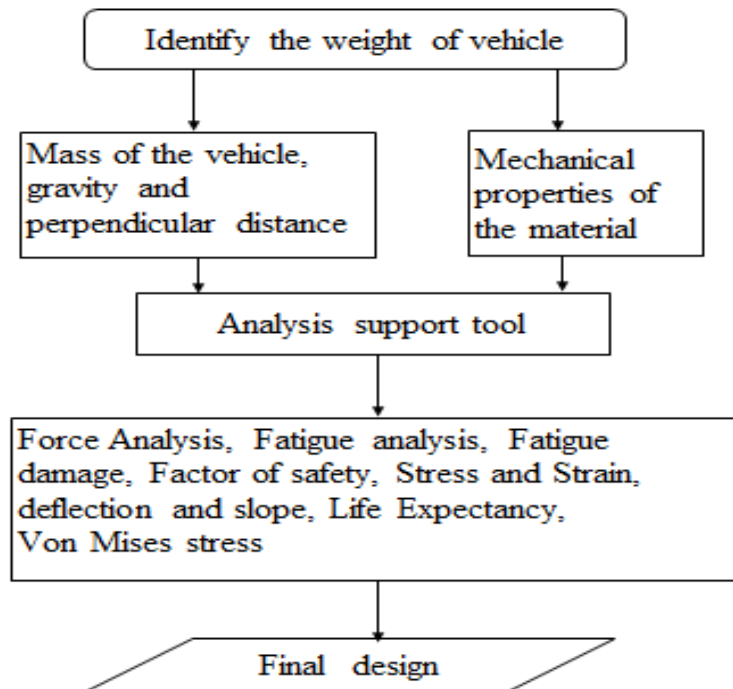


Figure-4. Flow chart Design support tool.

RESULT AND DISCUSSIONS

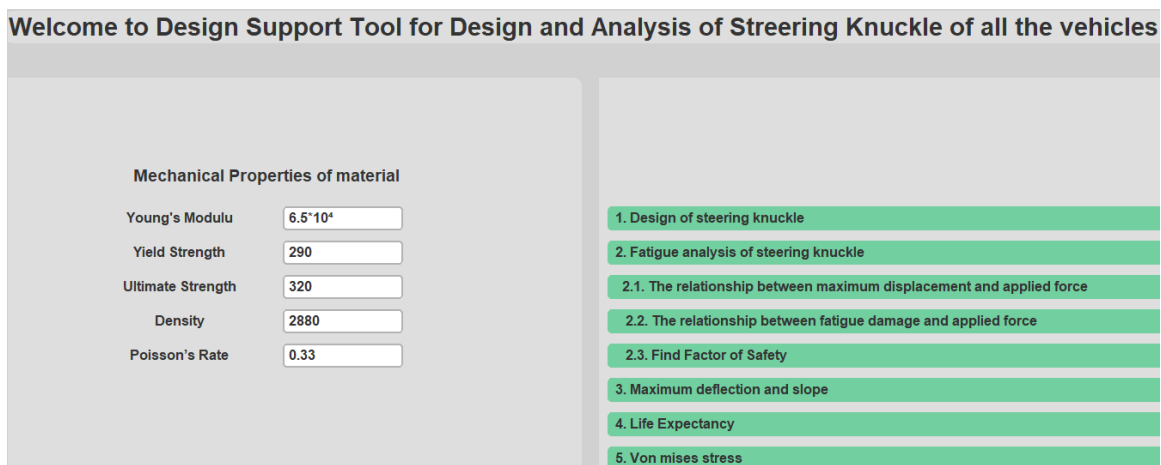
Description of Analysis Support Tool

The steering knuckle design and analysis support tool for all vehicles was developed via Python and the developed tool can be run on any computer without using the internet connection.

How It Works

To use the developed design support tool, the following steps are required: - first open the tool and feed the appropriate data then click on process. Finally, the result will generate and the first page of the tool is shown in the Table-1 below.

Table-1. Shows the first page of the developed tool.



The points that have been done inside the developed tool are as under.

Load Analysis on the Steering Knuckle of the Entire Knuckles

For the determination of forces and moments, required loading situations acting on steering knuckle

component which are used inside the tool indicated below and the result is shown in Table-2.



- a) Braking Load = 1.5 x mass x gravity
- b) Steering Load = 45 to 50 N
- c) Lateral Load = 1.5 x mass x gravity
- d) Moment = Braking Load x perpendicular distance
- e) Load on the X – axis = 1.5 x mass x gravity
- f) Load on the Y – axis = 1.5 x mass x gravity
- g) Load on the Z – axis = 1.5 x mass x gravity

Resultant Load = $\sqrt{F_x^2 + F_y^2 + F_z^2}$

Input: Mass of the vehicle, gravity and perpendicular distance

Output: All Forces and Braking moment

Input data to the tool

Modulus of elasticity (E), Fatigue strength coefficient (σ'_f), Fatigue strength exponent (b), Fatigue ductility coefficient (ϵ'_f), Fatigue ductility exponent (c), (σ_a) stress amplitude, (S_N) fully reversed fatigue strength at the desired number of cycles, Minimum stress (σ_m), Stress Ratio (R), Material dependent exponent (γ), True stress at fracture (σ_{ft}) ($2N_f$) number of reversals to failure in the strain life test

Output: Stress & strain amplitude which is used to determine the fatigue life

Table-2. Shows the result of loading condition on steering knuckle.

Load Condition Result		
Description	Result	
Braking force	4966.3125	
Lateral force	4966.3125	
Steering force	45.80N	
Load on knuckle hub in X- direction	9932.625	
Load on knuckle hub in Y- direction	9932.625	
Load on knuckle hub in Z- direction	3310.875	
Braking moment	147135.285	
Mount on the brake caliper Mount	691.605	
The resultant force on the hub	14431.769539695228	
Mass distribution on a wheel (m)	337.5	

Fatigue Analysis of Steering Knuckle

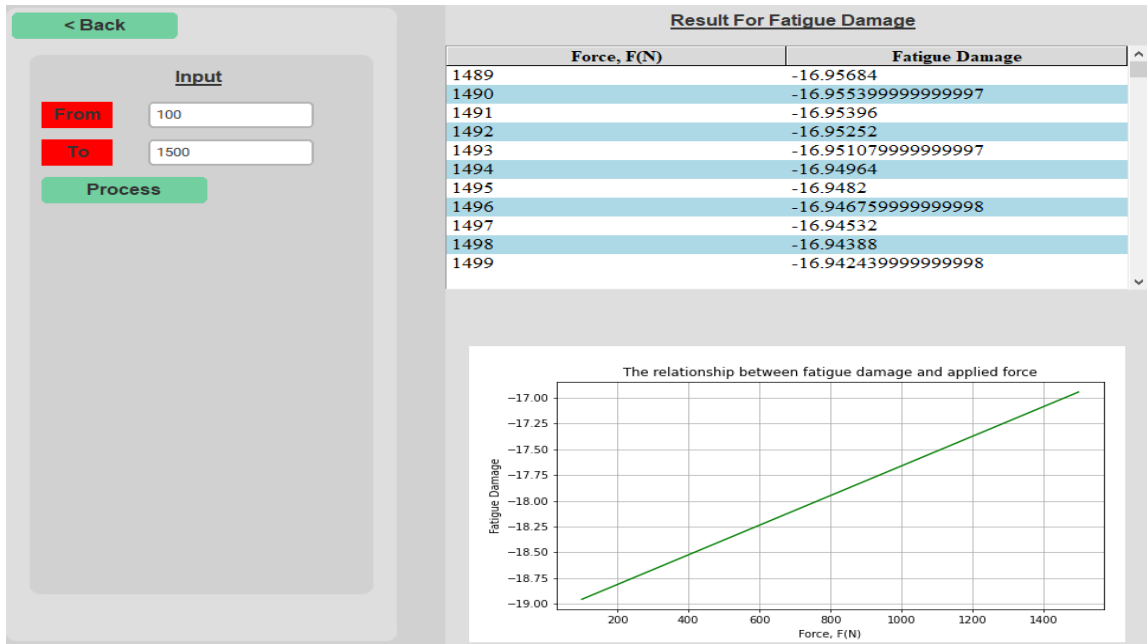
The detail stress and strain amplitudes are calculated by using Morrow, SWT and Walker equation to determine the life expectancy is shown in Table-3 and Table-3.1 below.

Table-3. Shows the stress and strain amplitude equation used inside the tool.

Relationship	Stress Life Equation	Strain Life Equation
Morrow	$\sigma_a = S_N \cdot \left(1 - \frac{\sigma_m}{\sigma_{ft}}\right)$	$\epsilon_a = \frac{\sigma'_f}{E} \cdot \left(1 - \frac{\sigma_m}{\sigma'_f}\right) \cdot (2N_f)^b + \sigma'_f \cdot \epsilon'_f \cdot (2N_f)^{b+c}$
Smith Watson Topper (SWT)	$\sigma_a = S_N \cdot \left(\frac{2}{1-R}\right)^{-1/2}$	$\epsilon_a = \frac{\sigma'_f}{E} \cdot [2N_f \cdot \left(\frac{1-R}{2}\right)^{1/2b}]^b + \epsilon'_f \cdot [2N_f \cdot \left(\frac{1-R}{2}\right)^{1/2b}]^c$
Walker	$\sigma_a = S_N \cdot \left(\frac{2}{1-R}\right)^{-\gamma}$	$\epsilon_a = \frac{\sigma'_f}{E} \cdot [2N_f \cdot \left(\frac{1-R}{2}\right)^{1-\gamma/b}]^b + \epsilon'_f \cdot [2N_f \cdot \left(\frac{1-R}{2}\right)^{1-\gamma/b}]^c$



Table-5. The relationship between fatigue damage and applied force.



Determination of Factor of Safety

The capability of a system’s structural capacity to be viable beyond its actual loads is called factor of safety (n) [41]. The result of factor of safety from Soderberg and Goodman equation is shown in Table-6.

a) Soderberg Equation [41].

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_y} = \frac{1}{n}$$

Where n is the factor of safety

b) Goodman Equation [41].

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_y} = \frac{1}{n}, \text{ Where n is the factor of safety}$$

$$S_e = k_a \cdot k_b \cdot k_c \cdot k_d \cdot S'_e$$

Input data

S_e : Endurance limit of a component subjected to reversed bending stress

S'_e : Endurance limit stress of a rotating beam subjected to reversed bending stress

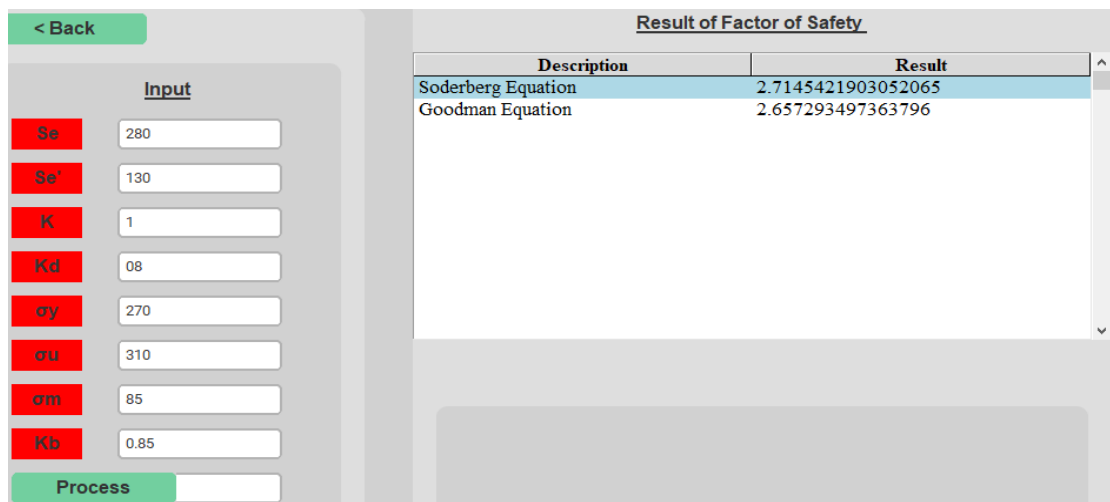
K_a : Surface Finish, k_b : Size factor, σ_m : Mean stress

K_d : Modifying factor of stress concentration

σ_y : Yield strength, σ_u : Ultimate strength,

Out Put: Result of Factor of Safety Soderberg and Goodman Equation

Table-6. Shows the result of factor of safety.





Maximum Deflection and Slope

Deflections are essential to the part's design and analysis. A part's excessive deflection damages the component as well as being visually difficult. It needs remain below the dimensional tolerances of the machined pieces. The slope and deflection are calculated using the basic formula below and the result stated under Table-7. Note that we used the standard SI unit.

$$\delta_{max} = \frac{FL^3}{3EI} \quad \text{and} \quad \theta = \frac{FL^2}{2EI}$$

Where δ_{max} is maximum deflection (m), is slope (*degree*), L- length (m), E- Modulus of elasticity (N/m²), F-Force (n), F is force (N), and I - Moment of inertia (m⁴).

Input data: length (L), modulus of elasticity (E), force (F), moment if inertia (I)

Out Put data: Maximum deflection (m) and slope (Radian)

Table-7. The result of Maximum deflection and slope.

Description	Result
Maximum deflection(M)	0.0027496216520606766
slope(RAD)	0.001649772991236406

Analysis of Equivalent Stress and Life Expectancy

The equivalent theories are based on the concept of converting a multiaxial stress state to a uniaxial one by finding an equivalent stress and the result is shown in Table-8.

Table-8. The result of Equivalent stress and Life expectancy.

Equeivalent Stress = 763.0769230769231
 Life Expectancy = 755.7375940050681

Von Misses Stress

The von Mises stress represents the equivalent stress state of the material before the distortional energy reaches its yielding point and the result can be found by using the Table-9.

Table-9. The result of Von misses stress.

Von Mises Stress = 793.9143530633515

Numerical Sample Load Calculation to Validate the Tool

Assume the total weight of the car is 1350Kg. in order to find the braking load acting on one wheel the total weight of the vehicle must be distribute for four wheels so that the weight of one wheel is 1350/4 = 337.5 kg.



$$a) \text{Braking Load} = 1.5 \times 337.5 \times 9.81 = 4966.3125 \text{ kg } \frac{m}{s^2}$$

$$b) \text{Lateral load} = 1.5 \times 337.5 \times 9.81 = 4966.3125 \text{ kg } \frac{m}{s^2}$$

$$b) \text{Moment} = \text{Braking Load} \times \text{perpendicular distance} \\ = 4966.3125 \text{ kg } \frac{m}{s^2} \times 94 \text{ mm} = 147135.285 \text{ kg } \frac{m}{s^2} \cdot \text{mm}$$

$$d) \text{Load on the X - axis} = \text{Load on the Y - axis} = 3 \times 337.5 \times 9.81 = 9932.625 \text{ kg } \frac{m}{s^2}$$

$$e) \text{Load on the Z - axis} = 1 \times \text{mass} \times \text{gravity} = 3310.875 \text{ kg } \frac{m}{s^2}$$

$$\text{Resultant Load} = \sqrt{F_x^2 + F_y^2 + F_z^2} = \sqrt{9932.625^2 + 9932.625^2 + 3310.875^2} \\ = 14,431.7695397 \text{ kg } \frac{m}{s^2}$$

CONCLUSIONS

The steering knuckle design support tool for all vehicles has been created using Python, and the tool can be used on any computer without an internet connection. The developed tool can calculate the results of all forces acting on the steering knuckle, stress and strain amplitude, stress & strain life approaches by using multiaxial fatigue loading. Additionally, for each applied force, the tool can calculate the maximum displacement produced as well as fatigue damage. Finally, based on the input data, the tool can calculate the factor of safety results using the Goodman and Soderberg equation. The tool can also calculate the steering knuckle's maximum deflection, slope, life expectancy, and equivalent stress. Open source literatures are used to validate the tool. All the developed codes are available with the author.

Data Availability

The data used in this study to support the findings of this research are available from the authors based on request including the developed software.

Conflicts interests

All the authors declare that there is no conflict of interest regarding the publication of this paper.

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