



# CONSTITUTIVE MODELLING TO PREDICT BEHAVIOUR OF PCC MIXES WITH TIRE DERIVED AGGREGATES

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## ABSTRACT

Due to the worldwide depletion of raw materials required for construction projects, it is becoming clear that alternative and renewable construction materials are needed to reduce the demand for raw materials. Sound concerns arise from the side effects of replacing normal aggregate with tire rubber waste, which is expected to affect mechanical properties of the Portland Cement Concrete (PCC) mixes. To be able to decide on suitable Tire Derived Aggregates (TDA) replacement percentages, a simplified model is proposed to predict the PCC mixes stress-strain diagram with different TDA percentages. 0, 10, 20, 40, 60, 80 and 100% TDA were investigated. Using homogenization and lab test results for PCC mixes, 0% and 100% TDA were used as upper and lower bounds, and a model was developed to predict Young's moduli and stress-strain curves for evaluated PCC mixes. Then the model results were verified using other test results. It was found that the proposed model seems to capture the behaviour of PCC mixes with TDA fairly well. However, peak stress for PCC mixes with 40% TDA results were slightly underestimated, but with reasonable accuracy. Furthermore, the proposed model well predicted the ultimate stresses, ultimate strains, and the elastic modulus.

**Keywords:** Portland cement concrete mixes, tire derived aggregates, homogenization, modelling.

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## 1. INTRODUCTION

There is no doubt that massive, widespread development and urbanization globally and regionally have negatively impacted the environment and increased the rate of depletion of natural resources. There is great concern regarding the effect of development and processing-related industries on climate change and global warming, due to their large carbon footprint, in addition to concern related to the enormous amount of solid waste generated by such developments. Typical solid wastes include organics, paper, glass, plastics, metals, wood, rubber tires, etc. The disposal of solid waste typically involves dumping in landfills or incineration for energy production, which may result in environmental problems and the degradation of air, water, and valuable land areas. However, if properly treated and processed, solid waste can become a sustainable source of useful construction materials.

Portland Cement Concrete (PCC) mixes are a primary component of construction projects worldwide. Due to the depletion of raw materials used in PCC mixes (e.g., rocks, water, limestone, shells, chalk, marl, and other materials) as a result of recent development and urbanization, as well as negative environmental impacts associated with the manufacturing processes, the search for alternative construction materials for greener, more sustainable solutions has been intensified.

Researchers have been investigating the utilization of different types of waste as construction materials. The evaluated waste materials included recycled construction materials, plastic waste, glass, rubber tire waste, and steel manufacturing by-products [1-6].

Molenaar [7] has suggested that the utilization of recycled and waste materials in road construction projects could reduce the carbon footprint, while generating economic and environmental benefits. However, concerns have been raised about the inclusion of waste materials in construction materials, especially in Portland Cement Concrete (PCC) mixes. For example, it has been suggested that the presence of waste materials might reduce bonding strength between aggregates and cement paste, which could lead to a reduction in strength in PCC mixes.

Many research studies have been conducted on adding rubber tire waste to PCC mixes. Findings have shown that with increased rubber content, PCC preferred mechanical properties decreased, but ductility and material toughness improved [8]. Bandarage and Sadeghian [9] reported that incorporating shredded rubber particles to replace fine materials resulted in failure patterns differing from those of a typical PCC mix. On the other hand, some studies showed that PCC mixes with rubber could be incorporated in concrete structural elements, providing sufficient strength and adequate service life [10] and [11]; with the added benefit that large amount of plastic energy could be absorbed by rubberized concrete under different loading conditions [12] and [13]. Abu Abdo and El Naggar [14] evaluated utilizing TDA in the construction of rigid pavement. They suggested that using TDA resulted in reduction of environmental impacts and overall costs and improvement in the flexibility of PCC mixes in rigid pavements and would achieve an eco-friendly and sustainable rigid pavement design.

El Naggar (2020) proposed a simplified relationship to describe the mechanical behavior of the



TDA concrete mixes. Alzabeebee (2022) studied using the TDA structural performance in buried concrete pipes subjected to soil load utilizing finite element method.

However, very limited research can be found on analytical modelling of the TDA concrete in literature. This paper aims at filling such a gap in this topic.

**2. OBJECTIVE**

The main objective of this study was to develop a simplified model to predict Young’s moduli and stress-strain curves for different TDA percentages as a substitution for coarse aggregates in PCC mixes.

**3. MATERIALS AND LAB TEST**

A typical PCC mix was used in this study, with a mix design of 25% cement, 10% water, 24% fine aggregates, and 41% coarse aggregates by weight. Waste rubber tires were shredded and cut to particle sizes ranging between 4.75mm to 19.05mm with a bulk density of 557 kg/m<sup>3</sup>. Seven different percentages of TDA were utilized: 0, 10, 20, 40, 60, 80 and 100% by weight of coarse aggregates. Three cylindrical samples were casted with φ150 mm x 300 mm high for each mix condition and then tested using ASTM C469M Standard Test Method for Static Modulus of Elasticity and Poisson’s procedure.

**4. ANALYSIS AND RESULTS**

To achieve the goals of this study, three uniaxial constitutive relations were used to simulate the stress-strain diagram of the normal concrete (0% TDA) and TDA concrete (100% TDA):

Desayi and Krishan [15] model:

$$\sigma = \frac{E\varepsilon}{1 + \left(\frac{\varepsilon}{\varepsilon_p}\right)^2} \tag{1}$$

where  $\sigma$ ,  $\varepsilon$  are stress and strain tensors,  $E$  is the Young’s modulus,  $\varepsilon_p$  is the strain at peak stress.

Saenz [16] model:

$$\sigma = \frac{E\varepsilon}{1 + \left(\frac{E}{E_p} - 2\right)\left(\frac{\varepsilon}{\varepsilon_p}\right) + \left(\frac{\varepsilon}{\varepsilon_p}\right)^2} \tag{2}$$

where  $E_p$  is secant elastic modulus at peak stress. and the simplified proposed model:

$$\sigma = E\varepsilon - aE_p\varepsilon^2 \tag{3}$$

Where  $a$  is a material adjustment parameter.

The three models were used to simulate the stress-strain diagram of normal concrete (0% TDA), and plotted in Figure-1, along with the experimental tests results of normal concrete standard cylinders. The ultimate stress reached about 40 MPa. The values used to plot the models are:  $E = 39112 \text{ MPa}$ ,  $E_p = 23723 \text{ MPa}$ ,  $\varepsilon_p = 0.001801$ , and  $a$  is found to equal to 400 in this case. It was clear that Desayi and Krishan [1] model underestimated the stress-strain diagram, while Saenz [16] model overestimated the test results. On the other hand,

the proposed simplified model captured the stress-strain behavior very well because of the use of the adjustment parameter. The discrepancy between the models started to be apparent at stress above about 50% of the ultimate stress.

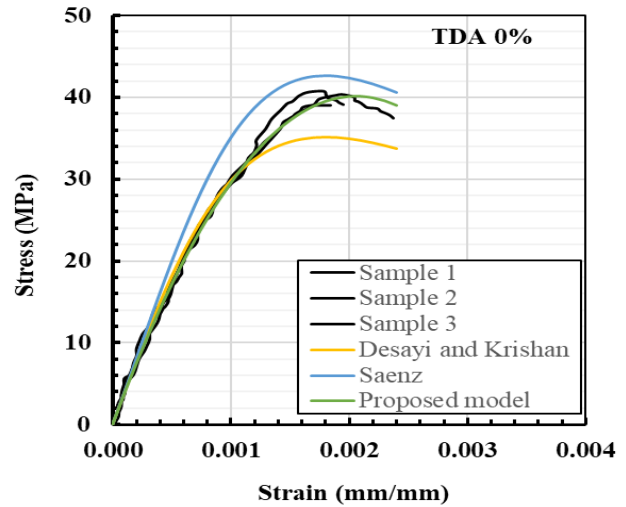


Figure-1. Stress-strain diagram of normal concrete (0% TDA).

Figure-2 shows the experimental results of testing 100% TDA concrete samples, along with the three aforementioned constitutive relationships. The material properties used to plot the models were:  $E = 6492 \text{ MPa}$ ,  $E_p = 3211 \text{ MPa}$ ,  $\varepsilon_p = 0.00263$ , and  $a = 400$ . The discrepancy between the three constitutive models here was less than the case of normal aggregate concrete (0% TDA). The proposed model shows a steeper strain softening after the ultimate stress is reached, when compared with the other two models.

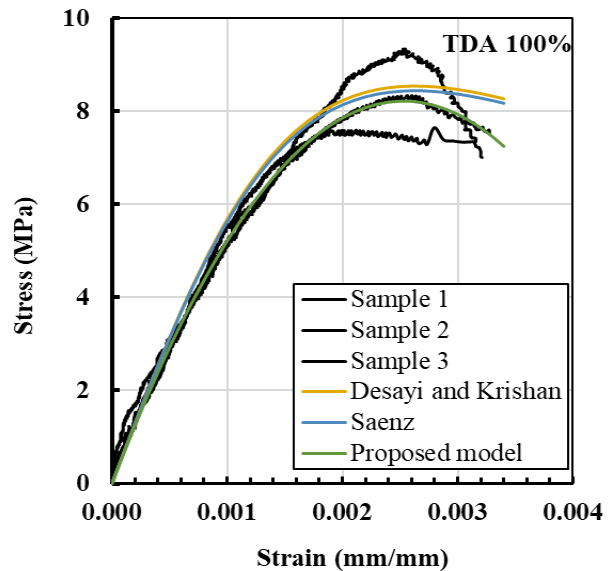


Figure-2. Stress-strain diagram of 100% TDA concrete.



In order to simulate the behaviour of concrete with different percentages of TDA replacing coarse aggregate, some kind of homogenization will be used to obtain the proper elasticity moduli (i.e.,  $E$  and  $E_p$ ). The most basic homogenization methods are the rule of mixtures (assuming uniform strain) and the inverse rule of mixtures (assuming uniform stress). In the rule of mixtures, the homogenized material property can be expressed as weighted sum of the associated properties of the material's constituents. For a two-phase composite, for instance, the elastic modulus of the mixture  $E_m$  is expressed by:

$$E_m = V_{TDA} E_{TDA} + V_{agg} E_{agg} \quad (4)$$

where the  $V_{TDA}$  is the volume fraction of TDA replacing normal aggregate,  $E_{TDA}$  is the modulus of elasticity of 100% TDA concrete,  $V_{agg}$  is the volume fraction of normal aggregate in concrete, and  $E_{agg}$  is the modulus of elasticity of normal aggregate concrete. The inverse rule of mixtures is similarly expressed by:

$$\frac{1}{E_m} = \frac{V_{TDA}}{E_{TDA}} + \frac{V_{agg}}{E_{agg}} \quad (5)$$

The two rules usually represent the upper and lower bounds of homogenization results. A more realistic model is Hirsch's Model [17], which is a combination of the previous two rules, and expressed as the following:

$$\frac{1}{E_m} = x \left( \frac{1}{V_{TDA} E_{TDA} + V_{agg} E_{agg}} \right) + (1 - x) \left( \frac{V_{TDA}}{E_{TDA}} + \frac{V_{agg}}{E_{agg}} \right) \quad (6)$$

where  $x$  and  $(1 - x)$  indicate the relative contributions of the uniform strain and uniform stress models. A value of  $x = 0.3$  was found suitable after many calibration trials, and the results are shown in Figure-3.

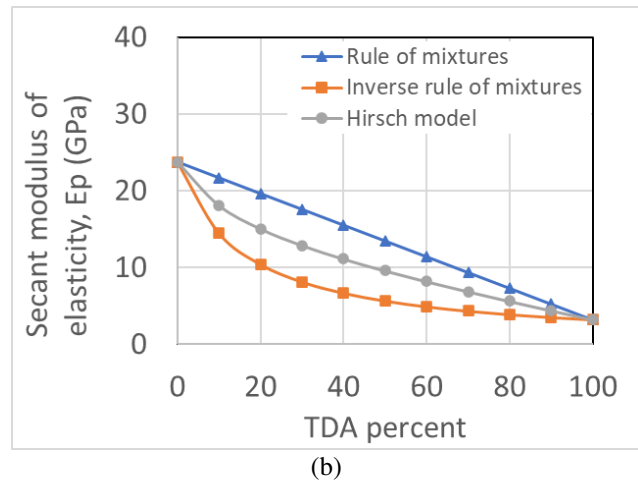
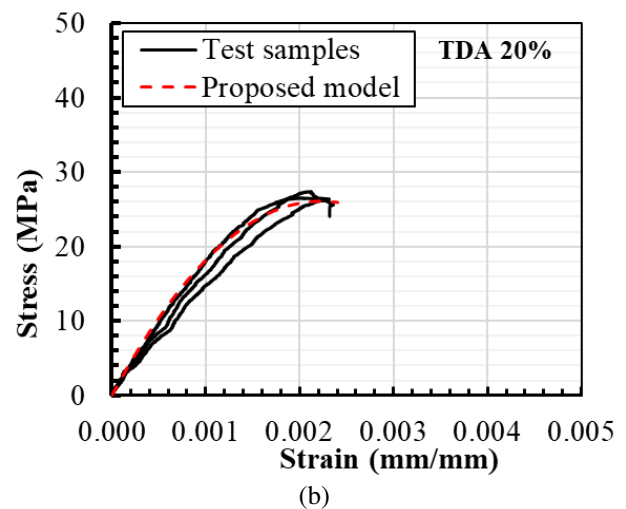
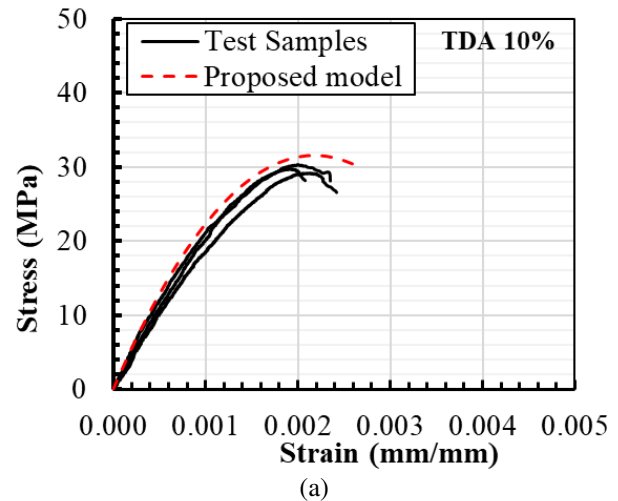
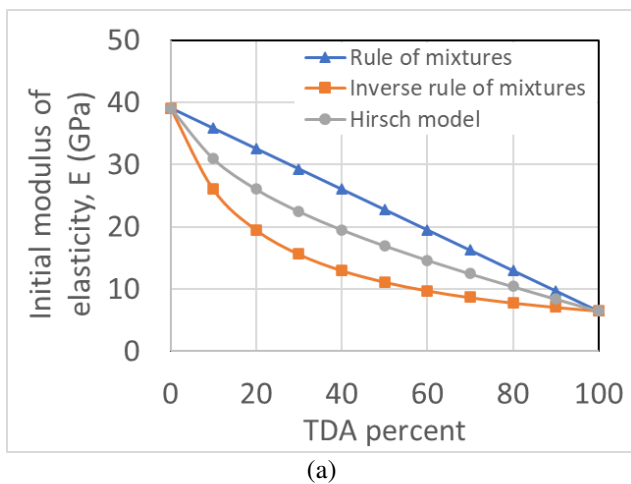
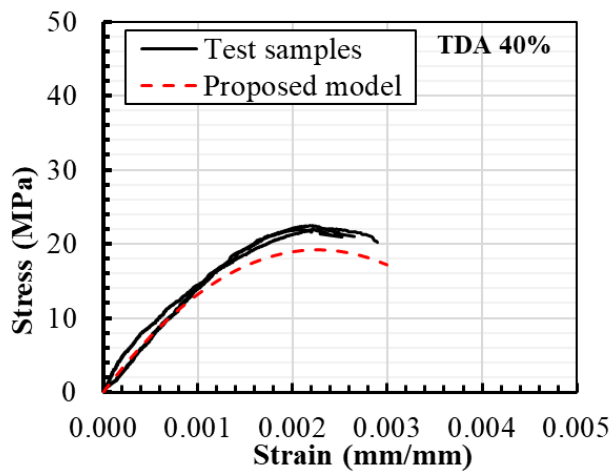


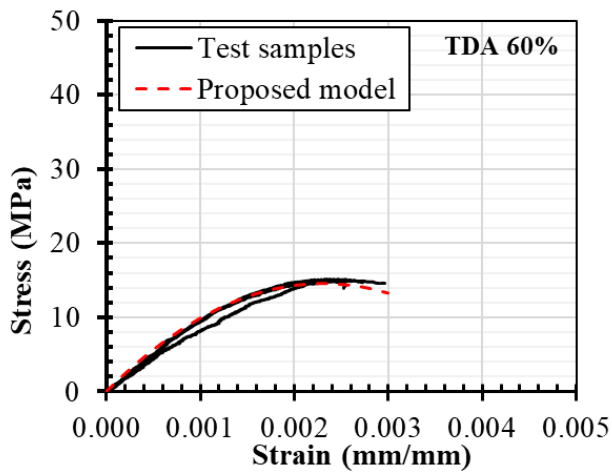
Figure-3. (a) Initial Modulus of Elasticity, and (b) Secant Modulus of Elasticity at Ultimate Stress, for Different Percentages of TDA in Concrete.

Modulus of mixture from Hirsh homogenization was used in the proposed model to simulate the stress strain diagram for concrete with 10%, 20%, 40%, 60%, and 80% coarse aggregate replacement with TDA. The results are shown in Figure-4.

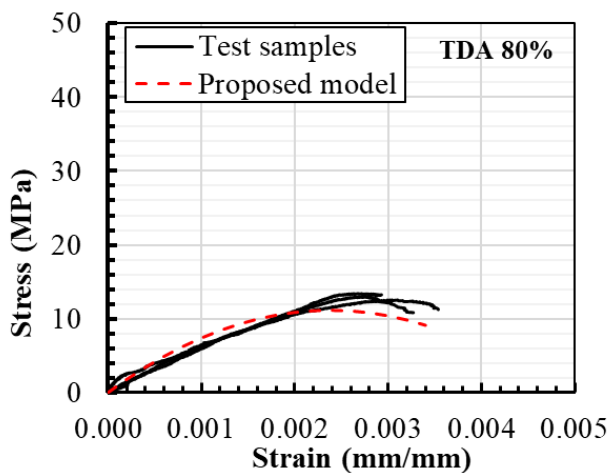




(c)



(d)



(e)

**Figure-4.** Stress-Strain Diagram for Concrete with (a) 10% TDA, (b) 20% TDA, (c) 40% TDA, (d) 60% TDA, and (e) 80% TDA Replacement of Natural Coarse Aggregate, Test Samples Along with Proposed Model.

#### 4. CONCLUSIONS

A simple model was developed to predict Young's moduli and stress-strain curves for PCC mixes with different TDA percentages using homogenization. Test results for PCC mixes without TDA and 100% TDA were used as upper and lower bounds of homogenization. Then the model results were verified using other TDA percentages. The proposed model seems to capture the behaviour of the material fairly well. It slightly underestimates the peak stress for 40% TDA replacement, but with reasonable accuracy. Both ultimate stresses and ultimate strains were well predicted, along with the elastic modulus.

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