



# COMBINED ECONOMIC AND EMISSION DISPATCH USING WHALE OPTIMIZATION ALGORITHM

M. Chandrashekhar<sup>1</sup> and P. K. Dhal<sup>2</sup>

<sup>1</sup>Department of Electrical and Electronics Engineering, Chennai, India

<sup>2</sup>Vel Tech Rangarajan Dr. Sagunthala R&D Institute of Science and Technology, Chennai, India

E-Mail: [pradyumna.dhal@rediffmail.com](mailto:pradyumna.dhal@rediffmail.com)

## ABSTRACT

Power plants give the most to environmental pollution, another important factor nowadays. Power stations must hold carbon credits and follow tight carbon emission restrictions. This is crucial for minimizing global warming and sustaining life. Electric power system planning and operation must meet load demand reliably, cost-effectively, and environmentally. Planners and operators use optimisation tools to attain these goals. In this study, the performance of two new optimisation methods, like the Whale Optimisation Algorithm (WOA), is compared to the performance of two older optimisation methods, like the Moth Flame Optimisation (MFO) and the Ant Lion Optimisation (ALO). When compared to the other two optimisation method, the results from the new optimisation method are better. It is obvious that there are competing goals that must be met. One cannot reasonably expect to achieve both the goal of reducing fuel costs and that of reducing gaseous emissions. In order to aid decision-makers in making the best choices, multi objective optimisation techniques are used to derive trade-off relationships between these incompatible goal functions. In this study, we examine the economic load dispatching issues that arise in the operation of power systems. The objective function of the issue is first analysed as a multi-objective function, with power dispatch and environmental considerations each being addressed as a distinct goal. Both the single- and multi-objective variants are examples of high-dimensional, nonlinear, non-convex constrained optimisation problems. Because of this, employing any optimisation strategy is extremely difficult. Several algorithms, including those that take their cues from nature, have been implemented to help us get as near as possible to optimum solutions tools.

**Keywords:** economic load dispatch (ELD), fuel cost, emission, whale optimization algorithm (WOA), moth flame optimization (MFO), ant lion optimization (ALO).

Manuscript Received 21 November 2023; Revised 11 February 2024; Published 29 February 2024

## 1. INTRODUCTION

In day-to-day life, it has become an important task for any engineer to get a good product out in the market at its minimum cost maintaining all its quality and competitive advantages. It's also a concern for him to maintain the same with unique qualities which provide him good sales points. This is the same case in the power system. In a power system scenario, we need to produce electricity at a minimal cost, maintaining all its quality and keeping all the parameters within its limit. The same thing is done in Economic Load Dispatch (ELD). In ELD generators are made to work in such a way that the total running price of the network is optimum. This is not a simple problem as its not only associated with making minimum fuel cost but also maintaining system parameters like voltage and frequency and making adjustments to the grid changes so that grid balance is always maintained. It's the prime problem in electricity production and distribution. As it's a key problem there are many principles and algorithms already in place for solutions. Some of the algorithms provide better results even today. However, there are also multiple things changing in the system and always gone improvements. There are also new guidelines and rules to adjust the system for the changes in the grid - so get compensated for covering up a problem in the grid and get penalized for making a problem in the grid. So, it's always important to explore

and evaluate better algorithms that can not only provide optimum results but also provide faster convergence.

Economic Dispatch (ED) optimization problem is one of the problems that needs prime attention in a power system. This includes a detailed plan of the output generated from each associated generator in a power system so that the complete cost remains at optimum, meeting all the consumer load requirements and ensuring the desired quality. Conventional electrical power systems minimise operational cost while meeting system limits. Fuel, labour, supplies, and maintenance comprise generator running expenses. Since labour, supplies, and maintenance are set percentages of fuel cost, we consider fuel cost the only variable cost for simplicity. Thermal and nuclear facilities use fuel, however, nuclear plants operate at fixed output levels, while hydro stations use free energy, therefore the operational cost is not important. This study considers solely thermal power plants.

Fossil fuels now power-generating units' rotor shafts. This releases a lot of CO<sub>2</sub>, SO<sub>2</sub>, and NO<sub>x</sub>, polluting the atmosphere. Emission control and environmental requirements for fossil-fired generating units have been prioritised. The 1990 clean air act amendments and growing public awareness of environmental pollution have driven utilities to change their thermal power plant designs and operations to decrease pollution and atmospheric emissions. Carbon credits are also regulated. These scenarios make pollution



finest and gasoline prices equal. Thus, the economic dispatch problem should take into account environmental contamination situations and optimize fuel costs. Carbon dioxide emissions must be prioritized. Thus, weighting functions with a good decision maker or a price penalty function must be used to change the goal function.

The ED problem is generically treated as a detailed optimization problem which targets in optimizing the complete cost for generating power, still maintaining all the system variables within its range. The solution of an ED problem involves dealing with a non-linear and highly convex problem including the impact of grids on the system. There are also constraints introduced by the environmental parameters. Because power plants are said to be a major contributor towards pollution, power-generating companies must adhere to the protocols set on various levels of emissions for components such as CO<sub>2</sub>, NO<sub>x</sub>, and SO<sub>2</sub>. This makes us to treat this problem as a combined economic and emission dispatch problem. So this problem should evaluate the inputs, and variables and provide an output of running the generators at specific levels of load to meet the total demand within the optimal cost. This cost will include all the costs like fuel costs, emission costs, and costs introduced for maintaining the system.

Chowdhury and Rahman [1] studied modern techniques related to economic dispatch and divided the same into 4. These include (i) power flow optimization, (ii) Automatic Generation Control (AGC) related ELD, (iii) dynamic-based dispatch & (iv) ELD with distributed energy resources including Wind and Solar. Farag *et al.* [2] developed extremely effective techniques and algorithms to accomplish the ideal shift in power dispatch linked to contingency states or overload circumstances in power system operation and planning phases for reliability, economy, and environmental conditions. He presented sequences with LP variables and section reduction and simplex algorithms. Sheblé and Brittig [3] adopted different method for further optimizing GA for the solution of ED problems. The algorithm made use of effective payoff patterns of perspective solutions to make improvements. This provided effective convergence and avoided the earlier problems to a great extent. Jabr *et al.* [4] made use of a simplified self-dual LP interior point algorithm that adapted to both (N-1) and (N-2) network security-constrained economic dispatch problem. They also considered an IEEE 24 bus test system to completely evaluate the same by the use of various system constraints. Feature of using interior point LP algorithm it has higher reliability and better convergence. He also made comparison with the predictor-corrector point algorithm for the same system.

Gnanadass *et al.* [5] considered the test system as a combined cycle cogeneration plant with three thermal plant systems. This is then fed to existing EP to solve the economic dispatch problems. This further improved the results both in terms of optimization and convergence. Yalcinoz *et al.* [6] introduced a whole new approach for GA which is based on arithmetic cross over on GA. This provides the GA based algorithm to calculate the real part,

arithmetic cross over mutation to calculate the imaginary. The best result was observed for the system with 20 generators. But this system did not show any improvement on the other buses with respect to the other systems. Attaviryanupap *et al.* [7] introduced combinational algorithm for making ED solutions better. They made use of earlier used and well-known technique of EP and sequential quadratic programming (SQP) to resolve Dynamic Economic Dispatch Problem (DEDP) having non-smooth fuel cost function. The EP is made use for providing correct direction to the optimal global region. SQP is made use in the steady state region to provide an excellent solution. This method of using two different algorithms at transient and steady state parts of system further leads to studies and many had adapted the same and come with better results.

Leung *et al.* [8] added a modified GA process for finding optimum solution to ELD problem. He introduced a new methodology named as 'selection & cross-over' methodology in GA. When comparison is done with previous algorithms, modified GA provided better results in terms of operation cost and convergence. Park *et al.* [9] added non-smoothing functions to PSO with the ELD problem which showed much better optimization. The major difference in this scenario is how the equality and inequality constraints were made in the adjustments with of each other's search point. For accelerating convergence, a dynamic search space reduction is used. Use of these methodologies had made this algorithm to show excellent results. Pothiya *et al.* [10] suggested multiple tabu search method for economic dispatch problem. They introduced local and global searches independently to improve tabu search. It exhibited greater convergence and optimisation than the usual GA approach. NNs were introduced by Mekhamer *et al.* [11]. He solved ELD using two methods. HNN and QP describe this. Real-time economic dispatch (dynamic ELD) is considered. This solution solves the economic dispatch's static and dynamic parts with an upgraded HNN and QP. They supplied a worldwide minimum.

Baskar and Mohan [12] used a hybrid of classical and improved PSO having line constraints to perform the economic dispatch problem. After the application of constriction factor and a new rule of velocity update, it showed better results for large to very large systems. Secui *et al.* [13] used a swarm intelligence approach for the solution of the economic dispatch problem. It applied the use of time-varying acceleration coefficients for improving global search in the initial phases of the optimization process to get better convergence. It showed better quality of solutions and lower computation time for the systems with larger number of generators, but for the systems with lesser generators this showed no improvements with the algorithms described above. Ahmad *et al.* [14] had given a breakthrough in economic dispatch problems by using PSO for solving the economic dispatch problem. For larger systems it has shown excellent results but for the systems with lesser number of buses no effect was seen. Pandian *et al.* [15] used a combination algorithm to make the results even better. He



used Evolutionary Programming (EP) and Efficient Particle Swarm Optimization algorithms (EPSO) and also considered transmission losses to solve the ED problem with two cost functions - (i) smooth & (ii) nonsmooth. The method used provided solutions far better than EPSO and had more computational time. However, it was found that it showed superior results in comparison with NN, EP, and algorithms that used a combination of NN and EP. Younes and Benhamida [16] came up with another approach and used a combination of GA and PSO to get better results for economic dispatch problem. This provided high stability and better optimization. The computation time was also good but can be further improved by using better ways of coding. Labbi and ATTOUS [17] used a similar approach of combining GA and pattern search method for solution. The approach provided solution for the problem with pattern search which did not had the provision for knowing the initial conditions.

Panigrahi *et al.* [18] studied in detail of simulated annealing technique for the solution of load dispatch. This consisted valve point effects too and tested the same using standard test buses to come up with good solutions. Swarup and Simi [19] came up with HNN for solving economic dispatch problems. He further presented the possibility of online learning for neural networks. This provided good results but still improvements were needed. dos Santos Coelho and Mariani [20] used an improved differential evolutionary algorithm for the solution of ED problem. He included non-linear features of the generators especially the ramp rate non-linearities of the system. It had two test systems with 6 generators and 15 generators respectively. This provided a real-time answer for economic dispatch problems. Papageorgiou and Fraga [21] modelled ED system with mixed integer quadratic programming method. This provided quick convergence properties but failed to produce a global minimum in terms of optimization. Balamurugan and Subramanian [22] indicated a differential evolution-based algorithm for the solution including valve-point effects. Pandi *et al.* [23] used a hybrid harmony search algorithm along with the particle swarm methodology to meet the real-time requirements of the system. This provided better methods of generator scheduling and provided near real-time adjustments to the system.

Reddy and Vaisakh [24] developed a triple hybrid method to solve economic dispatch problem using bacterial foraging optimisation algorithm, PSO, and differential evolution. Mohammadi-Ivatloo *et al.* [25] developed the imperialist competitive algorithm for economic dispatch problem. It worked well but had limitations. Vaisakh *et al.* [26] solved economic dispatch via differential evolution and bacteria foraging optimisation. He detailed valve point effects. Bijami *et al.* [27] proposed using imperialist competitive algorithm for economic dispatch problem. He considered system non-convexity well. Good approach, near global minima. Ding *et al.* [28] tried out the Local-best variant method to the existing PSO technique for solving economic dispatch problem. He had chosen the test system from very small to very big generators containing 5, 10 and 110 generator

units in the power system. This had shown that PSO method can be adopted from small to very large systems. Dubey *et al.* [29] checked various methods of bio-inspired algorithms for economic dispatch and compared their performances. In this PSO provided excellent results. Lu *et al.* [30] brought bee colony optimization algorithm with search replaced with chaotic local search method to provide better results. He also considered valve point effects of the system. Mandal *et al.* [31] came up with an entirely new nature-based technique of krill herd algorithm for the economic dispatch problems. This brought the results near to optimum and convergence characteristics were also better. Secui [32] made use of ABC algorithm for solving the emission dispatch problem. It also considered valve point effects. The conditions are verified with power systems having 6 units, 13 units, 40 units and 52 units.

Wang *et al.* [33] used hardware-based implementation of neural networks to come up with an effective system. They used a scalar factor function and represented two conflicting objectives using a weighing factor. He continuously adjusted weighting coefficients to derive the optimum solution for the system. The test system chosen was very small and contained only 3 generators. He also did not consider the transmission losses of the system Huang and Huang [34], considered Real time-oriented neural networks to avoid issues with the conventional NN. They used a hybridization of adductive reasoning and the decision-making approach of the neurons effectively to bring good results. He demonstrated the superiority of this approach with respect to conventional neural networks using a system having fewer generators. This provided near-optimum results, was good for real-time changes and provided good convergence factor. The continuous adjustment of the system in real-time was one of the biggest advantages. Zeynelgil *et al.* [35] applied a bi-objective function on conventional neural networks to come up with a solution for the system. It targeted on reduction of emissions of oxides of Nitrogen and also considered transmission losses of the system. It provided effective learning methodologies and made it suitable for real-time changes too.

Chen and Huang [36] used novel neural network theories to solve economic and emission dispatches. It solved problems through a goal attainment approach and an adaptive polynomial network. They added polynomial networks for large-scale systems. It analysed transmission losses and other characteristics to match real-time systems. Abido [37] completed another investigation comparing three evolution-based techniques. Non-dominated sorted GA, niched Pareto GA, and strength Pareto evolutionary algorithm. Pareto GA outperformed. Venkatesh *et al.* [38] solved the economic and emission dispatch problem using a hybrid EP-GA algorithm. Newton-Raphson was used for weighing factor solutions. It performed well in a large number of test systems. Jeyakumar *et al.* [39] employed EP method to compute emission and economic dispatches independently using single objective functions. They used a weighted factor to blend them for good outcomes.



Results from several IEEE systems were evaluated. Venkatesh and Lee [40] used the earlier method but done it focused on the pollutants. This study gave a detailed picture on how the pollutants can be reduced drastically by compromising little on the other costs. Purkayastha and Sinha [41] focused at non-dominated solutions in using Non-dominated Sorting GA II. The results were effective and tested using multiple generators available. The convergence characteristics were also good. Dixit *et al.* [42] allied new nature inspired algorithm known as ABC for solving economic and emission dispatch problem. This shown good results even in comparison with PSO. It provided higher quality of solution, excellent convergence characteristics and optimum cost. However, it did not give much focus on the emission part. Güvenç *et al.* [43] proposed an effective Gravitational Search Algorithm (GSA) for finding solution to combined emission and economic dispatch problems. It treated the problem as a multi-objective function itself and used a price penalty factor for emissions to make it easy. The cases of valve point effects, transmission losses and its combinations were considered in great deal. To analyze the effectiveness this algorithm was compared against the other proven algorithms. It provided excellent convergence and good results in terms of cost.

Generally, the heuristic methods like GA, NN, SA, PSO, Ant Colony, and ABC techniques and their derivatives have shown good improvement in the analyzing various aspects related to economic and emission dispatch problem and combination of both and came up with solutions that are good and suitable. However, the power systems are undergoing modernizations day by day and are subjected to the grid and rules driving the same. This makes the need for continuous improvement in getting quality solutions, higher control on emissions, better convergence, lower losses, treatment of all the constraints, and reaching global optimum solutions into combined emission and economic dispatch. So as per the literature review, the focus of the paper will be on the new algorithms and how the same can be adopted effectively in solving to combined emission and economic dispatch problems. This will need to get analyzed in comparison with the proven PSO algorithms as per the literature reviews. The parameters like valve point effect, transmission losses etc will also be treated for to combined emission and economic dispatch solutions which are also essential as per the literature review.

Here is the outline for the remainder of the paper. The formulation of the issue is covered in Section 2, the solution approach for the Combined Economic and Emission Dispatch Using Whale Optimization Algorithm is described in Section 3, and the obtained findings and their discussion are presented in Section 4. The conclusion is presented in Section 5.

## 2. PROBLEM FORMULATION

### 2.1 Economic Load Dispatch without Losses

The cost function is treated as the objective function here, which presents optimized cost of system

including all the parameters and objectives. As indicated earlier the operation and maintenance costs are also loaded to the same. As competitions are too high, it becomes first priority of any utility to generate power and deliver the same to customers at its optimum cost. There will be multiple generators available in power generating utility to achieve this and each generator will have its operating range. All the operations to be maintained in the range of loads permitted for the utility. All the generations need to be adjusted in a way that the total cost of running all the generators needs to be minimal. If we represent the same as a quadratic equation, the total cost of the fuel for running 'N' generators in parallel can be treated as the following.

$$FC_T = \sum_{i=1}^N FC_i(P_i) = \sum_{i=1}^N a_i + b_i P_i + c_i P_i^2 \quad (1)$$

In any economic dispatch problem we need to consider dual types of system constraints - Constraints related to Equality and inequality. Equality Constraints the equation indicating the power balance of a system is as given below:

$$\sum_{i=1}^N P_i = P_D \quad (2)$$

When we add loss-factor, contributing to various losses like the distribution losses, the equation becomes

$$\sum_{i=1}^N P_i = P_D + P_L \quad (3)$$

The inequality constraints for the power system can therefore be represented as:

$$P_{i \min} \leq P_i \leq P_{i \max} \quad (4)$$

$$Q_{i \min} \leq Q_i \leq Q_{i \max} \quad (5)$$

### 2.2 Economic Load Dispatch without Losses and with Constraints

Let's consider a maximum of N generators used to meet a demand in power represented by PD.  $P_i$  is considered as the power generated from ith generator. The concept of economic dispatch is that  $P_i$  need to be found out with criteria that the total cost of the system is its global minimum. This makes the optimization problem to be expressed as:

$$\text{Min } F(P_i) = \sum_{i=1}^N F_i(P_i) \quad (6)$$

And this needs to meet the following conditions

(a) Energy balance equation

$$\sum N_i = 1F_i(P_i) = PD \quad (7)$$

(b) Power capacity constraints

$$P_{i \min} \leq P_i \leq P_{i \max} \quad (8)$$

Parameter details are



$P_i$  - Active power of generator  $i$  PD -Total demand of power for the system N -Number of generators of the system  $P_{\text{mini}}$ -Minimum power range of the  $i$ th generator  $P_{\text{maxi}}$  -Maximum power range of the  $i$ th generator Considering  $F(P_i)$  as the real-time cost of the fuel for  $i$ th generator, the below quadratic equation represents the running fuel cost.

$$F(P_i) = a_i P_i^2 + b_i P_i + c_i \quad \text{rs/h} \quad (9)$$

### 2.3 Economic Load Dispatch with Losses

Transmission losses are a crucial part of getting power to far-off substations. Because of this, gearbox loss is an essential factor to take into account while calculating economic dispatch. However, the complexity of the economic dispatch problem rises as a result of this. B-Coefficients are taken into account while calculating losses. Below is the equation for the sum of losses.

$$\text{Mini } F(P_i) = a_i P_i^2 + b_i P_i + c_i \quad \text{rs/h} \quad (10)$$

The limitations and balance equation are applied further to this. The Energy Conservation Equation

$$\sum_{i=1}^N P_i = P_D + P_L \quad (11)$$

As it is impossible to have a power system without any losses, the transmission loss need to be considered for solving any economic dispatch problem. Taking into account transmission losses, the power network equation is as:

$$P_L = \sum_{i=1}^N P_i B_{ij} + \sum_{i=1}^N B_{0i} P_i + B_{00} \quad (12)$$

where,  $P_i$  - Load of  $i$ th generator in MW unit.  $P_j$  - Load of  $j$  th generator in MWunit  $B_{ij}$  - Coefficients indicating transmission losses between  $i$ th and  $j$ th generating unit in MW.  $P_L$ - loss in power represented in MW

### 2.4 Economic Load Dispatch with Valve Point Loading

Adding the valve point effect the objective function becomes

$$FC_T = \sum_{i=1}^N a_i + b_i P_i + c_i P_i^2 + d_i \sin(e_i (p_{i \text{ min}} - P_i)) \quad (13)$$

The  $i$ th generator unit's fuel price coefficients are  $a_i$ ,  $b_i$ ,  $c_i$ ,  $d_i$ , and  $e_i$ .  $P_i$  is the  $i$ th generator's output in megawatts.  $N_i$  represents the total number of generators.  $P_{\text{imin}}$  represents the minimal range of the  $i$ -th generator in a MW unit.  $F_C$  represents the eighth generator fuel cost function  $F_C$  represents the total fuel expenditure, in \$ per hour.

### 2.5 Emission Dispatch Problem

The power generator can be modeled with a quadratic relation that relates the pollutants emitted to the atmosphere and total generated power from the generators. The details of the same is depicted below.

$$E_i = \alpha_i + \beta_i P_i + \gamma_i P_i^2 \quad (14)$$

In this case, the coefficients of emissions are  $\alpha_i$ ,  $+\beta_i$ ,  $\gamma_i$  of generator  $P_i$  indicates the power generated from the generator unit, and is represented in MW  $E_i$  indicates the fuel cost function of the generator.

Pollutant output in a thermal power station that burns fossil fuels is proportional to the total output of the facility's generators. To reduce the complexity of the issue, we may express the overall system emissions as the sum of a quadratic function plus a term reflecting the active power generated by the units. The emission dispatch function may be thought of as one that reduces power grid pollution to a minimum. When the system's coefficients are taken into account, the whole emission function may be written as:

$$E_T = \sum_{i=1}^N E_i(P_i) \sum_{i=1}^N \alpha_i + \beta_i P_i + \gamma_i P_i^2 \quad (15)$$

The polluting emissions from the power plant are now being factored in. The resulting combined emission function may be expressed as:

$$E_T = \sum_{i=1}^N \alpha_i + \beta_i P_i + \gamma_i P_i^2 + \varepsilon_i \exp(C_i P_i) \quad (16)$$

As coefficients of the  $i$ th generating unit, the values  $i$ ,  $i$ ,  $ii$ , and  $i$  are interpreted as follows: Emission function of a unit, where  $P_i$  is the power output from the  $i$ th generator in megawatts and  $N$  is the number of generators. The system's total emissions are denoted in tones per hour (ET).

### 2.6 Combined Economic and Emission Dispatch

As said, economic dispatch and emission dispatch are different issues. Combining an emission limitation with an economic load dispatch problem allows emission dispatch. A price penalty factor can combine the two aims in this study.

$$h = \frac{\frac{F_T(P_i^{\text{max}})}{P_i^{\text{max}}}}{\frac{E_T(P_i^{\text{max}})}{P_i^{\text{max}}}} \quad (17)$$

Where  $h$  is the cost-of-living adjustment The most expensive fuel unit is the  $i$ . The maximum measurement of pollution output is  $j$ . The following equation can be thought of as a representation of the combined goal function of economic and emission dispatches:

$$\Phi_T = w_{\text{eco}} F_T + W_{\text{emi}} h E_T \quad (18)$$

where  $T$  is a weighted objective function that takes into account both  $w_{\text{eco}}$  and  $w_{\text{emi}}$ . There are a number of options for supplying the two weighting elements. When  $w_{\text{eco}} = 1.0$  and  $w_{\text{emi}} = 0.0$ , the classical economic dispatch problem is obtained; when  $w_{\text{eco}} = 0.0$  and  $w_{\text{emi}} = 1.0$ , the pure emission dispatch problem is obtained. By setting

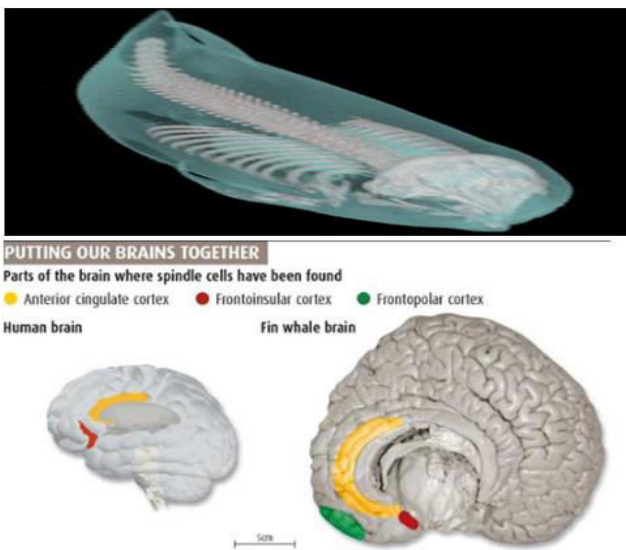


$w_{eco} = 0.5$  and  $w_{emi} = 0.5$ , we derive the economic and emission dispatch problem jointly.

**3. METHODOLOGY**

**3.1 Whale Optimization Algorithm (WOA)**

Whales are the biggest animal in the ocean and display peculiar behaviours. A fully grown up can be 30 meters in length and weigh up to 180 tons. Most of the species of whales are predators and they never take breath as they must take breath by coming to the ocean surface regularly for the same. They are considered as one of the intelligent animals who had survived on earth for so long and are also treated as animals with emotions. Mirjalili and Lewis [2016] had made extensive studies on whales and concluded that whales have common cells in some of the parts of their brain which are exactly like that of human beings. Because these cells are responsible for judgement, social behaviour and emotions they can be like humans in these areas. The presence of spindle cells makes them different from other creatures and adult whales have the same two times as compared to an adult human. This make them act much smarter even compared to other animals. Detailed studies conducted in this area proves that whales can also think, learn, judge, communicate. These features make them even as emotional as a human being, with a good level of smartness. Its seen and proven that most of the killer whales can communicate with its own unique dialect as well. Figure-1 shows the Comparison of Whale and Human Brain. Bubble Net Feeding by Humpback Whales shown in Figure-2.



**Figure-1.** Comparison of Whale and Human Brain.



**Figure-2.** Bubble Net Feeding by Humpback Whales.

**3.2 Mathematic Model WOA**

First let us consider the ability of Humpback whales to recognize the positions of fishes at surface of the sea and surround them with bubbles. The algorithm now assumes that the present best solution is at the fishes located on the surface of the sea. Now the iterations are done and at each iteration the best position gets updated based on various scenarios. This pattern is represented by the following equations of optimization.

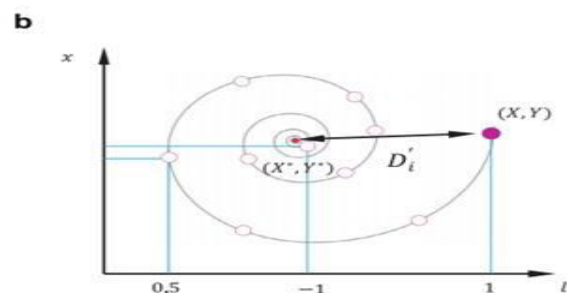
$$D = CX * (t) - X(t) \tag{19}$$

$$X(t + 1) = X(t) - AD \tag{20}$$

't' is treated as the present iteration, Coefficients for choosing between various scenarios - 'A' and 'C' represented in vector form X\*is the best solution vector which is there in the system so far X is treated as the present position vector X\*will get updated if X is better than previous X\*

**Shrinking encircle mechanism of feeding:** This pattern of hunting the prey by circling it in groups and shirking its circle, is obtained by verifying from two to zero. is a vector that contains the value between -a,a and chosen in random. When these flexibilities are added, the mathematical model itself gets multiple levels. The same is indicated in Figure-3 and it shows all the possible areas moving from (X,Y) towards (X\*,Y\*) in a two-dimensional area.

The entire problem can be treated under a three dimensional area too.



**Figure-3.** Spiral updating of bubble net feeding.

**Spiral updating position:** The bubble net feeding scenarios are depicted in Figure-6.3(b). In this



method the distance between the targeted prey and present location need to be calculated. The present location is treated as (X,Y) and target prey is at (X\*,Y\*). After this it need to be traversed through the spiral path to reach the prey. The details of the same are depicted in the equations given below.

$$X(t + 1) = D \cdot e^{bl} \cos(2\pi l) + X * (t) \tag{21}$$

b is treated as the constant for maintaining the spiral path. l is treated as a vector having random number between -1 and 1.

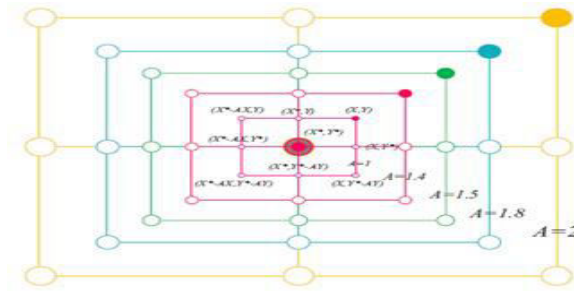
**Searching for prey (exploration phase):**

We can use the approach of the variation of the vector A for searching the fishes which is treated under an exploitation phase. Humpback whales do a random search for the prey by moving itself in the water. This updates both the position of the whale as well as the fishes to be caught. For making this into the model, we use 'A' vector having values between -1 to 1 in random. The unique feature of this phase is that, search agent positions get updated in the exploration phase according to a search agent that is randomly chosen. This is another very important phase that are indicated by the equations as given below.

$$\vec{D} = \vec{C} X_{rand} - \vec{X} \tag{22}$$

$$\vec{X}(t + 1) = X_{rand} - \vec{A}\vec{D} \tag{23}$$

X<sub>rand</sub> is treated as a position vector chosen in random from the present population. The possible positions of the various entities are as indicated below.



**Figure-4.** Exploration Phase as Per Whale Optimization.

Figure-4 shows Exploration Phase as Per Whale Optimization. All the above descriptions of exploitation and exploration with multiple adjustment possibility make whale optimization as an algorithm having flexibility for achieving global minimum. Because of these flexibilities its treated as right algorithm for economic dispatch. Even if we include requirements of emission dispatch and with n number of system constraints, WOA provides excellent results in terms of both convergence and optimizations. Flow-Chart of WOA is shown in Figure-5.

**3.3 PSEUDO-CODE OF WOA**

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Population of whales need to be initialized Xi (i = 1, 2, ..., n)
Derive the Fitness function of each search agent
Choose X* as best search agent, t = 1
while (t < highest number of iterations)
for every agent of search
Fill the present values of a, A, C, l, and p
if a (p<0.5)
if b (|A| < 1)
Position of search agent to be updated by Equation (19)
else if b (|A| ≥ 1)
Treat a random search agent ( )
Position of search agent to be updated by Equation (23)
end if b
else if a (p≥ 0.5)
Position of search agent to be updated by Equation (21)
end if a
end for
Check there are any overflows from the search space and amend if there are
Search agent fitness to be derived
Update X* with better solution if the same is available
t=t+1
end while
return X*
    
```

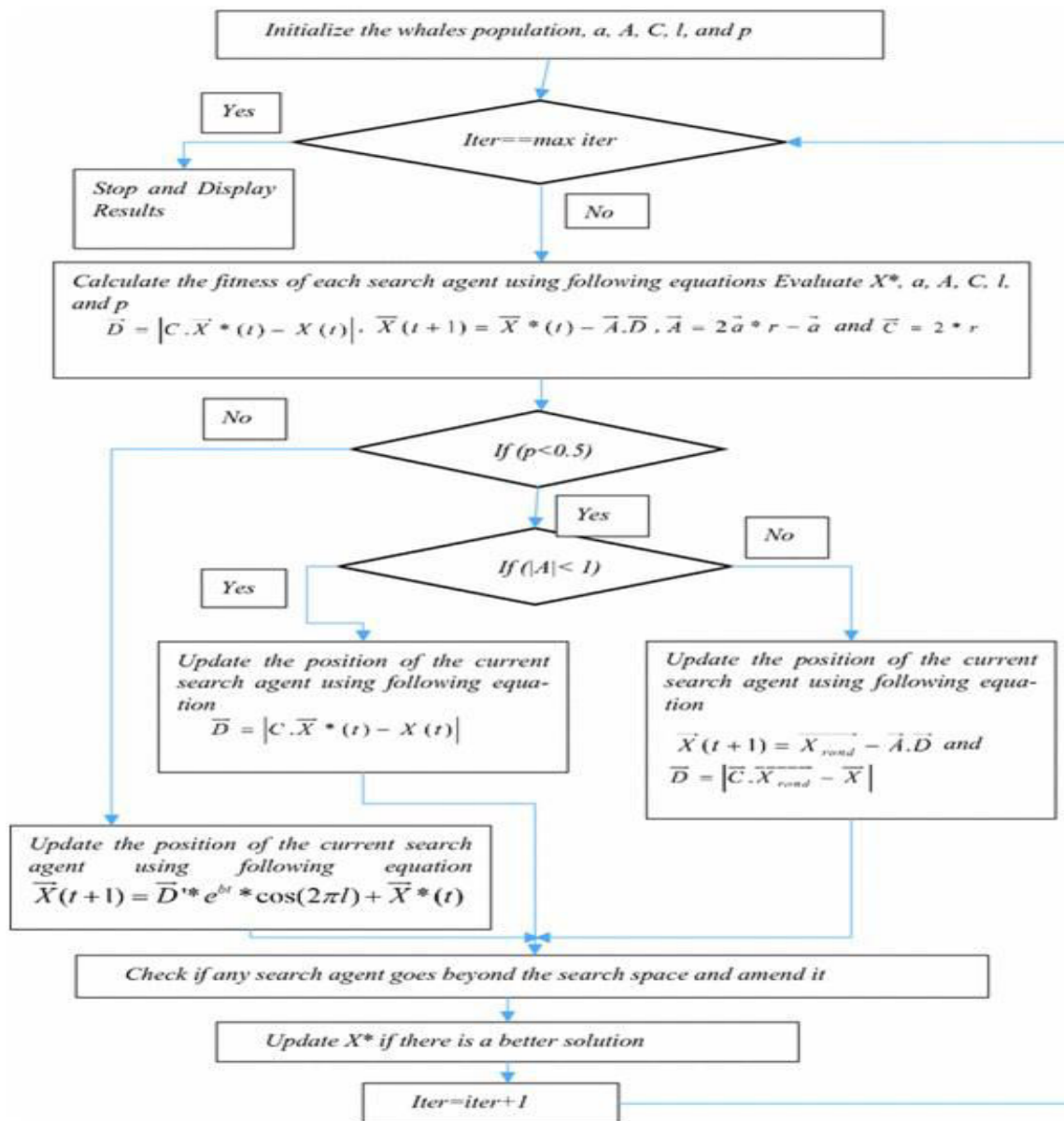


Figure-5. Flow-Chart of WOA.

### 3.4 Combined Economic and Emission Dispatch Problem with WOA

- Initialization of parameters: Whale population size:  $N$ , with random generation of the initial population; maximum iterations, valve point loading effect coefficients.
- Details regarding the different data's are inputted Emission and fuel cost coefficients for different generators.
- The fitness of each agent is derived to obtain initial best search.
- Choose a best search agent.
- When  $p < 0.5$  and  $|A_j| < 1$ ; the current position of individual whales are updated with pattern represented by equation (23); and if  $|A_j| \geq 1$ , a random individual whale  $X_r$  and is selected, and the current positions of individual whales are updated with equations (19)
- When  $p < 0.5$ , the current position of individual whales are updated with pattern represented by equation (21)
- The updated locations of all individual whale are checked to determine if their updated locations are beyond the search space, and corrected accordingly.





- h) The number of iterations is checked. The algorithm is stopped if the maximum number of iterations has been reached else continue
- i) The whale reaching the targeted prey and consuming the same will be treated as the final optimum position.

**4. SIMULATION RESULTS**

In this we are focusing on the ELD and combined economic and emission dispatch for various test systems including different buses. Standard coefficients would be provided for each system under test. The system considered would be provided with the full load that need to be generated. The range of power that can be generated with each generator of the system is also provided. These values were fed as input to optimization models of Antlion (ALO), Moth (MFO) and whale (WOA). The same case was also analysed with already existing PSO algorithm. The results of all the algorithm provide total cost of the system (fuel cost, emission cost and combined total cost) and the output (load) needed to be generated from each generator system.

**4.1 CASE 1: Six Generator Test System**

In this case, we are targeting on a six-generator bus system. Here the generators are kept at the bus

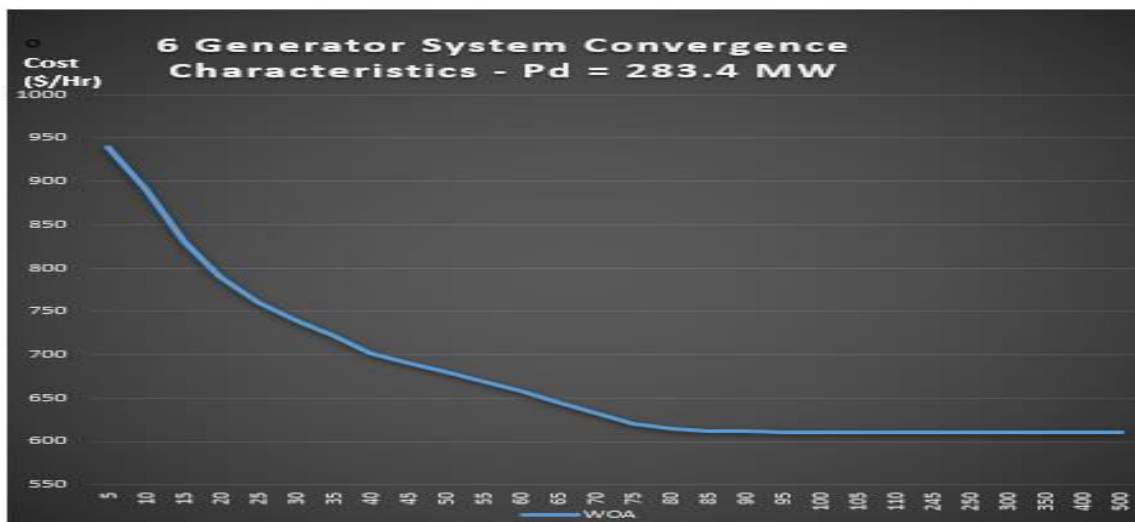
locations - 1st bus, 2nd bus, 5th bus, 8th bus, 11th bus and 13th bus to complete an IEEE-30bus-system.

**Six generator test system using WOA**

To get the results, the system with WOA was run with the standard factors and total demand of 2.83 pu. The results, which included the cost of fuel and the cost of pollution, were well optimised and showed good agreement. These are shown in the following Table-1 and Figure-6.

**Table-1.** Optimized cost for 6 Gen System Using WOA.

Generators	Individual Loads
P1	0.28753
P2	0.69927
P3	0.30624
P4	0.645
P5	0.86146
P6	0.54758
Fuel cost (\$ /hr)	611.034
Emission (t/hr)	0.1994



**Figure-6.** Convergence Aspects of the 6 Generation Network Using WOA.

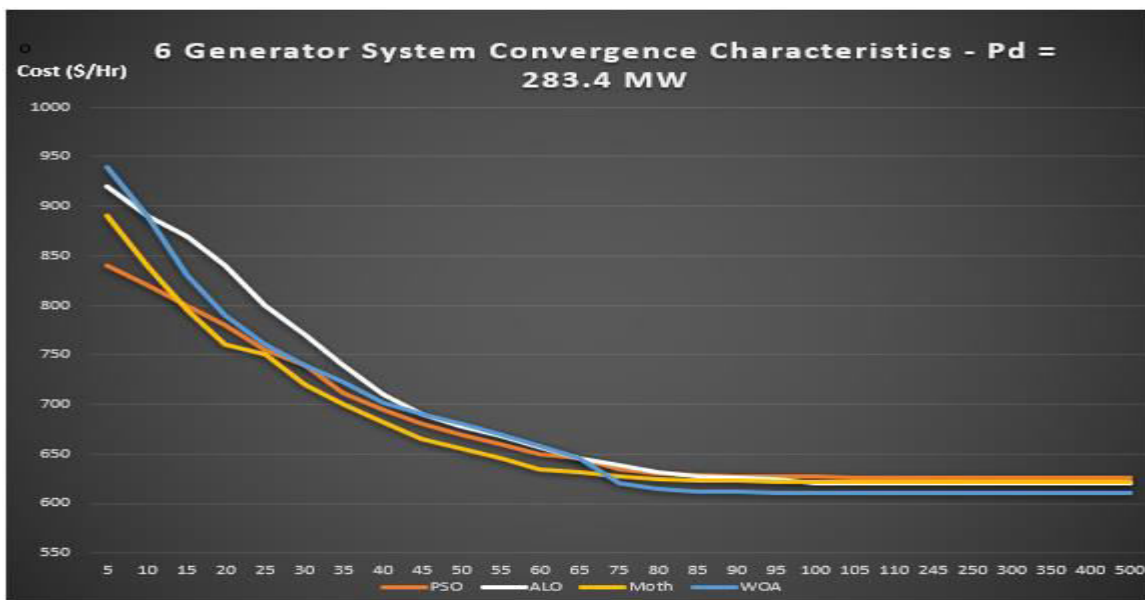
To get the findings, the system was run with the conventional coefficients and a total demand of 2.83 pu. The findings, which included fuel cost and pollution cost, were compared using all the methods. The outcomes are

contrasted with the tested PSO, MFO, and ALO algorithms. These are shown in the Table-2 and Figure-7 that are provided below.



**Table-2.** Six generator test system comparison with all algorithms.

	PSO	MFO	ALO	WOA
P1	0.32023	0.25006	0.76393	0.28753
P2	0.46546	0.46355	0.61654	0.69927
P3	0.5051	0.88804	0.39024	0.30624
P4	0.68179	0.62652	0.73887	0.645
P5	0.57512	0.24008	0.52557	0.86146
P6	0.28195	0.50145	0.76257	0.54758
Cost(\$ /hr)	639.248	621.995	620.565	611.034
Emission(t/hr)	0.21105	0.21095	0.21067	0.1994



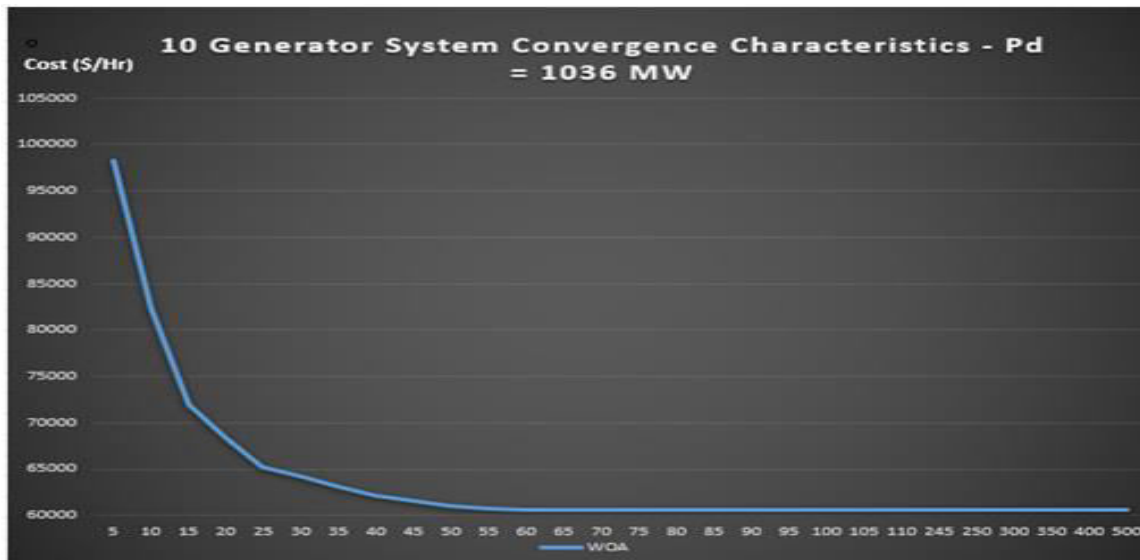
**Figure-7.** Convergence Aspects of the 6 Generation Network Using PSO, ALO, MFO & WOA.

**4.2 CASE 2: Ten Generator Test System**

In this instance research, we will focus on a bus system with 10 generators. Ten generators were stored in this location on an IEEE-39 bus system. It is assumed that the basis for generating is 100 MVA. The entire demand is estimated to be 10.36p.u. The findings, which included the cost of gasoline and the cost of emissions, offered good optimisation and also had great convergence characteristics. These are shown in the Table-3 and Figure-8 that are provided below.

**Table-3.** Optimized cost for 10 Gen Network Using WOA.

Generators	Individual Loads
P1	150
P2	135
P3	75.4778
P4	115.64
P5	122.4498
P6	93.1344
P7	121.1145
P8	64.6874
P9	52.567
P10	43.1234
Cost(\$ /hr)	61422.87
Emission (t/hr)	0.3980115



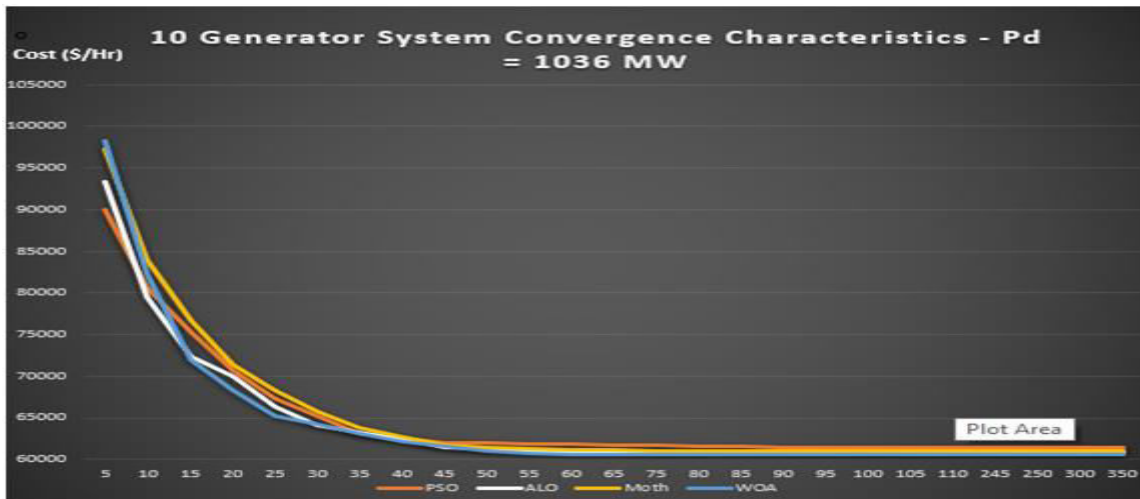
**Figure-8.** Convergence Aspects of the 10 Generation Network Using WOA.

To get the findings, the system was run with the standard coefficients and a total demand of 10.36 pu. The findings, which included fuel cost and pollution cost, were compared using all the methods. The outcomes are

contrasted with the tested PSO, MFO, and ALO algorithms. These are shown in the Table-4 and Figure-9 that are provided below.

**Table-4.** Ten generator test system comparison with all algorithms.

	PSO	MFO	ALO	WOA
P1	150	150	150	150
P2	135	135	135	135
P3	73	74.4377	79.402	75.4778
P4	107.505	61.505	60.044	115.64
P5	120.999	125.989	122.4517	122.4498
P6	135.145	35.145	93.114	93.1344
P7	55.009	55.009	120.115	121.1145
P8	43.718	35.718	53.2375	64.6874
P9	125.1355	80	80	52.567
P10	121.7054	54.234	43.2678	43.1234
Cost(\$ /hr)	61845.72	61660.84	61590.73	61422.87
Emission (t/hr)	0.387578	0.393512	0.39428	0.3980115



**Figure-9.** Convergence Aspects of the 10 Generation Network Using PSO, ALO, MFO and WOA.

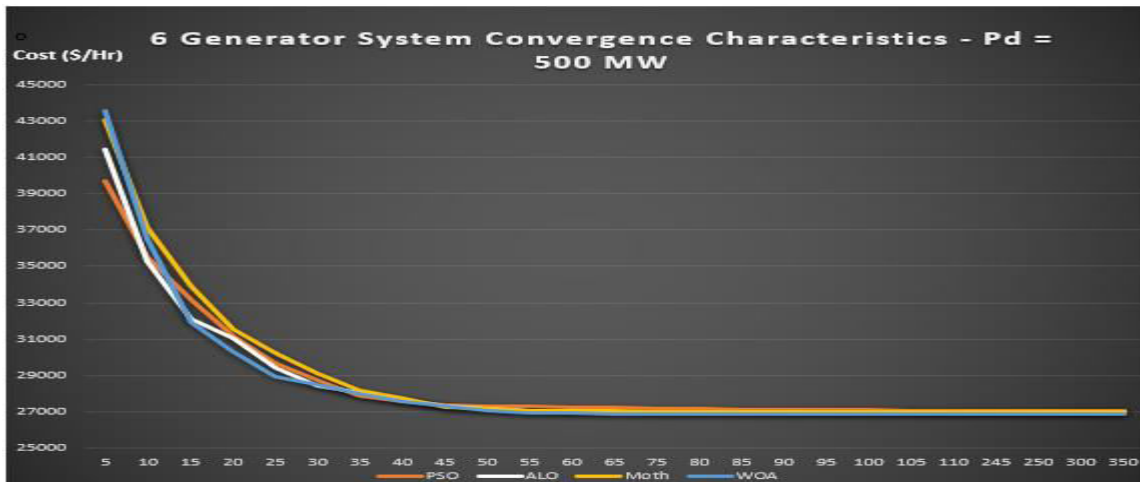
**4.3 CASE 3: Six Generator Test System with Total Power Demand 500MW**

In this case study, a six-generator unit in standard IEEE-30bus-system is tested. The total power demand is taken as 500MW and represented in per unit. The base is considered as 100 MVA. The algorithms are executed and corresponding graphs are plotted. In this case, 6 generator test system was being used with mathematical modeling

done as per ALO algorithm, Moth Flame Optimization and WOA. The system was executed with the standard coefficients and total demand as per unit, to come up with the results. The findings, which included fuel cost and pollution cost, were compared using all the methods. The outcomes are contrasted with the tested PSO algorithm. These are highlighted in the following Table-5 and Figure-10.

**Table-5.** Six generator test system comparison with all algorithms (PD=500MW).

	PSO	MFO	ALO	WOA
P1	1.1999	1.19856	1.19567	1.19786
P2	1.31723	1.31228	0.31056	0.32456
P3	1.93452	1.93456	1.93007	1.9319
P4	0.90064	0.91003	1.90023	1.9645
P5	1.43648	1.43678	1.42167	1.43678
P6	1.38542	1.34529	0.38235	0.38564
Cost(\$ /hr)	27,097.50	27,002.50	26,998.10	26,876.10
Emission(t/hr)	0.26189	0.261865	0.261805	0.2617



**Figure-10.** Convergence Aspects of the 6 Generation Network Using PSO, ALO, MFO and WOA with Total Power Demand 500MW.

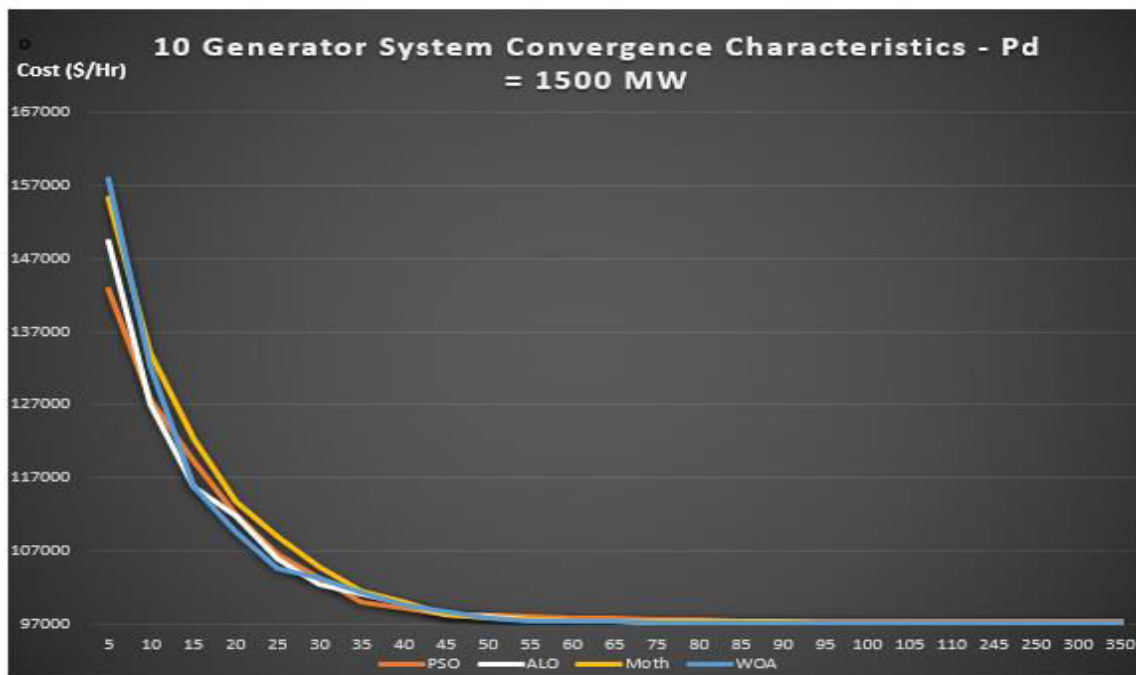
**4.4 CASE 4: Ten Generator Test System with Total Power Demand 1500MW**

In this case, 10 generator test system was being used with mathematical modeling done as per ALO algorithm, MFO Algorithm and WOA. The system was executed with the standard coefficients and total demand

as 1500MW to come up with the results. The findings, which included fuel cost and pollution cost, were compared using all the methods. The outcomes are contrasted with the tested PSO algorithm. These are highlighted in the following table 6 and Figure-11.

**Table-6.** Ten Generator Test System Comparison with all Algorithms with Demand Power 1500MW.

	PSO	MFO	ALO	WOA
P1	167.32	134.67	189.45	103.002
P2	178.9	189.9	199.67	234.88
P3	160	180	155.67	166.89
P4	34.67	67.89	45.89	50
P5	121.11	120	123.45	121.32
P6	145.9	120.76	123.56	123.88
P7	201	148.9	234.89	234.89
P8	123.45	245.63	289.01	256.09
P9	156.89	48.002	46.78	45.35
P10	256.78	324.89	254.09	205.39
Cost(\$ /hr)	97397.3	97348.4	97371.5	97328.7
Emission (t/hr)	0.80722	0.80722	0.81863	0.8072



**Figure-11.** Convergence Aspects of the 10 Generation Network Using PSO, MFO, ALO and WOA with Demand Power 1500MW.

## 5. CONCLUSIONS

After going through details of algorithms and its characteristics we found 3 nature based meta-heuristic algorithms meeting this purpose - MFO, ALO and WOA. These algorithms were mathematically modelled and applied against ED, emission dispatch and combined economic & emission dispatch use cases. Developed algorithms as per the mathematical model and implemented the same. The test systems are considered treating different IEEE generator buses as standard with multiple generators and standard coefficients. The inputs are given to these generator systems, and the outputs obtained were used to analyze against the results of proven PSO systems. The convergence characteristics were also plotted relating the result stability and iterations. All the three algorithms chosen provided excellent results in comparison with PSO and shown very good convergence characteristics making it suitable for the use with any ELD or combined economic emission dispatch problems. Among these WOA surpassed others in getting best results. WOA also shown high flexibility for considering the same to any complex systems and can treat any parameters well.

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