MATHEMATICAL MODELING OF THE CRITICAL BUCKLING LOAD FOR RC COLUMNS SUBJECTED TO ECCENTRIC COMPRESSION

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ABSTRACT

Given the column's importance in building, solving the reinforced concrete column stability requires nonlinear analysis, which makes the resolution very tedious. To simplify the analysis of slender columns, Eurocode 2 offers two simplified methods. However, these two simplified methods lead to very different results. Thus, to have accurate results, engineers must use the general method to analyze the nonlinear behavior of RC slender columns. Given the difficulty of resolving this problem, many designers use abacuses or software. From a research point of view, numerical methods are often used to predict the buckling phenomenon, while mathematical modeling has received no attention. In this article, we establish a new model to determine the stability domain for reinforced concrete rectangular columns subjected to eccentric compression by Eurocode 2. The model presented in this paper applies to columns with rectangular cross-sections and symmetric reinforcement distribution. It is quick and easy to use and will enable engineers to verify the stability of compressed columns without the use of computers. Moreover, it can also help them economically design columns as it evaluates the critical buckling load.

Keywords: buckling, non axial loading, reinforced concrete, slender columns, mathematical modeling.

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1. INTRODUCTION

In concrete buildings, the structure often consists of beams and columns. The reinforced concrete columns are commonly subjected to bending moments due to the eccentricity of the supported beams and the geometric imperfections in addition to the lateral loads such as those due to wind or earthquake.

Due to the importance of compression in columns for the stability of the structure, the design codes require that second-order effects must be considered [1-3]. Furthermore, it is necessary to take into account the nonlinearity of materials and the nonlinearity of the geometric characteristics. Thus, it is necessary to perform a complete nonlinear analysis, which requires numerical methods and computer programs.

Given the difficulty of resolving this problem and its dependence on several parameters, many authors have used numerical methods such as the finite element analysis or the finite difference method [5-8]. In contrast, others [9,10] looked at experimental studies on the behavior of reinforced concrete columns under the influence of axial loads, whether solid or hollow. However, mathematical modeling has received no attention.

Since solving the buckling problem is complicated and requires nonlinear analysis, the Eurocode 2 (EC2) offers, in addition to the general method, two simplified methods that do not need this complex nonlinear analysis: one based on nominal stiffness and the other based on nominal curvature [1]. However, these two simplified methods lead to very different results, and the stiffness method is much less economical than the curvature method [3, 4]. Moreover, this method is applicable only when the design load is small about the buckling load or the column is not very slender. The method based on nominal curvature, as presented in EC2, provides good results but is unconservative for columns under sustained loads [4].

By finding two distinct methods suggested in the same design code, the engineer imagines that they provide similar design solutions and he chooses the one that seems most convenient. Unfortunately, this is not the case. Thus, to assess most precisely the buckling resistance of columns, we have used the general method analysis for our previous studies [11, 12]. So, to simplify the buckling columns analysis for the engineer, we have proposed design charts [11] as well as a mathematical model [12] based on the EC2 general method. It allows engineers to easily check the stability of reinforced concrete rectangular columns subjected to axial compression.

In this paper, we will extend our study to rectangular columns subjected to eccentric compression. We will then make a mathematical model that allows designers to easily determine the stability domain of these reinforced concrete columns. It will not only allow engineers to verify slender column stability quickly but also to design them.

After the research significance paragraph, we will present the theoretical background along with the underlying assumptions. Then, we will model the limit of the stability domain, and finally, we will demonstrate the application of this model.

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2. RESEARCH SIGNIFICANCE

Since column buckling requires nonlinear analysis, solving the problem becomes very tedious and requires the use of computer software or charts. Moreover, the EC2 simplified methods lead to very different results. Thus, mathematical modeling is essential; however, it has received no attention from researchers. So the main objective of our work is to establish a simple mathematical model to study slender columns buckling and allow engineers to make rapid and efficient design and verification calculations.

3. THEOR ETHICAL BACKGROUND

3.1 Problem Presentation

As the simplified methods provide very different results [4], we compute the critical buckling load using the EC2 general method. It is based on a nonlinear analysis including:

- Geometric nonlinearity (second order effects),
- Nonlinear mechanical behavior of materials.

The EC2 also admits that for usual structures, regardless of the end conditions, the study of a compressed column can be brought to the case of a double articulated column of length l_f well known as model column [1-3]. The advantage of this model column is to rally the buckling problem into the study of the one cross-section at the ultimate limit state.

The fundamental assumption of the model column is that the deformation is sinusoidal [2, 3]. Therefore, the maximum deflection f and the curvature $\frac{1}{r}$ are tied by the following equation:

$$f = \frac{1}{r} \times \frac{l_f^2}{\pi^2}$$

Thus, the external eccentricity or the eccentricity of the axial load N_u in the middle cross section is:

$$e_s\left(\frac{1}{r}\right) = e_c + e_a + \frac{1}{r} \times \frac{l_f^2}{\pi^2} \qquad (1)$$

$e_c = \frac{M_u}{N_u}$ is the structural eccentricity.
M_u is the momentum applied to the column
N_u is the normal load applied to the column
$e_a = \frac{a_h . l_f}{400}$ is the accidental eccentricity due to execution imperfections [1-3].

3.2 Numerical Buckling Analysis

The stability analysis of the RC column consists of proving that there is a deflection of the element that equilibrates the design solicitations while taking into account second-order effects [1-3].

The analysis of the middle cross section (Figure-1) assesses the following parameters:

- The resisting solicitations N_i, M_i
- The corresponding deformation of the section at the middle span.

The internal solicitations or the resisting solicitations [11, 12] are given by:

$$N_i = N_c + A_{s1}\sigma_{s1} + A_{s2}\sigma_{s2}$$
$$M_i = M_c + A_{s1}\sigma_{s1}\left(\frac{h}{2} - d'\right) - A_{s2}\sigma_{s2}\left(d - \frac{h}{2}\right)$$

with $\sigma_s < 0$ if tension
N_c, M_c are the concrete-resistant solicitations.

Hence, the resistant eccentricity or the internal eccentricity is:

$$e_r\left(\frac{1}{r}\right) = \frac{M_i}{N_i} \qquad (2)$$

If the equation $e_r\left(\frac{1}{r}\right) = e_s\left(\frac{1}{r}\right)$ has a solution then the column will be stable, else it will buckle.

Whereas in the previous paper (case of column subjected to uniaxial compression) there was a value of N_{umax} which determines the resistance of the column to buckling [11, 12]. In this case, there is a curve $N_{umax} = f(M_u)$ below which the column will be stable and above which the column will be unstable.

3.3 Hypothesis and Materials Proprieties

3.3.1 Hypothesis

- The column is isolated and simply supported at both ends [1-3].
- The cross-section of the columns is rectangular.

b: Cross-section width,
h: Cross-section height in the buckling plane,
l_f: Buckling length of the column,

- The section contains bars put symmetrically: $A_{s1} = A_{s2} = \frac{A}{2}$

A is the total area of the steel reinforcements

3.3.2 Properties of steel bars

The stress-strain diagram chosen is the perfect elastoplastic one presented in article 3.2.7 of EC2 [1].
$E_s = 200$ GPa: modulus of elasticity
$f_{yk} = 500$ MPa: characteristic yield strength

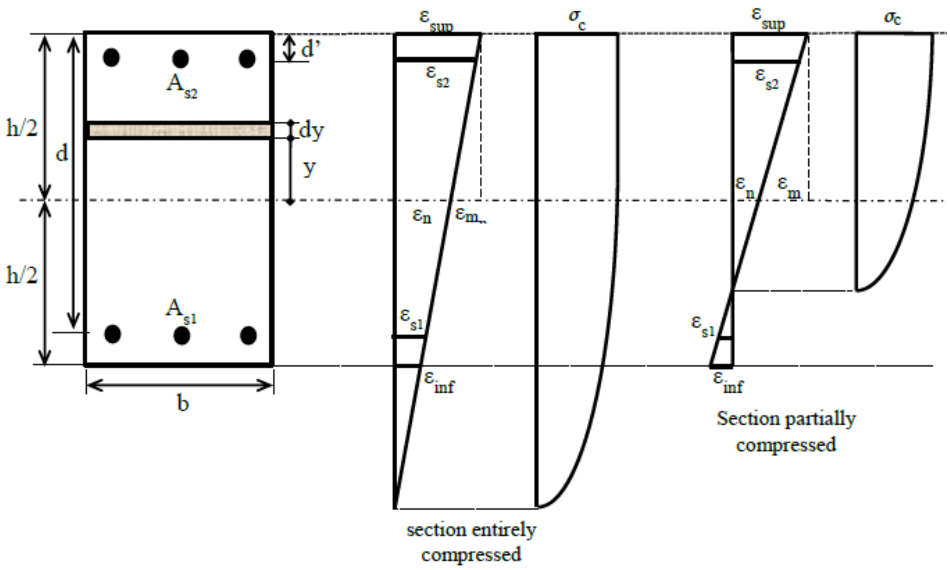

Figure-1. Strains and stresses diagrams.

3.3.3 Concrete characteristics

To compute a design value for the ultimate load, the stress-strain relation used is shown in article 5.8.6(3) of the EC2 [1].

$$E_{cm} = 22000 \times \left(\frac{f_{ck} + 8}{10}\right)^{0.3}$$

The concrete characteristic compressive strength f_{ck} is varying between 20 and 60 MPa.

3.3.4 Effect of the load duration

The creep [1] may be taken into account by multiplying all strain values in the concrete stress-strain diagram with a factor $(1 + \varphi_{ef})$.

3.3.5 Notations

$p = \frac{A.f_{yd}}{b.h.f_{cd}}$ is The mechanical percentage of reinforcement

f_{yd} is the design yield strength of reinforcement

f_{cd} is the design value of concrete compressive strength.

$\nu = \frac{N_u}{b.h.f_{cd}}$ and $\mu = \frac{M_u}{b.h.f_{cd}}$

3.4 Shape of the Curve $\nu_{max}(\mu_{max})$

$\nu_{max}(\mu_{max})$ is the critical relative normal load. It is the maximum relative normal load that can be applied on the column without failure due to instability. ν_{max} will be used to calculate N_{umax} which is the maximum normal load that can be applied in addition to M_u on the column without failure due to instability.

The critical relative normal load supported without instability ν_{max} is evaluated by an application developed on computer software.

The Figure-2 illustrates the curves $\nu_{max}(\mu_{max})$ established for a column where the cross-section width is b=0.6m, $f_{ck} = 25$MPa, $\varphi_{ef} = 1$ and for three different values of $\frac{l_f}{h}$.

After solving the problem for different values of p, φ_{ef}, f_{ck} and $\frac{l_f}{h}$, we have come to the following conclusion: "With a very good correlation, the curve $\nu_{max}(\mu_{max})$ is a straight line".

This line separates values of couples $(\nu\,;\mu)$ for which we have stability and for which we have instability.

3.4.1 Modeling of the stability domain boundary

The separation line between stable and unstable domains can be defined by two points (μ_1, ν_1) and (μ_2, ν_2). Therefore, it has been chosen (Figure-3) to look for the points $(0\,;\nu_1)$, and $(\mu_2; 0{,}1)$.

4. ν_1 EVALUATION

Under axial compression (μ=0), ν_1 is calculated using the formula (Eq. (3)) determined in the previous article [12].

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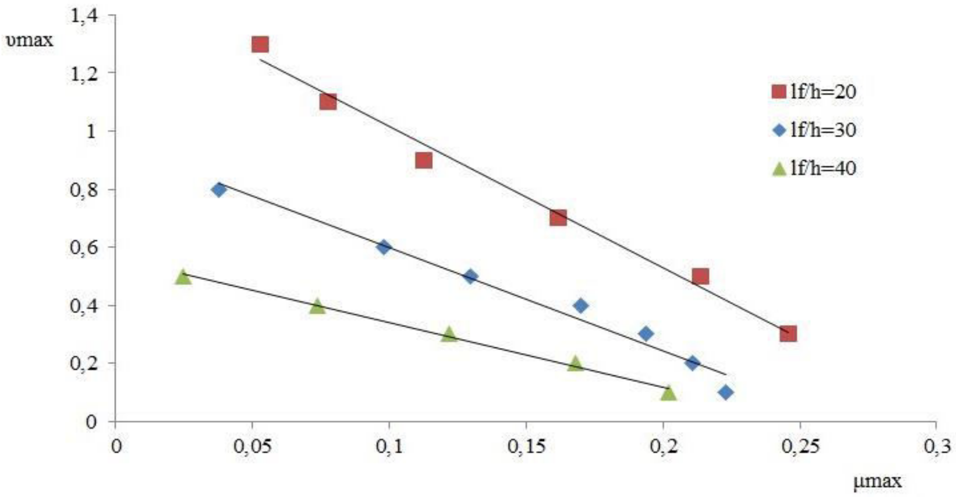

Figure-2. Graphic representing $\nu_{max} = f(\mu_{max})$.

$$\nu_1 = \gamma_\varphi \gamma_l (1{,}80p + 1{,}46) e^{-\frac{0.04 l_f}{h}} \tag{3}$$

with $\gamma_\varphi = 1 - \left(0{,}03 + \frac{p}{28}\right) \varphi_{ef}$
and $\gamma_l = 1 - 5{,}48 \times 10^{-3} l_f$

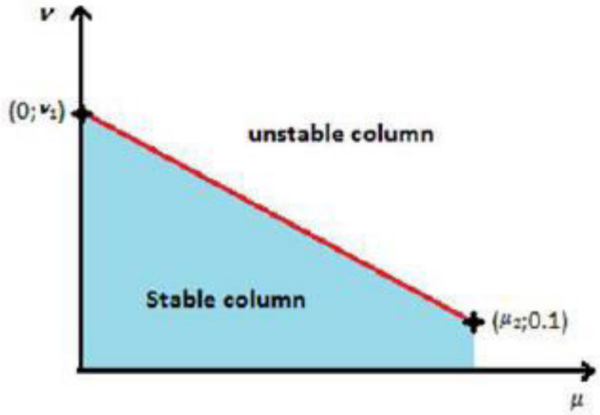

Figure-3. Separation line between stable and unstable domains

4.1 μ_2 Assessment

For determining μ_2, the problem is solved by another program that has been made in a computer which takes a constant value $\nu_{max} = 0{,}1$ and looks for μ_{max} the maximum value of µ without instability: $\mu_2 = \mu_{max}(0{,}1)$
The results allowed us to draw the following conclusions:

- μ_2 is independent of b, f_{ck} and φ_{ef} (Table 1- 3)
- μ_2 is linear about p as shown in Figure-4
- μ_2 is linear about h as shown in Figure-5
- μ_2 looks to be parabolic about $\frac{l_f}{h}$ as shown in Figure-6

Table-1. Illustration of the independence of μ_2 with respect to b.

b(m)	µ2 (m)
0,2	0,223
0,4	0,223
0,6	0,223
0,8	0,223
1	0,223
1,2	0,223

Table-2. Illustration of the independence of μ_2 with respect to f_{ck}.

f_{ck}(MPa)	µ2 (m)
20	0,223
30	0,223
40	0,223
50	0,223
60	0,223

Table-3. Illustration of the independence of μ_2 with respect to φ_{ef}.

φ_{ef}	µ2 (m)
0	0,220
0,5	0,224
1	0,223
1,5	0,222
2	0,222

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4.2 Modeling of μ_2

4.2.1 Influence of the mechanical percentage of reinforcement

As the curves (Figure-4) look to be linear, we fix their equation to: $\mu_2 = a.p + b$. By identification we find that the following equation is the best representation of these curves:

$\mu_2 = 0{,}372p$ μ_2 in meters

4.2.2 Influence of the cross-section height

As the curves (Figure-5) look to be linear, we fix their equation to $\mu_2 = 0{,}372p(a.h + b)$.by identification we find that the following equation is the best representation of these curves:

$\mu_2 = 0{,}372p(1{,}15h - 0{,}15)$

μ_2 and h in meters

4.2.3 Influence of the slenderness $\frac{l_f}{h}$

As the curves (Fig. 6) look to be parabolic we fix their equation to:

$$\mu_2 = 0{,}372p(1{,}15h - 0{,}15)\left(a + b.\left(\frac{l_f}{h}\right) + c.\left(\frac{l_f}{h}\right)^2\right)$$

By identification we find that the Eq. (4) is there best representation.

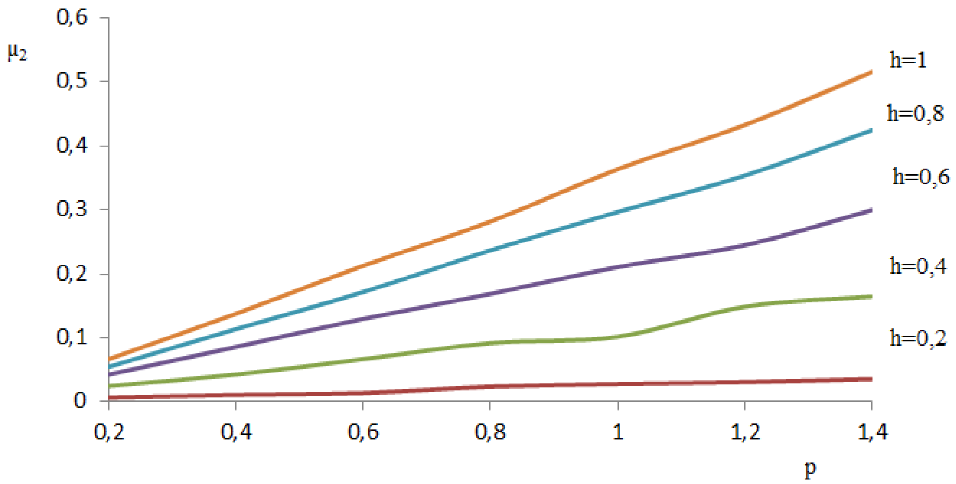

Figure-4. μ_2 in function of p for $\frac{l_f}{h} = 30$.

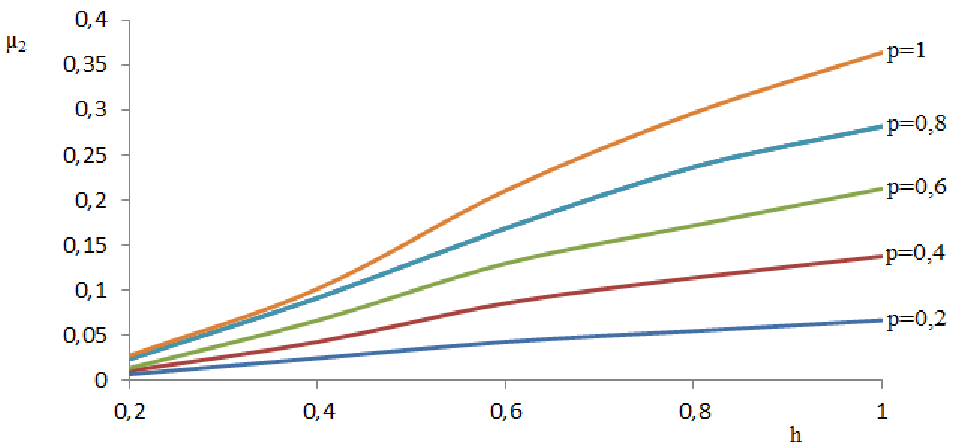

Figure-5. μ_2 in function of h for $\frac{l_f}{h} = 30$.

$$\mu_2 = 0{,}372p(1{,}15h - 0{,}15)\left(1 - 8.10^{-3}\left(\frac{l_f}{h} - 30\right) - 1{,}5.10^{-4}\left(\frac{l_f}{h} - 30\right)^2\right) \quad (4)$$

Witch can be right as: $\mu_2 = 0{,}372p(1{,}15h - 0{,}15)\left(1{,}11 + 10^{-3}\frac{l_f}{h} - 1{,}5.10^{-4}\left(\frac{l_f}{h}\right)^2\right)$

4.2.4 μ_2 Model validation

To validate the model, we have plotted (Figures7-8) the real curves (in color) superimposed on the curves given by the theoretical model (dashed line). We note that the theoretical and real curves are very close to each other, which validates the previous model.

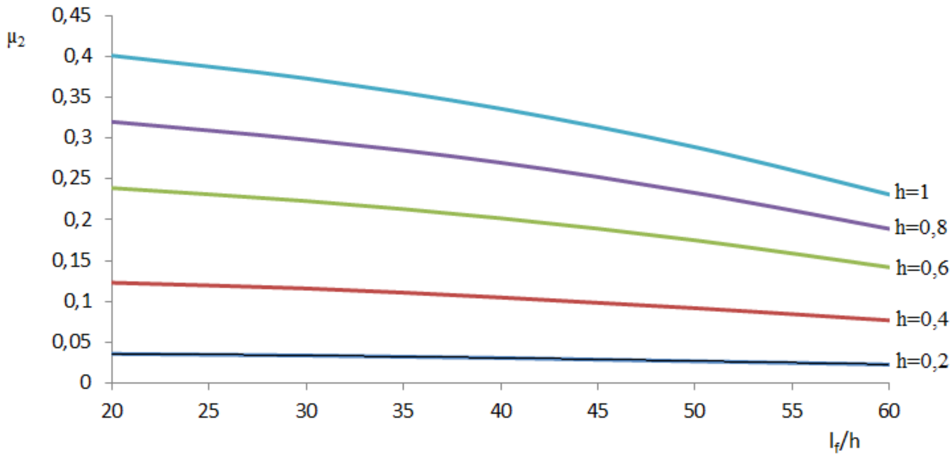

Figure-6. μ_2 in function of $\frac{l_f}{h}$ for p=1.

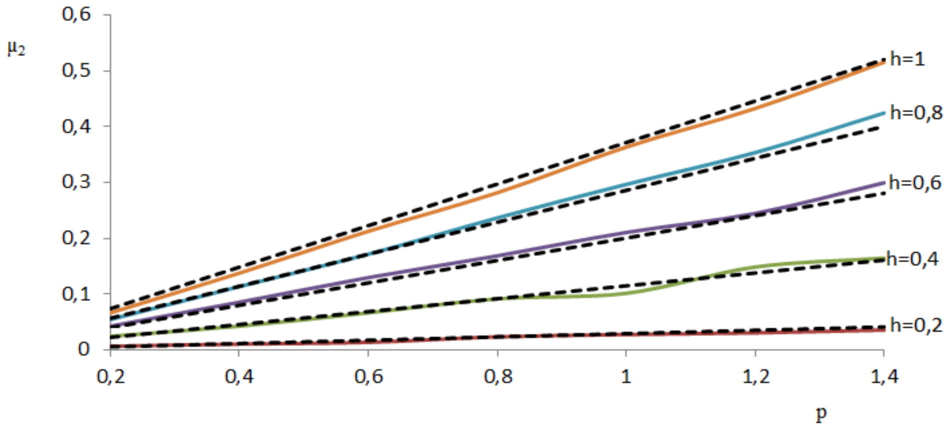

Figure-7. μ_2 in function of p for $\frac{l_f}{h} = 30$ and different values of h.

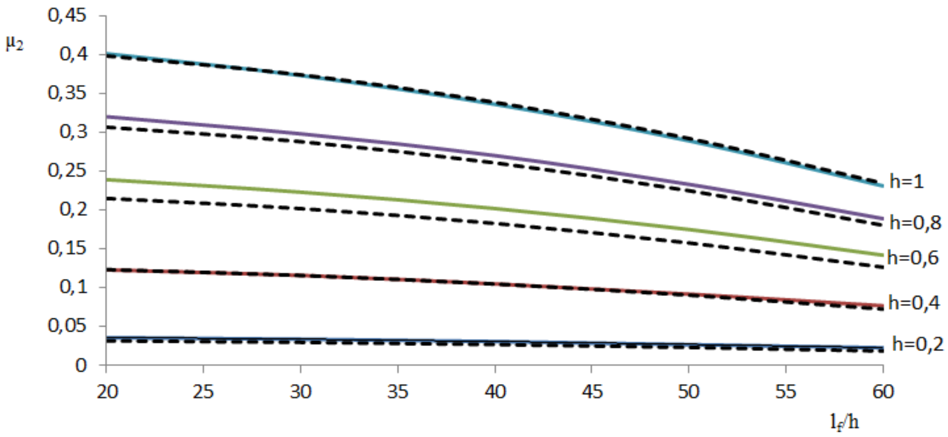

Figure-8. μ_2 in function of $\frac{l_f}{h}$ for p=1 and different values of h.

5. MODEL APPLICATION

We propose to verify, using our model, the stability of the column dimensioned by J. Roux [3]. The column with a cross-section B = 40x40 cm² and a buckling length $l_f = 2\ell = 12$ m, is subjected to an ultimate normal effort $N_u = 300$ KN eccentric by $e_1 = 11{,}6$ cm, and the creep coefficient is assumed to be $\varphi_{ef} = 2$. The characteristic yield strengths of materials are $f_{ck} = 25$ MPa and $f_{yk} = 500$ MPa.

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The reinforcement sections were found to be $A_{s1} = A_{s2} = 3.99\ cm^2$ by the curvature method and $A_{s1} = A_{s2} = 24.26\ cm^2$ by the stiffness method [3]. As reported by José Milton [4] we can notice that the simplified methods lead to very different results.

On our side, we will check the stability of the column reinforced with $A_{s1} = A_{s2} = 3.99\ cm^2$. The line limiting the stability domain of this column passing through the points $(0; \nu_1)$ and $(\mu_2; 0{,}1)$ where $\nu_1 = 0{,}436$ and $\mu_2 = 0{,}015m$, is plotted on the Figure-9. We can notice that the point P of coordinates (ν=0.112; μ=0.013m) is inside the domain of stability closely near the limit line. This means that the stability of the column is ensured and the dimensioning with the curvature method is economic.

6. CONCLUSIONS

Since the verification and the design of reinforced concrete columns are very tedious and time-consuming, a new simple mathematical model has been developed to assess the buckling resistance of RC rectangular columns subjected to eccentric compression in congruence with the prescriptions of Eurocode 2.

It applies to rectangular reinforced concrete columns under axial and non-axial compression with symmetric reinforcement distribution, 500 MPa yield strength of steel reinforcement, and a concrete characteristic compressive strength varying between 20 and 60 MPa.

The proposed model is very simple to use, and it is a very competitive approach in verifying the column's stability without the need to use computer programs or abacuses. Furthermore, it can also be used to design columns so that they don't face buckling instability.

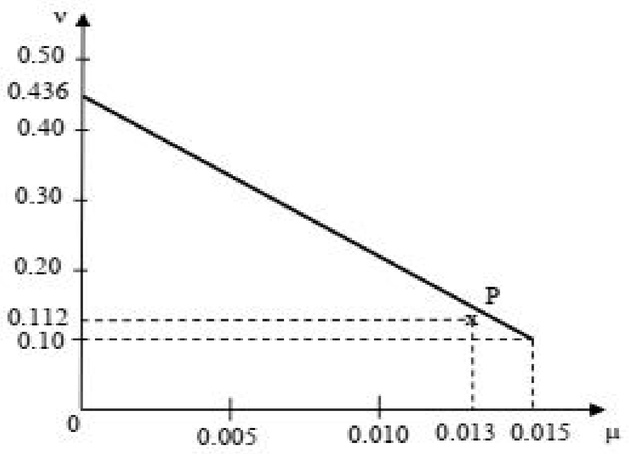

Figure-9. Checking the stability of the column.

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