VOL. 21, NO. 6, MARCH 2026 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

# ADVANCED MODELING OF FLEXIBLE MECHANISMS VIA VIRTUAL KINEMATIC CHAINS AND DAVIES METHOD

Gonzalo Moreno, James R. Vera Rozo and Juan Carlos Serrano
Department of Mechanical Engineering, University of Pamplona. Pamplona, Colombia
E-Mail: gmoren@hotmail.com

**ABSTRACT**

All mechanical systems are subjected to forces that generate stresses and deformations in their components. These effects are critical for structural design, material selection, and motion prediction. Flexible components are particularly challenging to analyze due to geometric changes under loading conditions. This paper presents a methodology for modeling flexible mechanisms using a virtual kinematic chain combined with the Davies method for static analysis. The proposed approach allows the determination of stresses, deformations, and motion characteristics in a unified framework. A case study is presented to validate the model, showing strong agreement with classical methods while providing enhanced insight into deformation effects.

**Keywords:** flexible mechanisms, virtual joint method, Davies method, static analysis, deformation modeling.

Manuscript Received 14 April 2026; Revised 25 March 2026; Published 20 May 2026

## 1. INTRODUCTION

Stiffness and deformation analysis are key indicators in the performance evaluation of mechanical systems. These parameters are essential for the design of mechanical components and control systems, particularly when system performance at specific positions is critical.

Traditional rigid-body models are often insufficient when flexible components are involved. To address this limitation, the Virtual Joint Method (VJM) introduces localized compliance by incorporating passive joints into rigid-body models [1].

However, some mechanical components exhibit not only axial deformation but also shear deformation. Although shear deformation is generally small, it can significantly affect the behavior of systems such as suspensions, tires, and hydraulic or pneumatic systems.

Based on the theory of buckling of columns presented by Popov [2], flexible components can be modeled by combining axial and shear deformation. Axial deformation is represented by a helical spring, while shear deformation is modeled using a revolute joint with a torsional spring.

In this work, these concepts are integrated into a virtual kinematic chain and analyzed using the Davies method [3], which provides a systematic framework for static analysis based on graph theory and screw theory [3-7].

The main objective of this work is to develop a flexible mechanism model using a virtual kinematic chain and to analyze it through the Davies method, providing an accurate and systematic approach for static analysis.

The remainder of this paper is organized as follows: Section 2 presents the proposed methodology, including deformation modeling and static analysis. Section 3 presents the case study and results. Finally, conclusions are given in Section 4.

## 2. METHODOLOGY

This section describes the modeling approach used to represent flexible mechanical components and the procedure followed to perform the static analysis using the Davies method.

### 2.1 Deformation Modeling

Elastic and plastic deformation of mechanical components can be caused by internal or external forces, as well as temperature variations.

Considering these aspects and based on the buckling theory of columns [2], a deformation model is proposed. In conventional modeling approaches, elastic deformation is typically represented using prismatic joints; however, this simplification neglects shear deformation, which can be significant in certain applications.

To overcome this limitation, this work adopts an approach based on column buckling theory [2]. Under an applied force, a structural element experiences:

- Axial deformation
- Shear deformation

These deformations introduce two degrees of freedom: translational displacement and rotational deformation (Figure-1).

The proposed model represents (Figure-2):

- Axial deformation using a helical spring
- Shear deformation using a revolute joint with a torsional spring

This combination enables a more accurate representation of flexible components.

ARPN Journal of Engineering and Applied Sciences




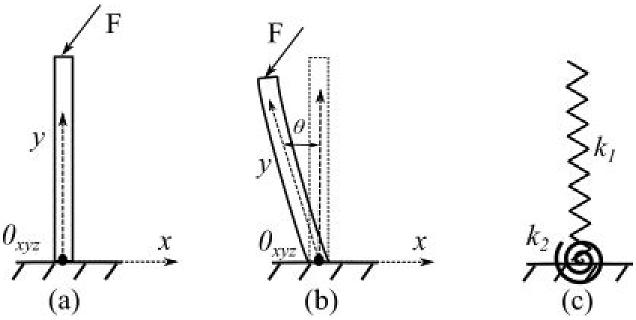


**Figure-1.** Deformation model of mechanical components.

Where $k_1$ and $k_2$ are the stiffness of the compression spring and the torsional spring, respectively ($k_2 \gg k_1$) [2], and $\theta$ is the rotation angle of the bar.

### 2.2 Virtual Kinematic Chain Representation

Using mechanism theory, helical springs are represented by prismatic joints in mechanism analysis [5] [8]. Figure-2(a) shows the proposed mechanism of the bar, which is composed of three links ($n = 3$) identified by the letters A (the base) and B-C (helical spring), and the two joints ($j = 2$) identified by numbers as follows: a revolute joint "R" (1) and a prismatic joint "P" (2).

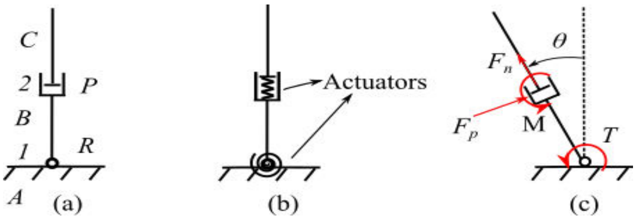


**Figure-2.** Proposed deformation model.

From the general equations of mobility (Eqs. 1 and 2) [9] [10] [11], in a planar space ($\lambda = 3$), the proposed mechanism has two degrees of freedom (2 DoF), and it makes the same movements (or behavior) as a buckling bar.

$$M = \lambda \times (n - j - 1) + j \tag{1}$$

$$\nu = j - n + 1 \tag{2}$$

Where $M$ is the number of degrees of freedom (DoF) or mobility of the mechanism, $\lambda$ is the number of degrees of freedom of the space in which the mechanism is intended to function, $n$ is the number of mechanism links, including the fixed link, $j$ is the number of mechanism joints, and ν is the number of independent loops in the mechanism.

The mechanism of the Figure-2(a) has two passive actuators that control the following movements of the bar: the first actuator is a prismatic joint, which controls the axial deformation of the bar, and the second actuator is a revolute joint, which controls the shear deformation, as shown in Figure-2(b).

Figure-2(c) shows the forces that act on the joints of the mechanism: $F_n$ and $F_p$ are the normal and perpendicular loads in the passive prismatic joint, respectively; $M$ is the moment that acts on the prismatic joint, $T$ is the torque that acts on the revolute joint, and $\theta$ is the rotation angle of the bar.

### 2.3 Deformation Due to Force and Temperature

In a continuous body, deformation results from displacement fields induced by forces or temperature changes.

The total deformation is expressed as:

$$\delta_{Total} = \delta_F + \delta_T \tag{3}$$

Where the deformation ($\delta_{Total}$) is influenced by the deformation due to forces ($\delta_F$) and the deformation due to temperature changes ($\delta_T$), as shown in Eq. (3).

Table-1 summarizes different deformation models reported in the literature, where all notations are adapted to those used in this paper:

**Table-1.** Summary of the related works and expressions for the deformation of mechanical components.

| Publications | Element | Deformation by | Model |
|---|---|---|---|
| Hibbeler [12] | Links | Force | $\delta = \frac{-F_n L}{AE}$ |
| Hibbeler [12] | Spring | Force | $\delta_S = \frac{\triangle F}{k_s} = \frac{F^{start} - F_n}{k_s}$ |
| Dhoshi *et al.* [13] | Leaf Spring | Force | $\delta_{LS} = \frac{3 \triangle F L_s^3}{8 E_s N B T^3} = \frac{\triangle F}{k_{LS}}$ |
| Taylor *et al.* [14] | Rubber - Tyres | Force | $\delta_t = \frac{3 \triangle F}{k_t + a} = \frac{\triangle F}{k_T}$ |
| Hibbeler [12] | Links | Temperature | $\delta_T = \alpha . \triangle T . L$ |
| Popov [2] | Torsional spring | Torque | $\theta = \frac{\tau}{k_{ts}}$ |

VOL. 21, NO. 6, MARCH 2026 ISSN 1819-6608
ARPN Journal of Engineering and Applied Sciences

www.arpnjournals.com

where $\delta$ is the elastic deformation, $\delta_S$ is the spring deformation, $\delta_{LS}$ is the leaf spring deformation, $\delta_t$ is the rubber axial deformation, $\delta_T$ is the thermal deformation, $L$ is the original length of the bar, $A$ is the cross-sectional area of the bar, $E$ is the modulus of elasticity of the material, $\Delta F$ is the algebraic change in the initial load, $F^{start}$ is the initial load, $k_s$ is the equivalent spring stiffness, $L_s$ is the length of the leaf spring, $N$ is the number of the leaves, $B$ is the width of the leaf, $T$ is the thickness of the leaf, $E_s$ is the modulus of elasticity for a multiple leaf, $k_{Ls}$ is the equivalent stiffness of the suspension, $k_t$ is the rubber stiffness, $a$ is the regression coefficient, $k_T$ is the equivalent rubber stiffness, $\Delta T$ is the algebraic change in the temperature, $\alpha$ is the linear coefficient of thermal expansion, $\tau$ is the torque, and $k_{ts}$ is the spring torsional coefficient.

### 2.4 Static Analysis Using the Davies Method

Several methodologies allow us to obtain a complete static analysis of the mechanism. In this research, the formalism presented by [3] was used as the primary mathematical tool to analyze the mechanisms statically. The Davies method appears in many publications in the literature, and further details regarding its use can be found in [3-7]. The Davies method was selected because it offers a straightforward approach to obtaining a static model of the mechanism, which can be readily applied in different mechanical systems.

The Davies method provides a systematic way to relate the joint forces and moments in closed kinematic chains [15]. This method is based on graph theory, screw theory, and the Kirchhoff cut-set law, and it can be used to express the static equilibrium of a mechanism in matrix form [15].

## 3. RESULTS AND DISCUSSIONS

### 3.1 Case Study

To validate the methodology, a mechanism consisting of a rigid bar supported by two springs is analyzed.

The rigid bar (AB) of the Figure-3(a) is supported by two springs at its ends A and B. If the bar is loaded with the forces $P_1$ and $P_2$ at point C, determine the loads and the deformations of the springs.

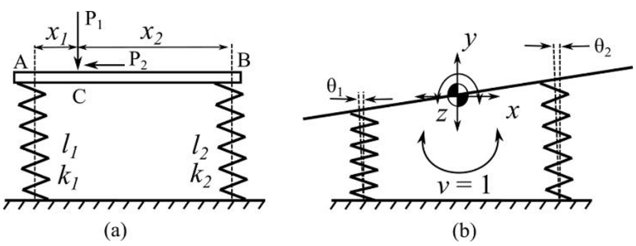


**Figure-3.** (a) Mechanism example 1. (b) Movement of the mechanism.

The mechanism allows three motions under the action of forces, displacement in the $x$- and $y$-direction, and a rotation about the $z$-axis, as shown in Figure-3(b).
To model the mechanism, the following considerations were taken into account:

- the mechanism of the Figure-3(a) has 3-DoF, the work space is planar ($\lambda = 2$), and the number of independent loops is one ($\nu = 1$); from the general equations of mobility (Eqs. 1 and 2), the kinematic chain of the mechanism is composed of six links ($n = 6$) and six joints ($j = 6$),
- applying the proposed methodology, the springs are modeled as flexible mechanical components, with axial deformation and a small shear deformation, and
- to allow the rotation of the bar AB in the $z$-axis, the link between the bar and the springs is made with revolute joints.

### 3.2 Model Development

Applying these considerations, the kinematic chain of the mechanism shows in Figure-4(a) is developed; the chain is composed of six links identified by letters $A$ (the base), $B$ and $C$ (spring 1), $D$ and $E$ (spring 2), and $F$ (rigid bar), and the six joints are identified by numbers as follow: four revolute joints *"R"* (1, 3, 4 and 6), and two prismatic joints *"P"* (2 and 5).

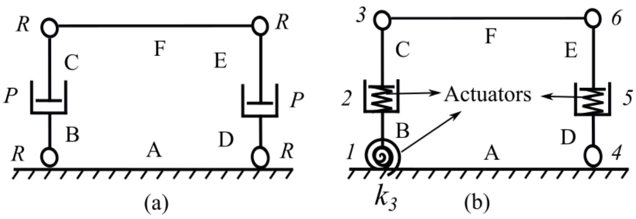


**Figure-4.** (a) Kinematic chain of the mechanism. (b) System including actuators.

The developed mechanism has *3-DoF*, and it requires three passive actuators to control its movement. The mechanism has one passive actuator in each prismatic joint (axial deformation), and one passive actuator in joint 1 (torsion spring - shear deformation), which controls the movement of the mechanism, as shown in Figure-4(b).
As it has been mentioned above, the static analysis of the mechanism can be carried out with the use of the Davies method, which uses screw theory and graph theory together with Kirchhoff's laws to build and solve the static and kinematic analysis of any mechanism.

The mechanism of the Figure-4(a) can be represented by the graph shown in the Figure-5(a), where the vertices correspond to the links and the edges correspond to the constraints of each joint and the external forces. Figure-5(b) presents the cut-set action graph, and Eq. (5) represents the expanded cut-set matrix $[Q]$, where each line represents a cut of the graph, and the columns represent the constraints of the joints as well as the external forces present in the mechanism.

ARPN Journal of Engineering and Applied Sciences



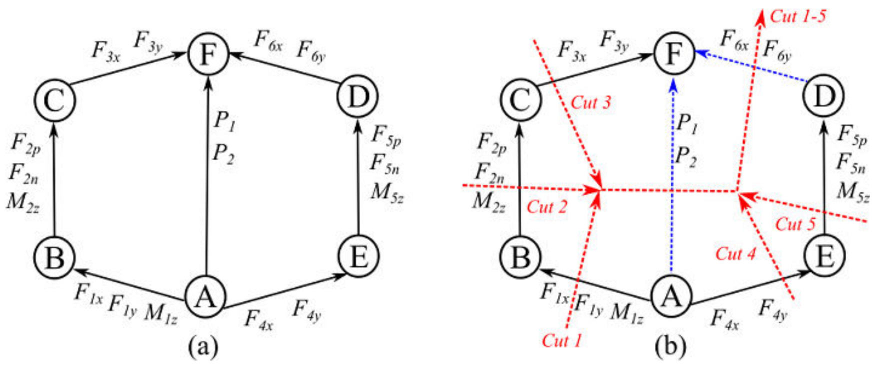


**Figure-5.** (a) Direct coupling graph. (b) Cut-set graph.

$$[\mathrm{Q}]_{5\times 17} = \begin{array}{c} \\ \text{Cut 1} \\ \text{Cut 2} \\ \text{Cut 3} \\ \text{Cut 4} \\ \text{Cut 5} \end{array} \begin{array}{c} \begin{array}{ccccccccccccccccc} F_{1x} & F_{1y} & M_{1z} & F_{2p} & F_{2n} & M_{2z} & F_{3x} & F_{3y} & F_{4x} & F_{4y} & F_{5p} & F_{5n} & M_{5z} & F_{6x} & F_{6y} & P_1 & P_2 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & -1 & -1 & 0 & 0 \end{bmatrix} \end{array} \quad (5)$$

Considering a static analysis in two-dimensional space, all the wrenches of the mechanism together comprise the action matrix $[A_d]$ given by Eq. (6).

$$[\mathrm{A_d}]_{3\times 17} = [\$^{\mathrm{A}}_{\mathrm{F_{1x}}} \quad \$^{\mathrm{A}}_{\mathrm{F_{1y}}} \quad \$^{\mathrm{A}}_{\mathrm{M_{1z}}} \quad \cdots \quad \$^{\mathrm{A}}_{\mathrm{F_{6y}}} \quad \$^{\mathrm{A}}_{\mathrm{P_1}} \quad \$^{\mathrm{A}}_{\mathrm{P_2}}] \quad (6)$$

The wrench can be represented by a normalized wrench and its magnitude. Therefore, from Eq. (6), the unit action matrix and the magnitude action vector are obtained, as represented by Eqs. (7) and (8).

$$[\hat{A}_d]_{3\times 17} = [\hat{\$}^{A}_{F_{1x}} \quad \hat{\$}^{A}_{F_{1y}} \quad \hat{\$}^{A}_{M_{1z}} \quad \cdots \quad \hat{\$}^{A}_{F_{6y}} \quad \hat{\$}^{A}_{P_1} \quad \hat{\$}^{A}_{P_2}] \quad (7)$$

$$[\Psi]_{17\times 1} = [F_{x1} \quad F_{y1} \quad M_{1z} \quad \cdots \quad F_{6y} \quad P_1 \quad P_2] \quad (8)$$

Using the cut-set law [15], the algebraic sum of the normalized wrenches, Eqs. (7) and (8), which belong to the same cut (Figure-5(b) and Eq. (5)), must be equal to zero. Therefore, the statics of the mechanism can be defined, as exemplified in Eq. (9):

$$[\hat{A}_n]_{15\times 17}[\Psi]^T_{17\times 1} = [0]_{15\times 1} \quad (9)$$

It is necessary to identify the set of primary variables $[\Psi_p]$ (known variables), among the variables of $\Psi$, starting from the system in Eq. (9). Once identified, the system is divided into two sets, as shown in Eq. (10),

$$[\hat{A}_{ns}]_{15\times 15}[\Psi_s]^T_{15\times 1} + [\hat{A}_{np}]_{15\times 2}[\Psi_p]^T_{2\times 1} = [0]_{15\times 1} \quad (10)$$

where $[\Psi_p]$ is the primary variable vector, $[\Psi_s]$ is the secondary variable vector (unknown variables), $[\hat{A}_{np}]$ are the columns corresponding to the primary variables, and $[\hat{A}_{ns}]$ are the columns corresponding to the secondary variables.

In this case, the primary variables vector is:

$$[\Psi_p]_{2\times 1} = [P_1 \quad P_2]^T \quad (11)$$

### 3.3 Results

Solving the system in Eq. (10) using the Gauss-Jordan elimination method, all secondary variables of the system being functions of the primary variables of the mechanism ($P_1$ and $P_2$), Table-2 shows the parameters used for the example and the result obtained:

**Table-2.** Parameters of Example 1.

| Parameters | Inputs | Test 1 | Test 2 | Units |
|---|---|---|---|---|
| $l_1$ | 0.4 | 0.275 | 0.2747 | m |
| $l_2$ | 0.4 | 0.3584 | 0.3586 | m |
| $x_1$ | 0.3 | | | m |
| $x_2$ | 1.2 | | | m |
| $\theta_1$ | 0 | 0 | ≈0 | degree |
| $\theta_2$ | 0 | 0.46 | 0.47 | degree |
| $P_1$ | 1000 | 1000 | 1000 | N |
| $P_2$ | 100 | 0 | 100 | N |
| $k_1$ | 6 | | | $kN.m^{-1}$ |
| $k_2$ | 6 | | | $kN.m^{-1}$ |
| $k_3$ | 12 | | | $kN.m^{-1}$ |
| $F_{2n}$ | | -750 | -775.3 | N |
| $F_{5n}$ | | -250 | -224.7 | N |
| $\delta_{11}$ | | -0.125 | -0.1253 | m |
| $\delta_{12}$ | | -0.0416 | -0.0414 | m |

It can be observed that the differences between Test 1 and Test 2 are minimal, indicating numerical consistency of the proposed formulation. Additionally, the deformation values remain within expected physical ranges, confirming the stability of the model under the given loading conditions.

This example demonstrates the effectiveness of the proposed methodology, and the obtained results show strong agreement with those derived from classical static analysis methods, confirming the validity and robustness of the proposed modeling approach.

### 3.4 Discussion

The proposed methodology offers several advantages over traditional approaches:

- It provides a more realistic representation of flexible components
- It incorporates shear deformation effects, which are often neglected
- It enables a unified treatment of rigid and flexible systems

The results indicate that shear deformation introduces measurable differences in displacement values. These differences become significant in systems requiring high precision.

Furthermore, the proposed approach enables a more comprehensive understanding of load distribution within flexible mechanisms. Unlike traditional rigid-body approximations, this method captures localized compliance effects, which are essential for accurately predicting system response under complex loading conditions. This becomes particularly relevant in applications involving precision engineering, vehicle dynamics, and robotic systems.

## 4. CONCLUSIONS

This paper presented a methodology for modeling flexible mechanisms using virtual kinematic chains combined with the Davies method.

The main contributions include:

- A modeling approach that incorporates both axial and shear deformation
- A systematic framework for static analysis

The methodology was validated through a representative case study. The results demonstrate that the proposed method provides accurate and consistent predictions, improving upon classical approaches.

The proposed methodology represents a valuable contribution to the field of mechanism analysis, particularly in applications where flexibility effects cannot be neglected. Its implementation can lead to more accurate designs and improved performance prediction in engineering systems.

Future work will focus on dynamic analysis and experimental validation of the proposed methodology.

**REFERENCES**


[1] Pashkevich A. and Klimchik A. and Chablat D. 2011. Enhanced stiffness modeling of manipulators with passive joints. Mechanism and Machine Theory. 46: 662-679. doi:10.1016/j.mechmachtheory.2010.12.008

[2] Popov E. 1998. Engineering Mechanics of Solids, Pearson, ISBN-10: 0137261598.

[3] Davies T. H. 1995. Coupling, coupling networks and their graphs. Mechanism and Machine Theory. 30(7): 1001–1012.

[4] Tsai L.-W. 1999. Robot Analysis - The Mechanics of Serial and Parallel Manipulators. John Wiley and Sons, Ltd., ISBN 0-471-32593-7, New York.

[5] Erthal J. 2010. Modelo Cinestático para Análise de Rolagem em Veículos. PhD. Thesis. Federal University of Santa Catarina, FLorianópolis, Brazil.

[6] Mejia L. and Simas H. and Martins D. 2013. Force Capability Maximization of a 3RRR Symmetric Parallel Manipulator by Topology Optimization. 22nd International Congress of Mechanical Engineering (COBEM 2013), Ribeirão Preto, SP, Brazil, ISSN 2176-5480.

[7] Moreno G. G., and Nicolazzi L., Vieira R. S., and Martins D. 2015. Three-Dimensional Analysis of the Rollover Risk of Heavy Vehicles Using the Davies Method. 14th World Congress in Mechanical and Machine Science (IFToMM2015), Taipei, Taiwan, DOI 10.6567/IFToMM.14TH.WC.PS4.006.

[8] LEE U. 2001. A Study on a Method for Predicting the Vehicle Controllability and Stability Using the Screw Axis Theory. PhD Thesis (PhD Thesis) - Hanyang University, Seoul, South Korea.

[9] Kutzbach K. 1929. Mechanische Leitungsverzweigung - Maschinenbau. 8(21): 710-716.

[10] Crossley F. R. E. 1964. A Contribution to Grübler's Theory in Number Synthesis of Plane Mechanisms. ASME Journal of Engineering Industry, Serie B. (86): 1-8.

[11] Tsai L.-W. 2001. Mechanism Design: Enumeration of Kinematic Structures According to Function. CRC Press, ISBN 0849309018.

[12] Hibbeler R. 2011. Mechanics of Materials, 8th Edition, Pearson - Higher Education, ISBN 13 9780136022305.

[13] Dhoshi N. P. and Ingole N. K. and Gulhane U. D. (2011. Analysis and Modification of Leaf Spring of Tractor Trailer Using Analytical and Finite Element Method. International Journal of Modern Engineering Research. 1: 719-722.

[14] Taylor R. K., Bashford L. L., and Schrock M. D. 2000. Method for Measuring Vertical Tire Stiffness, American Society of Agricultural Engineers. 42: 1415-1419.

[15] Davies T. H. 2000. The 1887 committee meets again. Subject: freedom and constraint. Ball 2000 Conference, Cambridge University Press. p. 56.