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# OPTIMIZED DESIGN OF MINING PRODUCTION PROCESSES UNDER RESOURCE CONSTRAINTS USING THE KNAPSACK PROBLEM

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**ABSTRACT**

Develop a new approach to designing mining processes under limited resources. Combinatorial optimization methods were applied to develop the new approach. The problem of designing technological processes can be solved by interpreting it as the "knapsack problem" with a private solution, "knapsack 0-1". Dynamic programming algorithms were used to search for the optimal solution. A new approach to designing mining management processes under limited resources has been developed. If the goal is to achieve maximum efficiency with minimal costs, the forward pass method is applied. If there are constraints on throughput capacity, energy characteristics, or specific costs, then the backward pass method is applied. In the final part of the research, verification of the proposed approach was carried out. The task involved developing recommendations for choosing the structure of the technological chain of longwall under given technological and mining-geological development parameters. The specific extraction cost was imposed as a constraint. As a result of solving the design problem using the backward pass method, recommendations were developed for replacing mechanical equipment. This allowed achieving the specified level of productivity and reducing specific costs by 10.6%. As a result of solving the problem of organizing the transport chain using the method described in the paper, an 8% reduction in electricity costs was achieved. This was achieved by reducing the total power of the motors that drive the conveyors. By imposing constraints on the size of the operating productivity, the optimal number of vehicles is chosen. The application of combinatorial optimization methods allowed obtaining an optimal solution within the specified parameters. The originality of the work lies in establishing the regularities of efficient formation in the processes of mineral deposits development. For the first time, combinatorial optimization methods have been applied to the design of processes accompanying the development of mineral deposits, determining their scope of application, and formalizing a series of dynamic programming problems and algorithms: reverse pass, spanning tree. Verification of the developed approach to designing mining production processes under limited resources has been carried out. The approach obtained allows finding optimal solutions with specified constraints at the design stage. Constraints may include: throughput capacity of the transport chain, energy characteristics of technological processes accompanying extraction, specific cost of extraction, etc. The design task can be solved in a direct manner - if the objective function is to maximize a parameter- or in reverse - if there are constraints.

**Keywords:** parameter optimization, objective function, technological scheme, resources, rational area.

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## INTRODUCTION

Production design is a rather labor-intensive and specific process that is associated with economic, organizational, technological, and resource factors. The combination of these parameters determines the ultimate efficiency of the production process, which is expressed in the volumes of production and its specific cost. At the same time, searching for interconnections in equipment technological chains is a recurring stage in all companies' operations. Therefore, dynamic programming methods can be applied to equipment selection, organization of transport chains, and so on. It is also important that under market economy conditions, the owner of the enterprise is guided not only by the types of equipment with the highest productivity but also by their price, cost, and labor intensity of operations. Thus, there is a problem of resource constraints.

It is worth noting that the presented work focuses on creating a new approach to designing technological processes accompanying extraction, transportation of valuable minerals, and supporting the mined space, which will allow for the allocation of limited resources. Limited resources - the presence of constraints on a certain parameter or condition, such as the amount of funds allocated for upgrading the longwall or the design throughput capacity of the transport chain, and so on. In other words, achieving maximum effectiveness under constraints. Thus, in this interpretation, it is nothing else but a combinatorial optimization problem. The general condition of the task of designing mining production under limited resources can be formulated as follows: it is necessary to find a set of mechanization means ("items") with maximum qualitative characteristics ("price") under

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conditions of limited resources (" capacity"). This involves several stages:

a) checking for the presence of the specified type of equipment ("item") in the technological chain;
b) tracking the state of resources ("capacity");
c) evaluating the qualitative characteristics ("its value") of the entire chain.

The described problem belongs to the class of NP-complete problems, and there is no polynomial algorithm that can solve it in a reasonable time, so a choice between exact and approximate solutions should be made.

Despite the significant time investment, the authors believe that exact methods for solving problems under resource constraints are preferable. Exact methods include exhaustive search, the branch-and-bound method, and dynamic programming. However, the time complexity of exhaustive search is 2N, which allows applying this approach only to a small number of alternatives [1], similar to the situation with the branch-and-bound method. Essentially, we exclude unfavorable alternatives in advance and then use exhaustive search.

The class of tasks solved using dynamic programming methods is quite versatile. The most comprehensive list is provided in the works of R. Bellman [2, 3]. In the works of V. Grinyov [4, 5], models and algorithms of dynamic programming are presented for rationalizing the parameters of critical and noble resource deposits exploitation, as well as for solving management tasks in the coal industry. However, despite the existing variety of models and approaches, during active military actions in the country's territory and, as a result, transformative processes in the industry, a number of new requirements arise for mining production design methods. Let's focus on them:

a) Problem Size: When designing any mining production process, there is not only a problem of having a large number of factors (over 20), but also the diversity in terms of the degree of influence and nature. If one factor can be optimized and a solution found along a certain curve, for two factors it becomes a plane, and for N factors, it becomes 2N planes. Existing computational capabilities do not allow for obtaining exact solutions. In this case, simulation modeling and algorithms for determining priority control factors and subsequent modeling of changes in the objective function can be applied [6]. However, this is a lengthy and laborious process. Therefore, it is advisable to use decomposition approaches [7], which involve isolating the most influential factor at each stage.
b) Interpretability and visualization. This requirement is essential because the obtained results are intended for a large number of users who have varying levels of knowledge in optimization methods. Additionally, there is a need for constant enrichment of the "project data bank," so it is worth applying combinatorial optimization methods. One advantage is the ability to obtain a solution regarding the optimal allocation of resources in a clear and compact format (such as a tree, graph, network model, etc.). It is worth noting that the obtained solutions can be programmatically implemented, which will significantly accelerate decision-making time and increase the size of the problems that can be tackled.
c) Ability to form databases and work with data streams: This requirement is a consequence of the previous one, as the decision-making process is influenced by the volume of data, their completeness, and the availability of a "bank of project solutions" within the country, enterprise, etc.

All of these requirements are met by combinatorial optimization methods, which involve the application of appropriate algorithms. Thus, creating a new approach to designing processes in mining production management under resource constraints is a relevant scientific task.

Various methods and models with different constraints and objective functions are used in designing mining production. Accordingly, there are several classifications that can describe approaches in design. In paper [8], a retrospective analysis and classification of design methods in mining production are provided. It should be noted that design methods can be classified by the type of economic-mathematical models, by the type of solution obtained, by the duration of planning, and by the method of obtaining the result, as with the development of information technologies, more accurate methods have become available. Considering economic-mathematical methods, the following methods are known: mathematical programming, which is divided into linear [9], quadratic [10], integer [11], stochastic [12], dynamic [13], geometric [14]; mineral reserves management [15], game theory [16], network models [17], correlation models [18], queuing theory [19], reliability theory [20], forecasting [21], simulation modeling [22].

The classification mentioned is more suitable for describing the process of building models but does not provide an answer to the question of whether this decision is qualitative. The category of "quality" is philosophical and is nothing more than the ability of an object to maintain parameters, quantities. In design, the concept of "quality" is not only the correspondence to a certain, specified parameter, but also the ability to reproduce characteristics within desired limits. That's why a classification of methods based on the type of solution obtained was considered and formulated [23]. According to this classification, methods are divided into optimization and informational. Optimization models provide an answer to the question of which solution is optimal from a quantitative point of view, while informational models address the qualitative aspect. Optimization methods include all the aforementioned

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methods of mathematical programming, as well as game theory, network, and correlation models. Informational models are based on the application of the Analytic Hierarchy Process (AHP) method [24] and its variations such as PROMETHEE [25], ELECTRE [26], TODIM [27], VICOR [28], Fuzzy-AHP [29], Gray-AHP [30], and fuzzy set methods [31]. The essence of these approaches is based on establishing priorities and hierarchies. The obtained solution will be qualitative, but it solely depends on the level of expertise. Quite often, there are situations where the value of one parameter significantly affects the decision-making process (eg, development costs of the deposit, equipment costs, etc.), and it is impossible to construct a connectivity matrix [32].

The application of methods based on hierarchy analysis and expert evaluations for the conditions of Ukraine is not always feasible due to the lack of reliable information on prices, the cost of services, and the impossibility of long-term planning.

To address the shortcomings of the first and second groups of methods (described in previous classifications), another classification [33] was considered, which is based on planning terms and tools. As mentioned earlier, methods can be conditionally divided into long-term and short-term planning methods - all of them are based on the ones described above [8-22, 24-31]. However, the author of the study highlights a separate group of approaches based on simulation modeling [34] of production parameters. These parameters can take into account qualitative or economic indicators, which will subsequently allow for the planning of production activities. The difference lies only in the simulation modeling methods. It should be noted that in work [35], emphasis is placed on distinguishing methods that are based on the use of software. The logic is that with the help of software, a voluminous task can be divided into local (infinitely small parts) ones, and by using algorithms for each local area, an optimal solution can be found.

From the analysis of recent works [32–35], the following generalization can be made, which forms trends in the creation of approaches regarding the design of mineral deposit development processes:

- the optimality criterion is "quality," but it is distinguished by various quantitative indicators;
- general tasks are divided into local ones, and only after optimizing the task at the first stage do they proceed to optimization at the second stage, and so on;
- when solving the tasks of completing and organizing the technological chain, the obtained solution must be integer-based.

Based on the analysis of the research, the goal and objectives of the study can be formulated.

To develop a new approach to the design of mining production processes under limited resources, it is necessary to complete a series of tasks:

a) Propose a new approach to the design of the technological process of mining production under limited resources based on the application of combinatorial optimization methods and dynamic programming algorithms. The research subject is the patterns of forming the efficiency of equipment technological chains in the mining face.

b) Verify the developed approach under the conditions of the mining faces of the Western Donbas.

c) Propose a decision support system (DSS).

In accordance with the research tasks, combinatorial optimization methods such as finding the maximum flow and finding spanning trees are applied. To solve the design problems of mining production under limited resources, the "knapsack problem" is interpreted, and the specific "0-1 knapsack" solution is applied.

The idea of the solution is that we have a volume of resources (a knapsack), and at each stage we analyze whether the specified item can fit into a given volume of resources; for this, there are recurrence relations.

## MATERIALS AND METHODS

To achieve the set goal, consider a specific task, which involves choosing the means of mechanization for the coal mine face. The technological chain consists of mechanized support, a shearer, and a scraper conveyor, that is, a set of heterogeneous types of equipment (items) planned to be purchased with a certain budget (capacity). This task can be solved using dynamic programming [36, 37].

To solve the problem, denote m as the maximum value of equipment, which is the quality of the equipment we want to improve, and *w* as the amount allocated for the purchase. Then it is necessary to find the maximum value$m[i,w]$ of the items obtained from the first i available items, whose total weight does not exceed w.

The idea of the solution is that we have a resource volume (knapsack) and at each stage, we analyze whether the specified item can fit into the given resource volume. For this, there are recurrence relations:

$m[0,w] = 0,$
$m[i,w] = m[i-1,w]$, for ,$w_i > w$
$m[i,w] = \max(m[i-1,w],$
$m[i-1,w_i - w] + v_i),$
for $w_i \le w$

where,$m[0,w] = 0$ corresponds to the initial situation where there are no items in the "knapsack", ie, no type of equipment is selected, according to the value (quality characteristic) of the equipment type $m_i = 0$ and the value of the entire technological chain,$m = 0$ while the cost (weight) for purchasing each type of equipment $v_i = 0$ and the entire chain consisting of items, .$w_i = 0$

The next relation corresponds to the situation where the weight (cost of purchase) of the equipment type exceeds the cost of the funds allocated for the purchase of the entire technological chain, hence the- th*i*item is not

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included in the chain, because compared to the $i-1$ item, the cost of its acquisition exceeds the fund limit (knapsack capacity). For example, we have chosen mechanized support and a mining combine with maximum resources, and it remains to choose a scraper conveyor with maximum resources $m_i$ and a value,$v_i$which does not exceed the remaining funds $w-w_i$.

The last relationship,$m[i-1, w_i - w] + v_i)$ corresponds to the situation where the cost of purchasing the equipment type (weight) $v_i$ allows for the acquisition of equipment with maximum resources, $m_i$while the cost (weight) of the entire technological chain does not exceed the allocated resources (capacity) $w$. By choosing mechanized support, the amount of funds for $m[i-1, w_i - w]$ purchasing other types of equipment decreases, while the resource of the technological chain, on the contrary, increases, ie.

The cost constraints can include restrictions on energy characteristics, throughput limitations by the volume of rock mass, and so on.

## RESULTS

The basis of the given ratios is Belman's principle of optimality, which indicates that no matter what the initial state and decision, the following decisions should constitute an optimal course in relation to the state obtained as a result of the first decision [2]. That is, the optimal strategy depends on the current state and goal and does not depend on previous decisions.

The decision process lies in a two-dimensional plane, where the number of types of equipment is placed on the OX axis, and the acquisition costs on the OY axis, and the line connecting the points in the XY system expresses the resource of the technological chain (Fig. 1, 2).

In Figure-2 shows the results of solving the problem of choosing the optimal structure of the mechanized complex. There are a total of 8 types of equipment, including 2 types of mechanized fastening with values (resource) $m_1$, $m_2$ (red lines), 4 types of shearers with values $m_3$, $m_4$, $m_5$, $m_6$ (dark orange lines), and 2 types of scraper conveyors with values $m_7$, $m_8$ (green lines). The weight (price) of the equipment $w_1 \ldots w_8$ is projected on the ordinate axis. Then, it is necessary to choose one fastener, shearer, and conveyor, while the cost of their purchase should not exceed $w$. A network model is built from these conditions. The value [0] corresponds to ratio (1), i.e., it is possible not to select the specified type of equipment; therefore, its value (resource) will be [0]. Otherwise, if selected, the resource (value) will be $m_1, \ldots m_8$, and the acquisition costs will be projected on the OY axis. In Figure-1 shows a graphical interpretation of the recurrence relation.

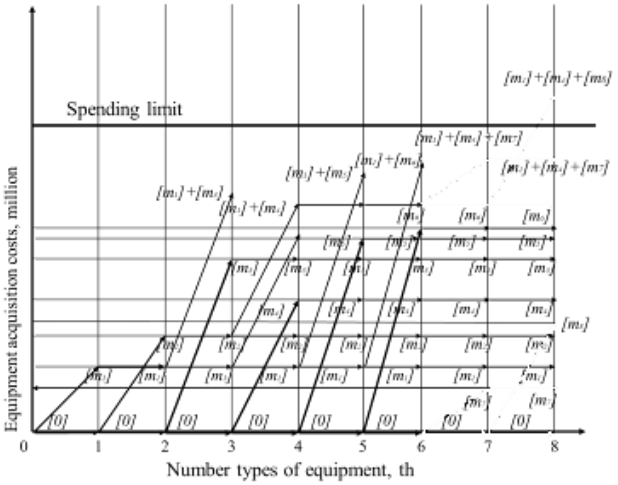



**Figure-1.** A network model that demonstrates the formation of the total resource of the technological chain and the reduction of funds for the purchase of equipment (the stage of research into the type of equipment belonging to the technological chain).

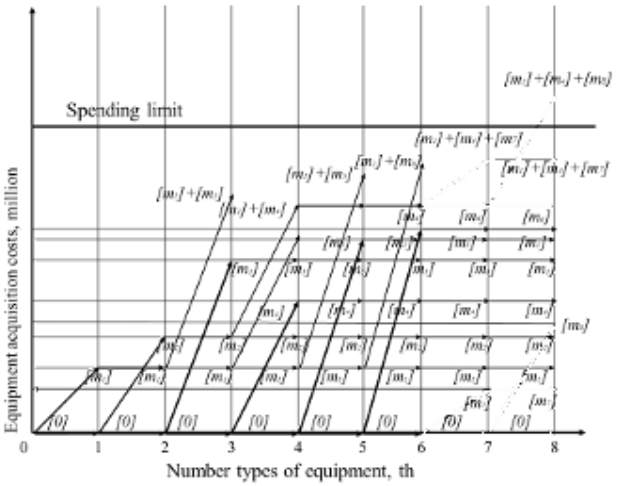



**Figure-2.** A network model that demonstrates the formation of the total resource of the technological chain and the reduction of funds for the purchase of equipment (after the execution of the algorithms).

Next, at each stage, we choose fasteners, a combine, and a conveyor. In this, there is always a choice strategy: to leave the current state, that is, costs and resources will not change, or to go to the choice of new equipment, which entails an increase in resources $[m_1 + m_3]$ and acquisition costs $w_3$. After comparing mechanized complexes by cost $w_1, w_2$, we exclude mechanized complex 2 from further calculations, i.e. $w_2 > w_1$, therefore, at the first stage, the optimal price will be $w_1$, and the resource of the chain will be $m_1$.

At the second stage, you should choose a shearer, for this, you need to connect a segment of length $w_1$ corresponding to the optimal type of fastening with segments of length $w_3 \ldots w_6$. As a result, the costs for the

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purchase of fasteners and a combine harvester will be$w_3 \dots w_6$, and the resource will be $[m_1 + m_3], [m_1 + m_4], [m_1 + m_5], [m_1 + m_6]$

After comparing $w_3 \dots w_6$, it can be seen that the chain of cleaning equipment consisting of attachment 1 and shearer 4 will be optimal, while the purchase costs will be $w_4$, and the resource will be $[m_1 + m_4]$.

Similarly, we will select the conveyor. After building the entire network model (Figure-2), the optimal technological chain will be the one consisting of fastening 1, shearer 4, and conveyor 7, therefore, the price will be $w_7$, and the total resource will be $[m_1 + m_4 + m_7]$. Moreover, as can be seen from Figure-2, the received alternative takes into account production possibilities, while at the same time, the resources $[m_1 + m_4 + m_8]$ are insufficient.

So, it also solves inverse problems. Consider the processes in a coal mine. After the design of the longwall, it is necessary to solve one more task, which consists of the transportation of minerals from the longwall to the bunker around the trunk yard. Let's analyze the following technological scheme - mined coal enters the scraper conveyor, then it is overloaded from it to the district belt conveyor, and after that it is loaded on the inclined belt conveyor. We must design the technological chain of transport so that the transport capacity is higher than the productivity in the longwall, while at each step the transport capacity must increase. This is due to the fact that only coal mined by the shearer enters the scraper conveyor and district belt conveyor, and additional flows of coal, for example, from other longwalls, can already enter the inclined belt conveyor.

For the application of dynamic programming methods, the task can be formulated as finding the minimum number of means of transport that will ensure the required throughput, i.e. to "reach" the given maximum as quickly as possible with a minimum of permutations.

To do this, we denote the transport capabilities as $y[i, V]$, where $V$ is the total value (given capacity of the technological chain) of the first $i$ objects (transporting devices). Then, the solution can be described by recurrent relations [36]:

$$y[0,0] = 0;$$
$$y[0, V] = W + 1$$
$$y[i, V] = y[i-1, V], \text{for} V < v_i$$
$$y[i, V] = \min(y[i-1, V],$$
$$y[i-1, V - v_i] + w_i,$$
$$\text{for} V \geq v_i$$

Similarly, it is possible to describe the tasks of equipping the longwall with equipment. The solution of the given ratios can be graphically interpreted (Figure-3).

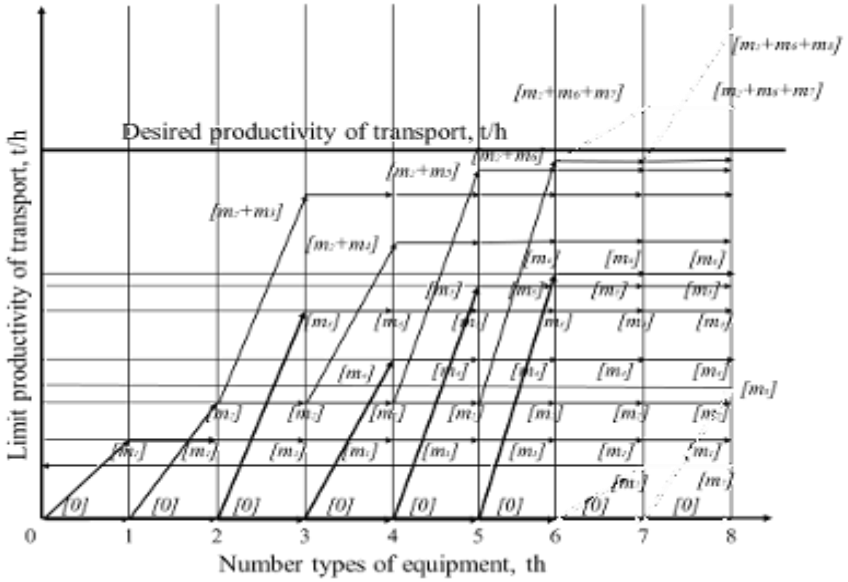



**Figure-3.** A network model that demonstrates the formation of the general traffic flow (after the execution of the algorithms).

The process of forming a transport chain consists of the analysis of alternatives. At the first stage, we choose the punching scraper conveyor with the maximum productivity from two possible ones, in our case, it is conveyor 2 (with characteristic $[m_2]$). Then you should proceed to the search for the optimal slaughter belt conveyor, i.e., choose from $[m_3 \dots m_6]$. To do this, let's compare the paths $[m_2 + m_3], [m_2 + m_4], [m_2 + m_5], [m_2 + m_6]$. As can be seen from Fig. 3, the chain consisting of conveyor 2 and bumpy belt conveyor 6 will have the highest throughput, i.e. $[m_2 + m_6]$. After that, we will choose an inclined conveyor. Since the use of

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conveyor 7 allows you to achieve the given transport capacity faster, a chain of scraper conveyor 2, belt conveyor 6 and inclined conveyor 7 will be rational, i.e. $[m_2 + m_6 + m_7]$. It can be assumed that this chain has the highest throughput, but this is not always true, it is explained by the fact that, at the stage of analysis of 8 types of equipment, after the 7th (by serial number) type (item), the desired goal was achieved, so the algorithm is completed. This is from ratios (1-3) and (4-6), i.e., we reach the optimum among the first $i$ subjects. Indeed, from Figure-3 shows that the throughput chain consisting of conveyors 2, 6, 8 is larger than that of 2, 6, 7, i.e. $[m_2 + m_6 + m_8] > [m_2 + m_6 + m_7]$.

The described situation demonstrates the possibilities of dynamic programming methods - it is possible to achieve the optimum both from the first stage to the final (direct run) and from the final to the first (reverse run). We considered the methods of direct running, that is, we believe that the productivity in the longwall is not limited by adverse conditions and we need to extract coal as much as possible, therefore, the requirements for the slaughter conveyor are increased (maximum theoretical throughput). In the reverse run, the situation is the opposite, that is, there are factors limiting productivity in the longwall, and additional flows of coal from other langwalls enter the inclined conveyor.

In this work, two tasks are considered that can be solved using dynamic programming methods, each of the tasks corresponds to the strategy of the "model" of development. In the first case, a limited amount of resources is given, and it is necessary to complete the cleaning pit so that the acquisition costs do not exceed the amount of allocated funds. In the second case, it is necessary to focus on the reproduction of necessary transport needs, i.e., to reach the maximum as soon as possible. It is worth noting that depending on the order of stages (running methods), the results will differ, this allows the designer to independently form a development scenario [38-40].

## DISCUSSIONS

### Initial Conditions Setting the Design Problem

To verify the given approach, we will solve the problem of choosing a technological chain of cleaning equipment in the structure of the mechanized complex, regarding the development of recommendations for the modernization of the structure. The "Zakhidno-Donbaska" mine of PJSC "DTEK Pavlogradvugillia" was accepted as the object of the study. This project of the planned activity during development of seam $C_5$ in blocks No. 1 and No. 3, as well as layer $C_8^B$ in the western part of block No. 3, preserves the existing pillar system of development at the mine with the development of notched longwalls by single benches along the rise [41-43].

Taking into account the experience of the mine in similar conditions, coal mining in the cleaning faces is expected to be carried out by mechanized complexes KD-90 (KD-80) with 1KA-200 extraction shearer and SP-251 conveyors.

The planned activity envisages niche-less mining of coal in the longwall using the shuttle scheme of the shearer. The method of managing the roof in longwalls is complete collapse. The pitch of the roof collapse corresponds to the pitch of movement of the mechanized fastening and is equal to 0.8 m.

In order to ensure a direct flow scheme of ventilation of the trenched areas (scheme 3-B, Figure-4) and with the aim of reducing the volume of excavation workings, the project envisages the support of the drift behind the longwall for its use as a development working for extract the adjacent pillar.

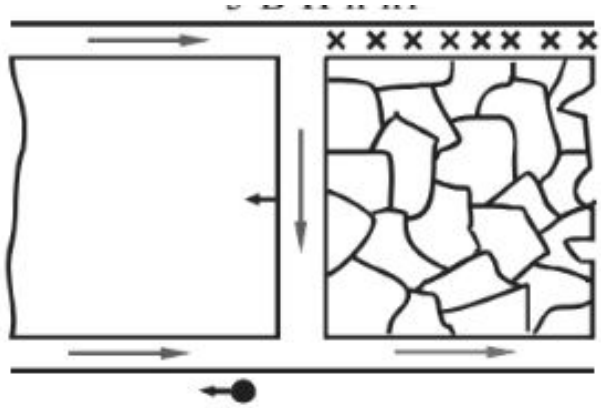


**Figure-4.** Ventilation scheme.

Taking into account that the management of the roof in longwalls is carried out by the complete collapse of rocks, as well as the absence of mass-produced foundation equipment and proven technology for laying the produced space on thin seams at the conditions of the Western Donbas, laying works in the mine are not foreseen.

Delivery of rock mass (coal and rock) during mining operations in the coal seam is expected to be carried out on general mine conveyor transport.

Production from preparatory slaughters is 350,000 tons per year. The pace of preparatory workings is up to 165 m/month.

The specific volume of preparatory workings is 5.0 m/1000 tons of mining.

In accordance with the calendar schedule for extracting reserves, development of the project capacity of the mine in the amount of 2.5 million tons of ordinary coal per year is expected in 2024.

According to the update of the mine development strategy until 2030, the introduction of the first longwall in the $C_5$ seam is planned in 2024 and is due to the need for the transition of the longwall from the $C_8^B$ seam in block No. 2 in order to exclude the counterfeiting of the $C_{10}^R$ seam reserves and to ensure its anticipatory development in the descending order of seams development adopted by the project [44-46].

As the reserves of the $C_8^B$ seam in block No.3 (year 2029) are refined, it is planned to put into operation

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the second bench on the $C_5$ seam in block No.1 to fill the front of the mine clean-up works.

Completion of reserves in this area of the $C_5$ seam in blocks No.1 and No.3 is planned in 2034.

This planned activity preserves the current system of full conveying, which ensures the transport of the main cargo flow of mining mass from the face to the loading facilities of the main shaft of block No.1 along the existing conveyor line.

In accordance with the plans to reduce the man-made load on the environment, the use of mining equipment is planned, which allows the extraction of coal seams to the full geological capacity with the provision of minimal rock output from the mine. Thus, it is necessary to solve two design problems with limited resources.

The first task: to choose equipment for the mechanization of works in the longwall with imposed restrictions on the specific cost of production (300 c.u. /t).

The second task: to organize a transport chain for the delivery of rock mass to the surface at a minimum level of specific energy consumption [47].

The starting conditions for designing are given in the Table-1.

**Table-1.** Initial data for the design of a longwall with limited resources.

| Indicator | Value |
|---|---|
| minimum extraction capacity of the seam | 1.05 m |
| the length of the longwall | 280 m |
| step of movement of the fastening | 0.8 m |
| extraction technology | shuttlecock, self-cutting |
| method of managing the roof | complete collapse |
| development scheme | columnar, with the recessing of long columns with single benches along the rise |
| Condition for task 1 (selection of equipment in the cleaning pit) | |
| objective function | cost minimization |
| spending limit | 300 c.u./t |
| average daily load on the breakdown | 2000 t/day |
| Condition for task 2 (organization of rock mass transport from longwall) | |
| cost limit (estimated operational productivity) | 500 t/h |
| maximum cargo flow on the drift conveyor | 15 t/min |

After the design problems have been formed, you can proceed to the solution.

**Solving the Problem of Choosing the Means of Mechanization of the Longwall**

In accordance with the initial conditions, we will apply the reverse run because the goal is to minimize the cost price.

According to the company's plans, during the development of the $C_5$ seam in blocks No.1 and No.3, as well as the $C_8^B$ seam in the western part of block No.3. Coal mining in cleaning faces is expected to be carried out by mechanized KD-90 (KD-80) complexes with 1KA-200 shearers and SP-251 longwall conveyors [48].

In accordance with the "Catalogue of rational types of equipment" for the conditions of Donbas [49], there are 9 alternatives of fastening for seams with a capacity of 0.90-1.1 m. Also, as the 10th alternative, we add a modern analogue - the DM fastening, the CLS-450 shearer, and the conveyor SP-251. Information about the types of equipment is given in Table 2 [50, 51].

**Table-2.** Rational types of fastening for given conditions.

| Configuration of cleaning equipment | | | |
|---|---|---|---|
| **No** | **Fastening** | **Shearer** | **Conveyor** |
| 1 | MKD80 | KA80 | SP251 |
| 2 | MDM | RKU10 | SP26 |
| 3 | MKD80 | KA80 | SP250 |
| 4 | 1 MKD90 | 1K101 | SP26 |
| 5 | MDM | UKD200 | SP326 |
| 6 | 1 MKD90 | 1K101 | SP326 |
| 7 | MKD80 | 1K101 | SP250 |
| 8 | MDM | K103 | SP250 |
| 9 | 1 MKD90 | UKD400 | SP326 |
| 10 | MDM | CLS-450 | SP251 |

As an optimization parameter, the "specific cost" parameter should be adopted; this parameter combines acquisition costs and equipment resources [52, 53]. The

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choice of the technological chain by the value of the unit cost allows you to balance the structure of the complex. The selected chain will meet two requirements: minimum acquisition costs and maximum operating time.

The specific production cost will be determined by the formula:

$$T = \frac{C_e(1 + r_t + r_s)}{R_q}$$

where

$C_e$ is the cost of a unit of equipment, million hryvnias;

$r_t$ – the share of costs for the purchase of spare parts;

$r_s$– share of maintenance costs;

$R_q$- resource of equipment, million tons - for shearer, conveyor; thousands of cycles – for mechanized fastening.

Given the fact that the shares of acquisition and maintenance costs for equipment in the complex are approximately the same, the unit cost should be determined. The specific cost of equipment is determined by the formula:

$$V = \frac{T}{R_q}$$

Resource indicators for shearers and conveyors can be taken from the operating manual, the resource of the fastening was calculated according to the formula [38, 54]. The resource of mechanized fastening depends on the standard size, seam parameters (density, extraction capacity), and is determined by the formula:

$$R_q = \frac{0{,}9 H_{max} l_{cp} L_{кр} \gamma}{10^6}$$

where

$H_{max}$ is the maximum structural height of the mechanized fastening section, m;

$l_{cp}$– the average length of the longwall, m;

$L_{кр}$- general advancement of mechanized fastening until the resource is completely exhausted, m;

$\gamma$– volumetric weight of fossil coal, t/m$^3$;

Based on the study of technological connections, it is proposed to replace the existing chain ("KD80 - KA200 - SP251") with a new one, which consists of "DM fastening - shearer CLS450 - conveyor SP251". The existing unit cost of production is 320 c.u./t, the project one will be 286 c.u./t, which allows to reduce the unit cost by 10.6%.

The expected annual economic effect from the introduction of a new coal mining technology, as well as a reduction in the cost of mining, will amount to 8.34 million c.u.

**Solving the Problem of Organizing the Transport of Rock Mass from the Longwall to the Bunker**

A complete conveyor system is used to transport the mineral through the seam from the longwall to the skip barrel bunker. After separation from the massif, the coal enters the scraper conveyor in the longwall, then onto the PTK-1 conveyor belt, from where the coal enters the district telescopic conveyor. District conveyors transfer the coal to the inclined conveyor, then to the crossbar conveyor, and from it the coal enters the hopper of the skip barrel.

In Figure-5 shows the accepted scheme of transportation. The task will be solved when the means of mechanization are selected.

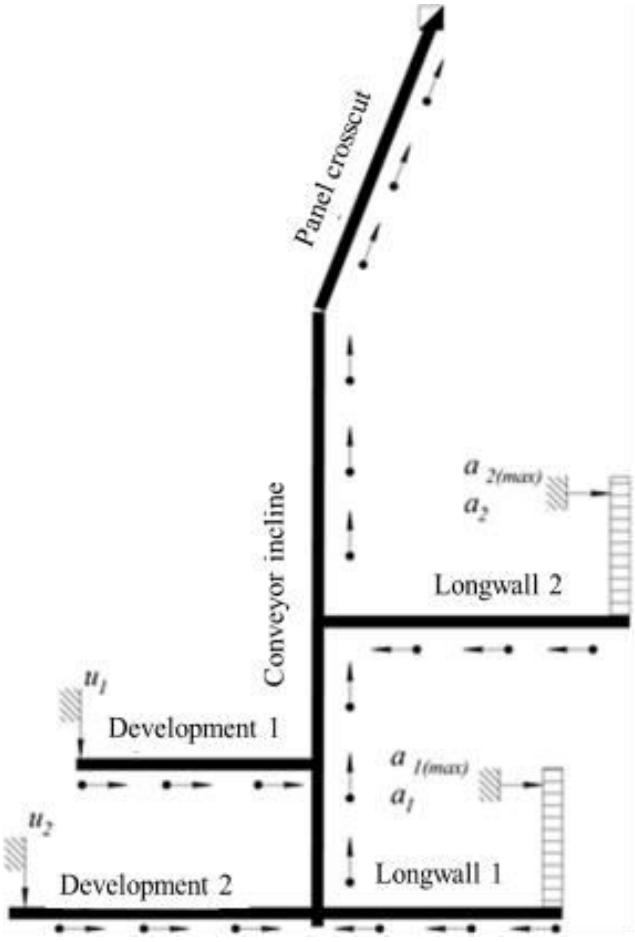



**Figure-5.** Transport scheme.

First, it is necessary to calculate the maximum values of cargo flows. Calculation data are given in Table-3.

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**Table-3.** Data on the calculation of cargo flows.

| Production | $q_1$(max), t/min. | $Q_{к.пр}$ | $B$, mm | $v$, m/s |
|---|---|---|---|---|
| Conveyor drift of longwall 1 | 4.30 | 5.06 | 800 | 1.60 |
| Conveyor drift of longwall 2 | 4.30 | 5.06 | 800 | 1.60 |
| Conveyor drift of preparatory face 1 | 0.96 | 0.56 | 800 | 1.60 |
| Conveyor drift of the preparatory face 2 | 1.00 | 0.58 | 800 | 1.60 |
| Conveyor slope | 8.44 | 9.93 | 1000 | 1.60 |
| Crosscut of the panel | 8.44 | 9.93 | 1000 | 1.60 |

where:
$q_1$(max) is the maximum cargo flow entering the conveyor;
$Q_{k.pr.}$– volumetric cargo flow entering the conveyor;
$B$ – conveyor belt width;
$v$ is the speed of the conveyor belt.

As can be seen from the Table-3 all cargo flows coming from production are within the given condition. After that, you can proceed to the selection of means of transportation. In Figure-6 shows the scheme of cargo flows entering the slope conveyor.

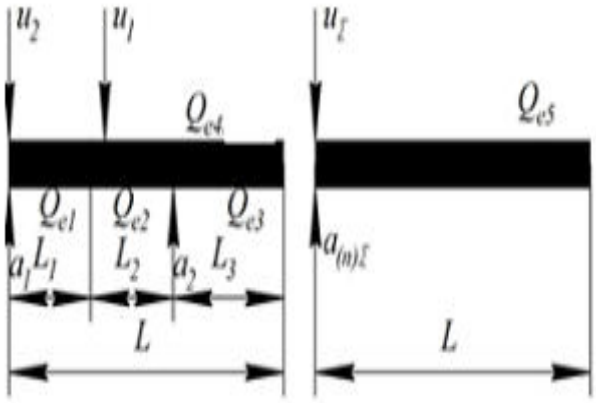



**Figure-6.** The scheme of the arrival of cargo flows on the slope conveyor.

The task is to choose conveyors that will ensure the load of the slope conveyor (Figure-6). At the same time, taking into account the given volumes of rock mass, the number of conveyors that create flows $a_1$, $a_2$ - from the longwall, $u_1$, $u_2$ - from the preparatory hole can be different (from 1 to 3). According to the results of the simulation of bandwidths by the method of backtracking, means of transport were selected (Table-4).

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**Table-4.** Results of solving the problem of choosing a means of transportation.

| Production | Conveyor type | Width of tape B, mm | The speed of the tape *v*, m/s | Operating productivity $Q_e$, t/h | Drive power *N*, kW | Conveyor length *L*, mm | Installation angle *β*, deg |
|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| Conveyor drift of longwall 1 | 2LT80U | 800 | 2.00 | 191.88 | 165 | 1100 | 0 |
| Conveyor drift of longwall 2 | 2LT80U | 800 | 2.00 | 181.50 | 165 | 900 | 0 |
| Conveyor drift of preparatory face 1 | 1LTP80 | 800 | 2.00 | 57.60 | 40 | 880 | 0 |
| Conveyor drift of the preparatory face 2 | 1LTP80 | 800 | 2.00 | 60.00 | 40 | 600 | 0 |
| Conveyor slope | 3L100U | 1000 | 2.50 | 349.45 | 550 | 1135 | 10 |
| | 3L100U | 1000 | 2.50 | 396.48 | 550 | 1135 | 10 |
| Crossbar of the panel | 1L100 | 1000 | 1.60 | 434.40 | 200 | 500 | 0 |

Based on the results of modeling the transportation process with limited resources, a complete scheme of rock mass transportation was developed.

The general transportation chain of the mineral consists of two cargo flows: the main one - coal mined by cleaning combines; the additional one- due to the separation of coal from the rock that comes from the preparatory workings. A complete conveyor system is used to transport the mineral through the seam from the longwall to the skip barrel bunker. After separation from the massif, the coal enters the scraper conveyor in the longwall, then onto the PTK-1 conveyor belt conveyor, from where the coal enters the district telescopic conveyor. District conveyors transfer the coal to the inclined conveyor, then to the crossbar conveyor, and from it the coal enters the hopper of the skip barrel.

The rock from the preparatory faces (development 1 and development 2) enters the conveyor, then, at the transshipment point, the rock is dumped into trolleys with cutters (knives). The rock is transported by carts along the slope from the transshipment points to the acceptance-dispatch site of the slope, and from the reception-dispatch site of the slope to the bunker of the trunk yard. The rock is transported in carts by locomotive rolling OK-2 tippers, which are used to unload VG-2.5-900 carts.

According to the results of the calculation of the economic effect, it was established that the cost of electricity will decrease by 8% due to a decrease in the total power of electric motors.

At this point, the solution to the problem of choosing means of transportation can be considered complete.

## CONCLUSIONS

In the course of the research, a new approach was developed for the design of mining production processes with limited resources. For this, combinatorial optimization methods were applied by interpreting the knapsack problem, and the private solution "knapsack 0-1" was applied.

It was established that forward and reverse run algorithms can be used during design. If the task of obtaining the maximum effect with minimum costs is solved, then a direct run is used. In the event that there are limitations - bandwidth, energy characteristics, etc. - backtracking methods are used.

In the final part of the study, the verification of the developed approach was carried out under the conditions of the mine "Zakhidno-Donbaska" of PJSC "DTEK Pavlogradvugillia". With the given technological parameters of operation, a limit was imposed on the level of specific cost. The backtracking method was used for the solution. Based on the study of technological connections, it is proposed to replace the existing chain ("KD80 - KA200 - SP251") with a new one, which consists of "DM fastening - shearer CLS450 - conveyor SP251". The existing unit cost of production is 320 c.u./t, the project one will be 286 c.u./t, which allows reducing the unit cost by 10.6%.

Also, with the help of the given method, the problem of choosing the means of transportation of the rock massif from the longwall to the bunker was solved. Operational productivity and maximum minute cargo flow were set as limitations. On the basis of the calculation, the optimal transport chain with the lowest installed power of electric motors was determined, which allows reducing electricity costs by 8%.

The expected annual economic effect from the introduction of a new coal mining technology, as well as a

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reduction in the cost of mining, will amount to 8.34 million c.u.

In the future, an appropriate decision support system will be developed, which will allow automating the decision-making process.

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