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GEOSTATISTICAL APPROACH FOR MAPPING THE TRANSMISSIVITY OF THE SENEGALESE DEEP AQUIFER SYSTEM

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ABSTRACT

Accurate estimation of hydrodynamics parameters of aquifers is extremely important for the exploitation and the proper management of aquifers systems. Transmissivity is one of these parameters and the withdrawal into an aquifer depends highly on the transmissivity. So, by the methods of geostatistics, this paper studies the spatial pattern of transmissivity of the Senegalese deep aquifer system. The results showed that the semivariogram of the transmissivity coefficient could be described by spherical model. Based on the information obtained from the field of study, the isocline maps of the transmissivity coefficient were created by the geostatistical software Surfer with Kriging interpolation, and the resulting maps gave a clear indication about the spatial patterns of the transmissivity.

Keywords: Senegal, maestrichtien aquifer, transmissivity, geostatistic, kriging, mapping.

INTRODUCTION

In Senegal, urbanization and the population growth have been rising very fast and all projections indicate that drinking water demand will continue to increase. This increase in demand is due to the fact that we also have a good portion of the population who do not have access to drinking water despite all the hydraulic policies put in place by successive Governments. In many cases, satisfying this demand and making drinking water available in needy areas has made the use of groundwater, sometimes nonrenewable, mandatory.

Consequently problems arise at quantitative and qualitative levels. Indeed, if the volume of the reservoirs of groundwater in Senegal is pretty much known (figure 1), their renewal rate is less than that, especially for confined groundwater [1, 2].

The groundwater resources in Senegal are very abundant but the current trend leads to a continuous decrease of the groundwater level.

The chances of accessing the groundwater are also very unequal; they depend on the region site and vary based on capacity, depth and levels of groundwater mineralization. The main constraints to the groundwater exploitation include insufficient knowledge of hydrodynamic parameters of some aquifers which is a limiting factor for proper planning of its exploitation.

This work contributes to a better knowledge of the deep Senegalese aquifer system which is the largest in the country and covers 4/5th of the territory. This aquifer only concerns the sand formation Maestrichtien [3]. It affects a huge demand of the masses' water supply and is therefore of great interest for Senegal.

The (constant) increase of the number of wells capturing the aquifer, will inevitably lead to a water management problem. Thus a rational and intelligent use of this groundwater is necessary. This begins with the knowledge of the aquifer's geometry but also its hydrodynamic characteristics of which transmissivity is an essential part [4].

The objective of this work is to carry out a mapping of the transmissivity of the aquifer, using geostatistical methods with kriging as an estimator. Indeed, the kriging provides a quantitative mapping of optimal, one-off or zonal estimates [5]. It is a method of weighting the available data, where the weighting mode is strictly linked to the characteristics of the field recognized in a geostatistical description [6, 7, 8]. Geostatistical methods are widely used in the field of hydrogeology and the science of water in General [9, 8, 10, 11, 12] and they will no doubt allow to better understand the potential of this slick and thus provide a valuable management tool for decision-makers.

A brief summary of the Hydrogeology of the aquifer system is presented in the next section. The other three parts that follow successively, describe the geostatistical approach, the determination of the theoretical and experimental variograms and mapping of the transmissivity of the aquifer over the entire national territory.

MATERIALS AND METHODS

Description of the aquifer

Senegal is characterized by two large geological sets that are distinct in structure and dimensions. One located at the extreme east of the country, is made up of ancient, Paleozoic or Precambrian often crystalline or metamorphic rocks. The other is a large sedimentary (Senegalo-Mauritanian basin) Mesozoic and tertiary basin that covers the major part of the Senegalese territory [1, 2]. This geological configuration determines the nature and the type of deposit of groundwater in Senegal where there are in fact two large sets [13].

- Discontinuous aquifers of the cracked rock.
- Generalized intergranular aquifers of permeable sedimentary formations whose exploitation potentials depend on the characteristics of these formations, and are generally high due to their great extension.

Many hydrogeological studies in Senegal for several decades helped to identify most of the aquifer levels constituting these two sets: the sedimentary basin with its generalized intergranular type of aquifers and the socle with its discontinuous to semi continuous cracks aquifers. These aquifers are grouped into four major systems corresponding to the main geological formations.

- The shallow aquifer system that combines formations in dominant sandy clay and sand of the Quaternary, the Continental Terminal and the Oligo-Miocene.
- The intermediate aquifer system: includes mainly limestone, karst in some location and marno-limestone formations of Paleocene and Eocene.
- The deep aquifer system which concerns only the Maestrichtien sands formation.
- The fractured aquifer system which includes the discontinuous cracks aquifers and alteration of granitic and metamorphic rocks formation of Senegal.

In this paper, we focus only on the deep aquifer system because of its importance in the groundwater resources of Senegal. This aquifer consists of thick sandstone and sandy series sandy clay of Maastrichtian era (Upper Cretaceous) in direct contact with overlying formations of the Eocene and Paleocene. We meet these formations in the entire sedimentary basin. They are tapering beveling on the pedestal to the east of the country while they thicken considerably and deeper towards the Atlantic coast while remaining more clay, except in the Ndiass Horst, Figure-1.

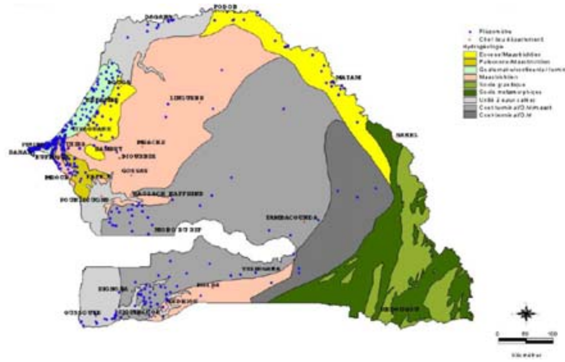

Figure-1. Map of the aquifer units of Senegal (Source, DGPRE).

On the eastern edge of the basin, the limestone series disappears and is only found in sandy formations, coarser and relatively thin (tens of meters) that can be attributed either to the Continental Terminal or the Continental Terminal and the Maastrichtian, or a variation facies of the Eocene that makes this border a common recharge zone for three sedimentary aquifers of Senegal [2]. The roof of the water table increases from a few dozen meters (average 50 m) to more than 400 to 500 meters. With an area of over 150,000 km2, a useful thickness of 150 m on average and a storage coefficient of 10% in unconfined aquifer (Ndiass and Senegal River) to 0.6% in confined aquifer, the Maestrichtien aquifer represents a reserve of the order of 300 to 400 billion m^3 of water. It is also by far the most currently exploited aquifer with approximately more than 700 boreholes. Exploitable flow rates in boreholes vary from 10 to 250 m^3/h, but the vast majority of them are between 50 and 100 m^3/h. Specific rates are on average about 5 to 10 $m^3/h/m$, but coastal areas and the southeast of the sedimentary basin (Tambacounda) have specific rates below 5 $m^3/h/$ m. The transmissivity obtained from the interpretation of pumping test are good to excellent between 10^{-3} and 5.10^{-2} m^2/s

In total, the Maastrichtian aquifer supplies about 40% of all flows withdrawn on Senegalese sedimentary basin aquifers [2].

Geostatistical approach: Kriging

Geostatistical methods aim to describe natural phenomena correlated in space and possibly in time, to quantify the uncertainty of estimates of these phenomena made from a very fragmentary sampling.

The estimation method derived from this theory is called kriging. It is a spatial interpolation method that takes into account the geometric configuration of the observed points and the estimated variable's own spatial structure. In addition to the estimate itself, kriging assesses the uncertainty associated with this estimation.

Unlike all other methods, the kriging also allows us to calculate the estimation error. Thus, in order to apply the results of the random functions theory, one must use what is called statistical inference.

This is only conceivable a priori if we already have a sufficient number of achievements. Statistical inference then requires the introduction of additional assumptions on the random function for lifting the impossibility. Two hypotheses are often issued to make statistical inference: the assumption of ergodicity and second order stationarity where you can then assume that each particular project is sufficient to account for all possible achievements and the intrinsic hypothesis where it will be enough to assume that, for any vector "*h*", the difference $Z(x+h)-Z(x)$ has zero mathematical expectation and a variance independent of the point *x* with $Z(x)$ the random variable.

$$\begin{cases} E[Z(x+h)-Z(x)]=0 \\ Var[Z(x+h)-Z(x)]=2\gamma(h) \end{cases} \quad (1)$$

The function $\gamma(h)$ is called "semi-variogram" often referred to as variogram. Thus the random function describing our aquifer Transmissivity will be regarded as an intrinsic random function.

Geostatistics primarily uses descriptors such as the variogram and the covariance function, which are also called structure functions. The variogram of intrinsic random function is thus defined by:

$$\gamma(h)=\frac{1}{2}Var[Z(x+h)-Z(x)] \quad (2)$$

Given the intrinsic hypothesis, the variogram $\gamma(h)$ is estimated by the experimental variogram $\gamma^*(h)$ from pairs of experimental points available on the single accessible achievement:

$$\gamma^*(h)=\frac{1}{2n(h)}\sum_{i=1}^{n(h)}[z(x_i+h)-z(x_i)]^2 \quad (3)$$

Where $n(h)$ is the number of pairs of data in intervals of length "*h*" and $z(x_i)$ the data.

RESULTS AND DISCUSSIONS

Experimental data and variogram

The input data are a sample of 222 points. They were obtained from the Direction of Senegalese water resources management and planning (DGPRE). The distribution of these samples throughout the national territory is shown in Figure-2.

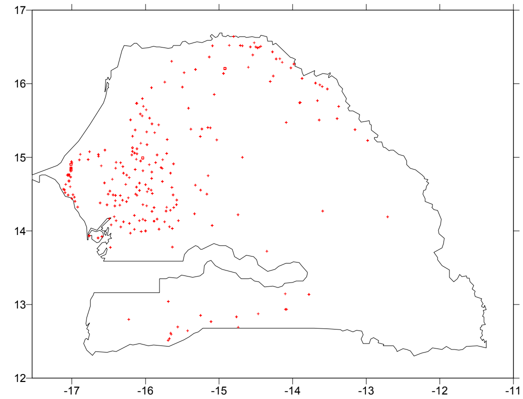

Figure-2. Sample data distribution on the studied area.

The values of transmissivity of the sample were obtained by pumping test. The statistical characteristics of the sample are presented in Table-1.

Table-1. Statistical characteristics of the sample.

Statistical characteristics of the sample	Value
Sample size	222
Range	0.180
Average	0.090
Minimum	1.1E-05
Median	0.0061
Maximum	0.18
Standard deviation	0.0237
Variance	0.000562

Construction of experimental variogram

To determine the variogram from the sample, groups of classes are created based on the distances between measurement points. All possible combinations of pairs of measuring points form a set of pairs of points which are then grouped into previously-defined classes. The estimate of the variogram is obtained for the average distance of each class from the half sum of increments observed on pairs of this class:

$$\gamma^*(h)=\frac{1}{2n(h)}\sum_{i=1}^{n(h)}[z(x_i+h)-z(x_i)]^2 \quad (3bis)$$

Where

$\gamma^*(h)$ = The variogram estimator;

h = lag distance; (interval length)

$z(x)$ = z Value at x_i;

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$z(x+1)$ = z Value at x_{i+1};

$n(h)$ = Number of pairs within the class.

In order to build the experimental variogram, a constant step "***h***" equal to 8.8 km has been selected. This step represents the difference between two distances for which a value is calculated in the variogram. Thus we obtained 25 distance classes represented by their mean values on the variogram. Each point is obtained from a number *n(h)* of the sample pairs of points. We notice as well that the variogram being built could be limited to 100 km because the number of pairs of points starts to decrease from this distance. Thus, we have a decrease in information beyond this distance (Table-2).

Table-2. Values of the experimental variogram in terms of the average distance.

n (h)	**Average distance** (km)	**γ** (h)
125	5,635533125	0,000247977
299	13,38127133	0,000410965
433	21,99354724	0,00049324
501	30,99729868	0,000425235
656	39,79776266	0,000593273
655	48,5807774	0,000634009
801	57,28225376	0,000607585
843	66,02033444	0,000632403
863	74,80306337	0,000518213
885	83,65988892	0,000660157
866	92,35628748	0,000615886
878	101,3392246	0,00068057
869	109,9047504	0,000546554
805	118,9719409	0,000572184
786	127,3902344	0,000617976
701	136,4725597	0,000869211
665	145,1721996	0,000747405
650	153,9349839	0,000795522
640	162,5092195	0,000586999
533	171,2902784	0,000637574
506	180,4672376	0,000569875
450	189,3470931	0,000509109
452	198,2477097	0,000633369
439	207,0324208	0,000601417
452	215,5987361	0,000619011

Variogram modeling

The structure of a given variable is studied using the variogram. The set of calculated points is the experimental variogram that is only an estimate of the real theoretical variogram moreover unknown. The adjustment of the points of the experimental variogram is obtained through the weighted least squares method. Thus, for our study of the transmissivity, the theoretical variogram ($\gamma(h)$ [m²/s]²) that is obtained, follows a spherical model combined with a nugget effect of the order of 0.00013, with a variance, a priori, of 0.00047 and has a plateau at 40 km. In addition, since the deep aquifer Maestrichtien consists of sand almost everywhere, we choose an isotropic model (Figure-3).

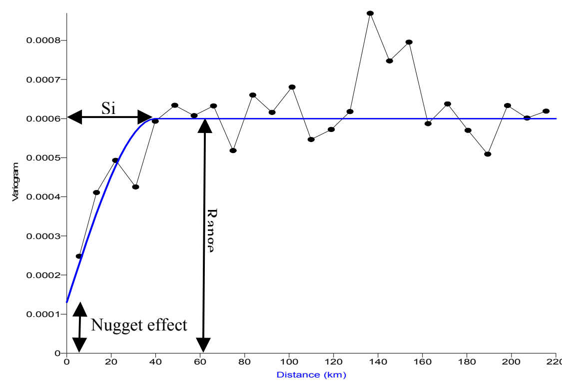

Figure-3. Plotting of the theoretical variogram.

The spherical model that takes into account these peculiarities, is written:

$$\begin{cases} \gamma(h) = 10^{-5} \times [13 + 47(3750h - 0.78h^3)] \text{if } h \leq 40km \\ \gamma(h) = 60 \times 10^{-5} \quad \text{if } h > 40km \end{cases} \quad (4)$$

Kriging and transmissivity estimation

The estimator used in this work is kriging which provides a quantitative mapping of optimal, specific or zonal estimates. It is a method of weighting the available data strictly linked to the characteristics of the field recognized in a geostatistical description. The resulting representation consequently keeps the observed data from the measurement points and assigns a qualitative criterion based on the data of the mapping properties. Kriging also allows us to calculate the estimation error.

Thus, to better visualize the scope of the transmissivity and its spatial variability in this subject area, the numerical values are replaced with iso-value maps.

The results of the Cartographic estimation of transmissivity and the resulting standard deviations from the ordinary mono variable kriging, are presented in Figures 4 and 5.

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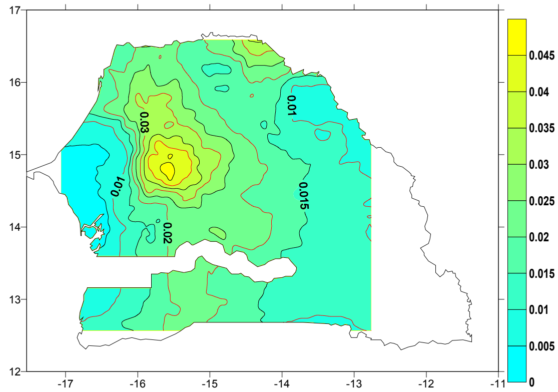

Figure-4. Map of estimates of transmissivity.

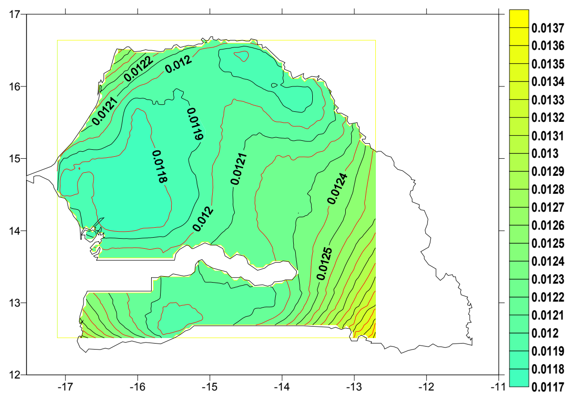

Figure-5. Map of standard deviations of estimates.

So the maps of the transmissivity fields reflect well a sort of “field logic” in the spatial distribution of data. The data show that the highest values are found at the center of the country and they decrease as one moves away from the center. The values are between 0.015 and 0.045 m²/s in the most part of the deep aquifer system located in the center. They become very low in the West and Southeast of the country with values less than 0.015 m²/s. This contrast reflects the fact that coastal areas, with relatively small thickness of aquifer, and the Southeast part of the sedimentary basin, bordering the fractured rock area, have specific flow rates less than 5 m^3/h/m while they are on average between 5 to 10 m^3/h/m for this layer.

The map derived from the kriging standard deviations, Figure-5, quantifies the post-uncertainty associated with the map of transmissivity, figure 4 in addition to the cross-validation analysis presented Figure-6.

This map is obtained using the error variance calculated from the probabilistic kriging interpolation method. The results characterize the variability of the interpolation error in units of transmissivity.

There is a low kriging standard deviation towards the center indicating that, on average, the interpolated values are close to observed values and that demonstrates good mapping accuracy. We have to note that this central zone represents an area of high density of data. Conversely, there is a higher kriging standard deviation towards the coastal areas, and the Southeast part of the sedimentary basin, where the data density is relatively low, the number of data points determining the precision of the estimate [14].

Finally, we must also note that the types of kriging variance maps are to be interpreted with great caution because they are very sensitive to the type of variogram model we select for the interpolation, especially when it has a "nugget" structure from the start, as in the present case.

Validation of the structural model adopted

This operation represents a cross-validation and allows checking the fit between the data and the variographic model used. It involves estimating each data of known value by eliminating it from the input data.

Comparing the estimated value to the observed or true value is done via a scatter plot whose configuration around (the line of best fit) first bisector attests to the robustness of the model: the more the scatterplot is tightened around the (the line of best fit) bisecting line, the better the estimated values by kriging.

The results of cross-validation, Figure-6 shows that variogram models built to assess and map the transmissivity scope by kriging are satisfactory with a correlation coefficient of about 0.94.

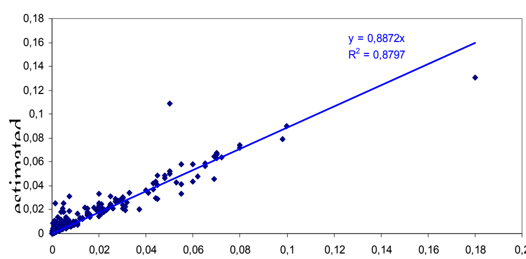

Figure-6. Cross-Validation.

CONCLUSIONS

Given that the Maestrichien aquifer represents 40% of all water flows from the sedimentary basin aquifers of Senegal and that the transmissivity is one of the fundamental hydrodynamic characteristics of an aquifer, mapping it, indeed seemed necessary in order to better understand the capabilities of this aquifer. The results we have obtained through this study are in sync (agreement) with the field data and they allow us to understand the deep aquifer transmissivity in the whole country.

The results of cross-validation keep us at ease with the choice of variogram model built to assess and map the transmissivity distribution by kriging.

Therefore this study allowed us to put at the decision-makers’ disposal, a relatively precise cartography of the deep aquifer transmissivity as well as a map of the errors of estimation.

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